Growth of discharges on polluted insulation

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Synopsis

The voltage required to maintain local discharges on polluted insulation may increase with increase in discharge length, and if this voltage exceeds the supply voltage, the discharges extinguish without causing a flashover. An analysis based on this mechanism has been made for the simplified case of an insulator containing a constant surface resistance, \( r_c \), per centimetre of leakage path. This has shown that the flashover stress, \( E_c \), is proportional to \( p^{0.43} \), where \( p \) is the resistivity of the pollution, and that the maximum leakage current which can flow when flashover is impossible is 233 \( \times 10^{-3} \) amperes, \( E_c \) being in volts (peak) per centimetre. It has been found that these conclusions apply to conventional bushings subjected to wet-flashover tests, and to power insulators subjected to natural pollution under service conditions. Further confirmation of the analysis was obtained from experiments made by the authors and previous workers with artificial pollution, deposited so as to maintain \( r_c \) constant along the surface.

The agreement between this analysis and the experimental results of several investigators indicates that the mechanism discussed here controls flashover in many instances.

1 Introduction

The flashover of outdoor insulation due to pollution constitutes one of the most difficult high-voltage problems. Its practical importance can be assessed from the fact that during 1951–55 the number of faults which it caused on the 132 kV lines of the British Grid was second only to those due to lightning. With the increase in system voltages the importance of pollution, relative to that of lightning, is likely to increase.

The performance of polluted insulation has been the subject of extensive research and development, and the mechanism of flashover has been elucidated qualitatively. A detailed account of this work is given in References 1–6, but the mechanism can be described briefly as follows. Insulation which operates in polluted atmospheres becomes covered with dry pollution. This does not in itself affect the performance of the insulation significantly, but if the surface become moist, owing to dew or fog, for example, the pollution becomes conducting and greatly distorts the electric field along the insulation. The stress may then exceed locally the electric strength of air, and sparkover occurs, causing discharges along small portions of the insulation. These discharges are maintained by current flowing through the charge-free, but polluted, portion of the surface. Owing to the heat generated at the discharge roots, the pollution dries out in their neighbourhood, and ceases to conduct. To maintain conduction, the discharge root must travel along the surface to a region which is still moist. The discharges therefore elongate, and flashover occurs if they span the distance between electrodes. However, discharges do not necessarily lead to a flashover, and polluted insulators often exhibit discharges which extinguish after having spanned only a fraction of the insulation surface. The phenomena which cause extinction are not fully understood, but it is possible that, as the discharges grow, they reach a critical length at which the voltage, \( V \), required to maintain conduction between the terminals of the insulator exceeds the supply voltage, \( V_s \). This mechanism of extinction is the subject of the paper.

It will be shown that an analysis of this mechanism yields quantitative criteria which define conditions in which flashover cannot occur. These criteria have been expressed in terms of the mean stress applied to the insulation and the resistance of the pollution, and also in terms of the stress and the maximum leakage current. The validity of these criteria has been confirmed by experimental evidence obtained by the authors and by previous workers.1, 4, 5, 6

This mechanism has also been postulated by Obenaus,2 who derived a relation between the stress applied to the insulation and the maximum leakage current. Obenaus's analysis is discussed in Appendix 7.4, where it is shown that his relation is only valid under certain conditions, and this limitation does not apply to the criteria derived in this paper. While this paper was being prepared, Nasser1 published a paper which contained a qualitative discussion of the mechanism of extinction considered here, but he did not derive criteria.

