CHAPTER 10

The Application of Image Theory in Geosteering

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10.1 INTRODUCTION

From Chapter 9, Theory of Inversion for Triaxial Induction and Logging-While-Drilling Logging Data in One- and Two-Dimensional Formations, we can see that the inversion in either one-dimensional (1D) or two-dimensional formations are rather mathematically complicated and time consuming due to the fact that the forward modeling is called numerously in each minimization step. Jacobian matrix is usually computed using numerical differentiation, which also uses forward modeling. Unfortunately, the forward modeling of the induction or logging-while-drilling (LWD) tools are relatively slow. In the forward modeling the most time-consuming part is the numerical integration. For real-time inversion, fast forward modeling and inversion method must be used. In this chapter, we will discuss a fast forward
modeling and inversion algorithm that uses image theory to avoid numerical integration. This method is an approximate solution but with an accuracy acceptable for most geosteering applications.

We know that the geosteering is a real-time process used to control and adjust the direction of the drilling bit in a horizontal or deviated well in order to keep the drilling in the target layers as shown in Fig. 10.1. One of the most challenging issues in geosteering is the boundary detection, which calculates the tool distance away from the upper or lower boundary from the measured resistivity data. To implement the real-time control of the drilling process, the forward modeling must be fast enough with compromise on the accuracy and complexity of the formation. In this chapter, the complex image theory in lossy media is introduced to simplify the forward model, which reduces the simulation time and improves the real-time controllability of the geosteering system. This method is implemented in both two-layer and three-layer cases. The accuracy is tested at different frequencies and conductivity combinations. Compared with the results from the full solution, the complex image method has satisfactory accuracy and much less computation time. The error only appeared in the small area when the tool is too close to the boundaries.

Directional resistivity has been applied in geosteering in past years to interpret the measurements used to obtain the distance to bed, dipping angles and anisotropy, among others [1]. Li et al. presented a differential measurement approach, based on the standard propagation resistivity tool, placing two tilted antennas on a drill string, to obtain the bed information by the ratio of two signals at different tool azimuth angles [2]. The measurements contain both the direction-sensitive information and

Figure 10.1 The geosteering system is a guiding device to provide accurate information of the downhole drilling so that the drill bit is kept inside the production zone.
direction-insensitive information, by using postprocessing. In 2006 Wang proposed a new approach that employs an orthogonal transmitter and receiver antennas [3]. The voltage signals from a main receiver antenna and a bucking antenna directly represent directional sensitive information.

Due to the requirement of real time, a fast forward modeling method is desirable. Currently, most of the forward modeling is based on the full solution. In 2005 Omeragic proposed a model-based (parametric) inversion method to detect distances to both upper and lower shoulder beds [2]. In 2006 Wang showed the inversion of distance to a bed based on a full 1D forward model [3]. In 2007 Wang first adopted the image theory to interpret the directional resistivity measurement and showed that the image theory could be used as a quantitative computation method.

The conventional image theory is used to simplify the inhomogeneous problem to the homogeneous problem when the source is over a perfect electric conductor (PEC) or perfect magnetic conductor (PMC) interface. In 1969 Wait extended the approximate discrete image theory to a finite conducting interface [4]. Then Bannister further developed this extension to arbitrary sources [5]. In 1984 Lindell and Alanen studied the continuous exact image source over a dissipate plane [6].

The application of the image source could extremely simplify the forward modeling and speed up the calculation. Wang published more cases verifying the feasibility of the complex image in well logging [7]. This method is powerful for both qualitative and quantitative analysis of a logging response in a stratified formation.

Generally, image theory is transferring the inhomogeneous problem to a homogeneous problem by setting up an image source. Then the homogeneous space Green’s function can be used to solve the field distribution, which is much easier and faster than the full solution.

### 10.2 THEORY OF FORWARD MODELING USING IMAGE THEORY

#### 10.2.1 Review of traditional image theory