2 Analysis of the growth of discharges

Discharges may burn in series and parallel on a polluted insulator, so that a rigorous expression for the burning voltage, \( V \), would be extremely complex; furthermore, it would vary in time even for a given insulator, and would depend on the random distribution of pollution. To make an analysis possible, it is necessary to consider a simplified model, and, because of the simplifications, such an analysis can only be deemed valid if confirmed by experiment. The model, shown in Fig. 1, consists of a cylindrical insulator of length \( L \), with electrodes on the flat ends. A discharge, of length \( x \), burns from one electrode, and tends to elongate as explained in Section 1. The resistance in series with the discharge is \( R \), and the current in \( R \) is \( i \). If \( V_d \) is the voltage across the discharge,

\[
V = V_d + iR
\]  

(1)

Fig. 1

Model used for the analysis of the growth of discharges

The main symbols are defined in this Figure. The suffix \( c \) is used in the text to denote critical values of current, voltage, stress and discharge length


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To derive quantitative criteria from this equation it is necessary to express $V_d$ and $R$ in terms of $x$. It will be assumed that the discharge current is constant along the length of the discharge; in other words, there is no electrical contact between the discharge and the pollution, except at the discharge tip. The justification for this assumption is that the heat generated in the discharge dries out the pollution with which the discharge is in prolonged contact, and dry pollution does not conduct. Again, it will be assumed that the region across which the discharge is initiated is free of pollution; such pollution-free, or at least high-resistance, regions, occur on insulators in service. Current can therefore flow to the polluted region only through the discharge, and consequently the discharge current equals $i$. It follows that $V_d$ is the voltage required to maintain a discharge of current $i$ and length $x$ in air, and from measurements described in Appendix 7.3 the relation between $V_d$, $i$ and $x$ is taken as

$$V_d = x A i^{-n} + B$$

where $A$ and $n$ are constants. $B$ is less than 400 V, and can be ignored if the voltage applied to the insulator exceeds a few kilovolts.

Eqn. 1 can therefore be rewritten

$$V = x A i^{-n} + i R$$

The resistance, $R$, depends on the resistivity of the pollution and the size and shape of the insulator. To make the analysis manageable, the resistivity will be taken as constant, so that the resistance of the polluted surface has a constant value, $r_c$, per unit length along the axis of the cylinder. If the length of the polluted discharge-free area was great compared with the diameter of the cylinder, the electric field would be uniform over the greater part of the length, so that, to a first approximation,

$$R = r_c (L - x)$$

The effect of field non-uniformity is discussed in Appendix 7.1.1. From eqns. 3 and 4 the burning voltage becomes

$$V = x A i^{-n} + i r_c (L - x)$$

To determine conditions in which the discharge will extinguish, consider first the voltage required to maintain conduction for any given value of $x$. This voltage depends on the current, and the $V/i$ curve has only one turning-point, which occurs when

$$i = \left[ n A x r_c (L - x) \right]^{1/(n+1)}$$

and the corresponding value of $V$ being

$$V_m = (n + 1) A x r_c (L - x) r_c x^{n/(n+1)}$$

From this expression, $V_m$ has been sketched to a base of $x$ in Fig. 2. $V_m$ has a maximum, given by

$$V_m = Al^{(n+1)} r_c^{(n+1)}$$

This occurs when $x$ equals

$$x_c = L/(1 + n)$$

Now let a voltage $V_c$, to be applied to the insulator, the discharge being quite short initially. The discharge could grow until $x = x_c$ (see Fig. 2); it cannot grow any further, because the voltage required to maintain conduction exceeds $V_c$ if $x > x_c$. However, if the discharge length were to exceed $x_c$ initially, further increase would reduce the burning voltage so that the discharge could grow to flashover.

A similar argument applies to any value of $V_c$ less than $V_c$. Flashover is therefore impossible if the applied voltage and the initial length of the discharge are less than the values given by eqns. 8 and 9. When these equations hold, conditions will be said to be critical, and the values of individual variables under critical conditions will be termed 'critical values'.

The condition given by eqn. 8 can be expressed in terms of the mean applied stress, by dividing that equation by the leakage length, $L$, which gives

$$E_c = A l^{(n+1)} r_c^{(n+1)}$$

This equation defines the critical relation between the applied stress and the resistance of the pollution. It follows that, for a given $r_c$, flashover is impossible at any stress, $E$, which is less than the critical stress $E_c$. Similarly, for a given stress, $E_c$, flashover is impossible for any resistance, $r$, greater than the value $r_c$ given by eqn. 10. In general, flashover cannot occur if

$$E < A l^{(n+1)} r_c^{(n+1)}$$

and, taking the values of $A$ and $n$ from Section 7.3, this can be written

$$E < 10 \cdot 5^{0.45} \text{volts/cm}$$

Since $E_c$ is the maximum value at which flashover is impossible it follows that, on power frequency, the peak cyclic value of the stress must be used for $E_c$ and $E$.

If the value of $n$ is inserted in eqn. 9, the condition that the initial length (which will be designated by $x_i$) must not exceed $x_c$ becomes

$$x_i < 0.57 L$$

It appears improbable that $x_i$ does, in fact, exceed 0.57L in service. It will be remembered that $x_i$ is the distance covered by the initial sparkover which triggers off the discharges, and which is due to the distortion of the electric field by the pollution (see Section 1). The stresses at which outdoor insulation is energized are so low\(^1\) that, even if the supply voltage appeared entirely along $x_i$, the mean stress along the sparkover distance would be only a few hundred volts (peak) per centimetre if $x_i > 0.57 L$; this is comparable with some 4 kV (peak)/cm for flashover between point electrodes,\(^7\) and upwards of 24 kV (peak)/cm for flashover in uniform fields.\(^8\) Again, if the initial sparkover could span distances comparable with the leakage length $L$, flashover should, on occasion, occur suddenly, owing to field distortion, without being preceded by discharges. As far as the authors are aware, such sudden flashovers have not been observed, except perhaps...
for grease-coated insulators. Apart from such insulators, therefore, it appears likely that conditions in which flashover is impossible are adequately defined by expression 12.

It must be emphasized that the expressions 12 and 12a define conditions under which flashover is impossible, but not conditions under which flashover will necessarily occur. The reason for this is that, even if  \( V_s > V_c \), flashover may nevertheless be inhibited owing to the operation of mechanisms which are not discussed in this paper; for example, because the energy dissipated at the discharge root is insufficient to dry out the pollution (see Section 1). It is shown in Appendix 7.1 that, if certain of the simplifying assumptions made above were invalid, the critical stress could exceed the value given by eqn. 10, even if flashover were controlled by the mechanism discussed in this paper.

2.1 The critical leakage current

The critical value, \( i_c \), of the current can be obtained by substituting the critical value of \( x \) from eqn. 9 in eqn. 6. This gives

\[
i_c = (A/E_c)^{1/(n+1)}
\]

and \( A \) or \( r_c \) can be eliminated using eqn. 10, so that

\[
i_c = (A/E_c)^{1/n}
\]

and

\[
i_c = E_c r_c
\]

It is demonstrated in Appendix 7.2 that \( i_c \) is the maximum current which can flow on an insulator energized at a stress \( E_c \), under conditions in which flashover is impossible. Substituting numerical values for \( A \) and \( n \), the maximum current becomes

\[
h_{\text{max}} = 233 E_c^{-1.31} \text{ amperes}
\]

\( E_c \) being in volts per centimetre.

3 Discussion of measurements on polluted insulation

3.1 Experiments with artificial pollution

It has been explained that expression 12a is likely to be satisfied in practice. This has, in fact, been confirmed by high-speed photographs taken by the authors\(^5\) on insulated sheets, up to 400 cm long, covered with artificial pollution, and by similar observations made by Nasser\(^6\) on actual insulators. Since inequality 12a is satisfied, it follows that, provided that flashover is controlled by the mechanism discussed in the paper, it must occur at the critical stress, \( E_c \).

From eqn. 10, \( E_c = A r_c^{1/(n+1)} \), where \( K = A^{1/(n+1)} \). In order to simplify the analysis, no account was taken of the cyclic variation of stress at power frequency, of the distortion of the electric field near the discharge root, or of the possible occurrence of several discharges, in series or parallel. These factors are discussed in Appendix 7.1, where it is shown that the distortion of the field, and the occurrence of parallel discharges, would be reflected by an increase in the constant \( K \); the effect of all factors would be to increase \( E_c \), for a given \( r_c \).

To assess the effect of these factors, an analysis was made of results obtained, by the authors and other workers,\(^5,9\) from experiments with artificial pollution. In these experiments, the voltage applied to the insulation was gradually increased until flashover occurred (except as explained in Fig. 4), and the stress at flashover, i.e. flashover-voltage/leakage-length, was taken as the critical stress, \( E_c \). The resistance of the pollution was roughly constant along the insulator surface, as assumed in Section 2. \( E_c \) was then plotted against the

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**Fig. 3**

Flashover characteristics for a flat glass specimen, contaminated with moist industrial pollution

The pollution was spread uniformly on a rectangle between parallel electrodes, as shown in the inset.

- 12 cm long sample; resistance per square constant at 170 k\(\Omega \).
- Changing sample width from 3 to 6 cm.
- 12 cm long sample; resistance per square constant at 237 k\(\Omega \).
- Changing sample width from 4 to 10 cm.
- 10 x 10 cm sample; resistance per square and \( r_c \) varied by increasing water content.

The equation of the straight line is \( E_c = 15.6 r_c^{0.43} \).

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**Fig. 4**

Flashover characteristics for porcelain cylinders, 100 cm long, contaminated with pollution containing salt. Experimental data obtained by von Cron\(^9\)

Voltages of constant amplitude were applied to the insulators. Blacked-out points indicate stresses which gave flashovers, and blank points stresses which were withstood.

- \( \Delta \) 7.5 cm-diameter cylinder
- \( \bigcirc \) 15 cm-diameter cylinder

The equation of the straight line is \( E_c = 21.5 r_c^{0.43} \).

---

\( r_c^{0.43} \); \( n \) was taken as 0.76 (Section 7.3), so that the plots were to a base of \( r_c^{0.43} \). Figs. 3 and 4 were obtained in this way, from experiments made by the authors and by von Cron.\(^9\) Despite the scatter inherent in this work, the
experimental points lie near straight lines, and this confirms
the validity of the equation \( E_c = K r_p^{(n+1)} \). The constant,
\( K \), is given by Figs. 3 and 4 as 15·6 and 21·5, respectively,
while its calculated value is 10·5 (from eqn. 10). The fact that
experimental values of \( K \) exceed its calculated value confirms
the quantitative criterion derived in Section 2, in that flashover
does not occur when the inequality 12 applies.

\[ r_c \] is proportional to the resistivity, \( \rho \), of the pollution, so
that the critical stress equals \( \rho^{0.43} \), where \( K' \) is a constant.
\( E_c \) has been plotted against \( \rho^{0.43} \) in Fig. 5, and the proximity
of experimental points to the straight lines (i) and (ii) provides
further confirmation of the analysis.

In the experiments discussed so far, special cylindrical or
flat insulators were used, and the solid pollution was deposited
so as to give a constant \( r_c \) along the surface. Line (iii) of
Fig. 5 has been drawn through experimental points obtained
on conventional high-voltage bushings, subjected to 60c/s
wet-flashover tests. It will be seen that, in this case also,
\( E_c \) is proportional to \( \rho^{0.43} \).

3.2 Measurements under natural pollution

It has been deduced analytically in Section 2.1 that,
if flashover is impossible, the current cannot exceed
\( I_{\text{max}} = 233E_c^{-1.31} \) (see eqn. 15). The values given by this expression
will now be compared with those obtained by Forrest,
Lambeth and Oakeshott,1 who carried out experiments on a
wide range of power insulators in polluted atmospheres. No
artificial pollution was used, and the insulator surfaces
became conducting owing to the natural deposition of pollution.
The insulators were operated at 85 to 231 kV (see Tables 1
and 2 of Reference 1), and the applied stress was between
270 and 750 V (peak)/cm. The current flowing along each
insulator was fed to relays, which operated at 25 mA and
again at 150 mA. The current was found to exceed 25 mA
on numerous occasions, but on no insulator did it exceed
150 mA on more than three occasions; in some cases, a
current of 150 mA was attained only on flashover. It appears,
therefore, that the maximum leakage current is of the order
of 150 mA at these stresses.

Maximum currents, corresponding to the stresses applied
to the insulators, were calculated using eqn. 15, and were found
to be between 40 and 150 mA. This indicates that, despite its simplifications, the analysis takes into account the main phenomena governing the maximum current which can flow when flashover is impossible.

4 Conclusion

A study has been made of a mechanism which may
arrest the growth of discharges on polluted insulation, and
so prevent a flashover. It is possible that other mechanisms,
which have not been discussed in this paper, may also prevent
flashover, and, if that is so, insulators may operate safely
under more onerous conditions than those derived here.
However, the agreement obtained between the analysis given
in the paper and the experimental results of several independent
investigators indicates that, in many instances, the mechanism discussed here controls the onset of flashover.

An analysis based on this mechanism has been made for
an insulator having a constant resistance, \( r_c \) ohms per centi-
meter of leakage length, and its results explain the observa-
tions made by the authors and by previous workers in experiments with artificial pollution, in which \( r_c \) was kept constant. It was found that the flashover stress, \( E_{oc} \), is proportional to \( \rho^{0.43} \); the constant of proportionality depends
on the insulator, but \( E_{oc} \) is not less than 10·5\( \rho^{0.43} \) V(peak)/cm
at power frequency. \( E_{oc} \) is also proportional to \( \rho^{0.43} \), where \( \rho \)
is the resistivity of the pollution, and this proportionality
was also found to hold for conventional bushings subjected to
wet-flashover tests.

The analysis has shown also that the maximum current
which can flow along an insulator when flashover is impossible
is 233\( E_{oc}^{-1.31} \) amps, and this agrees with observations made
on power insulators subjected to natural pollution under
service conditions.

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1263
The root is small compared with the length of the polluted root, because the current flow-lines diverge from the root. Hence resistance given by eqn. 4.

Some factors affecting the analysis of Section 2

1. Number of discharges

There are $m$ discharges in parallel, and let them carry the same current. This is tantamount to dividing the insulator surface into $m$ parallel strips, each having a resistance $mr_e$ per centimetre. The critical stress is obtained by substituting $mr_e$ for $r_c$ in eqn. 10:

$$E_c = A^{1/(n+1)}\rho_0(n+1)\rho_0(n+1)$$

Comparison with eqn. 10 shows that the effect of increasing the number of discharges from one to $m$ in parallel is to increase the constant of proportionality between $E_c$ and $\rho_0(n+1)$ by the factor $mn/(n+1)$.

Application to insulation energized at power frequency

Eqn. 10 defines the peak stress at which flashover is impossible, and $E_c$ has therefore been taken as the peak cyclic voltage at power frequency. However, the stress remains at its peak value for only a small fraction of a cycle, and this may not allow sufficient time for the discharge to grow to flashover before the cyclic fall in stress extinguishes the discharge. This cyclic extinction may inhibit a flashover on alternating current, even if the stress exceeds the critical value given by eqn. 10. This view is supported by Forrest, Lambeth and Oakeshott, who found that the leakage length on insulators operating at the same voltage had to be 30% greater on direct current than on alternating current.

7.2 Maximum leakage current

This will now be demonstrated that the current cannot exceed $i_c$ if flashover is impossible. The current is given by eqn. 5, and the solution of that equation will be discussed for a constant applied stress $E_c$ and variable discharge lengths $x < x_c$. Longer discharges will not be considered, because flashover is possible if $x > x_c$.

The solution will first be discussed for an insulator whose surface resistance has the critical value, $r_c$. At the stress $E_c$, $V = E_cL$: again, from eqn. 10, $A = E^{(1+n)}r_c^{-n}$. If these expressions are substituted in eqn. 5, and its terms are rearranged, that equation yields

$$\frac{(ir_c - E_c)}{L} = \frac{x}{L}(ir_c - E_c^{1+n}(ir_c)^{-n})$$

Let the left-hand side of eqn. 22 be designated by $u$, so that

$$u = (ir_c - E_c)$$

and the right-hand side of eqn. 22

$$w = \frac{x}{L}(ir_c - E_c^{1+n}(ir_c)^{-n})$$

Let $u$ and $w$ are sketched against $i$ in Fig. 6a; they must intersect at all values of $i$ which satisfy eqn. 22. $w$ is a function of $x$ as well as $i$, and two curves have been sketched, for $x = x_c$ and $x < x_c$.

Inspection of eqn. 22 shows that it is satisfied by $i = E_c/r_c$ for all values of $x$, so that the three curves intersect at this value of $i$. Comparison with eqn. 14a shows that this current equals the critical current, $i_c$; substituting its value in eqns. 23 and 24 gives $u = w = 0$.

$u$ is a straight line of slope $r_c$. Differentiating $w$ with respect to $i$ gives

$$w' = \frac{\partial w}{\partial i} = \frac{x}{L}(ir_c^{1+n}(ir_c)^{-n} + 1)$$

This expression is not zero for any value of $i$ ($i$ can only be positive), so that $w$ has no turning-point. It will be seen that $w'$ is positive for all values of $i$, and decreases as $i$ increases. Again, $w'$ increases with $x$, so that for any $i$, its maximum
value occurs when \( x \) has the greatest permissible value, \( x_c = L/(n+1) \). Substituting this in the expression for \( w' \) gives

\[
w_c = \frac{r_c}{n+1} \left[ nE_c^{n+1}i^{-n+1} + 1 \right]
\]

and this equals \( r_c \) at \( i = i_c \). Now \( r_c \) is also the slope of the straight line \( u \) (see eqn. 23), so that the line is tangential to \( w \) at \( i = i_c \), as shown in the Figure.

If \( x < x_c \), the slope of \( w \) is smaller than \( w_c \), and so the curve of \( w \) falls below that corresponding to \( x = x_c \), for \( i > i_c \). The Figure shows that \( u \) and \( w \) diverge for \( x < x_c \), as \( i \) increases above \( i_c \). Hence \( w \) and \( u \) cannot intersect at \( i > i_c \), and no current greater than \( i_c \) can satisfy eqn. 22. It has been shown that \( i = i_c \) does satisfy that equation, and therefore this is the maximum current which can flow when \( r = r_c \).

Consider now that the resistance is increased, so that \( r > r_c \). Values of \( r \) less than \( r_c \) need not be considered, because flashover is possible at those values. For convenience, let \( r = kr_c \), where \( k > 1 \). The stress is still \( E_c \), so that eqn. 5 yields

\[
(ir_k - E_c) = \frac{x}{L} [ir_k - E_c^{n+1}(ir_k)^{-n}] \quad \ldots \quad (25)
\]

Let the left-hand side of eqn. 25 be designated by \( y \), so that

\[
y = (ir_k - E_c) \quad \ldots \quad \ldots \quad \ldots \quad (26)
\]

and the right-hand side by

\[
z = \frac{x}{L} [ir_k - E_c^{n+1}(ir_k)^{-n}] \quad \ldots \quad \ldots \quad \ldots \quad (27)
\]

These curves are sketched in Fig. 6b.

Substituting \( i = i_c = E_c/r_c \) in eqns. 26 and 27 gives

\[
y_0 = E_c(k - 1)
\]

\[
z_0 = \frac{x}{L} E_c(k - 1)
\]

and since \( x < L \) and \( k > 1 \), \( y_0 > z_0 \). Again, \( y \) is a straight line of slope \( r_k \). The slope of \( z \) is

\[
z' = \frac{x}{L} (r_k + nE_c^{n+1}i^{-n-1}r_c^{-n}) \quad \ldots \quad \ldots \quad \ldots \quad (28)
\]

and at \( i = i_c = E_c/r_c \), this becomes

\[
z'_0 = \frac{x}{L} (k + n) \quad \ldots \quad \ldots \quad \ldots \quad (29)
\]

Eqn. 29 shows that \( z'_0 \) increases with \( x \); its maximum value occurs at \( x = x_c = L/(n+1) \), and, substituting this in eqn. 29,

\[
z'_0 = \frac{r_c}{n+1} (k + n) = r_k \frac{1 + n/k}{1 + n}
\]

Remembering that \( k > 1 \), it follows that \( z'_0 \) is less than \( r_k \), i.e. \( z' \) is less than \( r_k \) when \( x = x_c \) and \( i = i_c \). Now, eqn. 28 shows that \( z' \) decreases as \( x \) decreases, and as \( i \) increases; consequently \( z' \) is less than \( r_k \) for all permissible values of \( x (x < x_c) \), and for all values of \( i \) greater than \( i_c \).

Hence the slope of \( z \) is less than that of \( y \), and the curves diverge as \( i \) increases; consequently eqn. 22 cannot be satisfied by any value of \( i \) greater than \( i_c \).

It has been proved, therefore, that no current greater than \( i_c \) can flow on the insulator, for all values of \( x \) and \( r \) at which flashover is impossible (\( x < x_c, r > r_c \)).

Fig. 6b shows that \( y \) and \( z \) converge as \( i \) decreases, and it can readily be shown that they intersect at a current less than \( i_c \). Consequently, the maximum current, \( i_c \), flows when \( r \) has its critical value, \( r_c \), but only smaller currents can flow if \( r > r_c \). It can be shown by a similar argument that, if \( r \) is less than \( r_c \), \( i \) can exceed \( i_c \) if \( x < x_c \).

7.3 Discharge characteristics

An experiment was made to determine the discharge characteristics in air, at 0.1 - 1.5 A. Power-frequency currents were passed between two 3-7 cm copper discs with rounded edges, spaced up to 20cm apart, the discharge being vertical. Measurements were taken only when the discharge appeared straight when viewed at right angles. Current and voltage were recorded oscillographically. Glow-arc transitions occurred frequently, as would be expected under these conditions.10 From the oscillograms, the peak cyclic value of the current (in amperes) was plotted against the corresponding instantaneous voltage (in volts). The voltage/current characteristic was obtained for the peak current, because \( E_c \) in eqn. 10 is the peak applied stress, and this corresponds to the peak current.

Curve fitting showed that the relation between voltage and current was of the form

\[
V_d = Ax_i^n + B \quad \text{eqn. (2)}
\]

and \( A \) and \( n \) were found to be 63 and 0.76, respectively, the units of \( V_d \), \( x \) and \( i \) being volts, centimetres and amperes, respectively. \( B \) was approximately 370 V for the glow condition, and approximately 50 V for the arc condition. Eqn. 2 is similar to those obtained by other investigators.11 \( A x_i^n \) and \( B \) represent, respectively, the voltage drops along the discharge column and near the electrodes.

7.4 Discussion of Obenaus’s analysis

Obenaus derived a relation between \( E_c \) and the maximum leakage current, but his work was not extended to relate \( E_c \) and the resistance of the polluted surface. Again, in discussing the growth of discharges, Obenaus explains that the resistance in series with them decreases as the discharges elongate: this is consistent with eqns. 4 and 16. However, in pursuing his analysis, Obenaus introduced a current \( I_p \) which he takes to be independent of discharge length, and which he defines as \( I_p = V_d/R \), where \( V_d \) is the supply voltage and \( R \) the resistance in series with the discharge. It appears therefore that Obenaus’s analysis is based on the assumption of a constant resistance in series with the discharge, and this seems to be inconsistent with his initial premise that the resistance varied.

The constant-resistance condition could be looked upon as an extreme case of the conditions considered in Section 7.1.1, and would hold if \( R' \gg r_k (L - x) \). The critical stress is bound to be greater than that derived in Section 2, since the condition \( R' \gg r_k (L - x) \) implies a much greater resistance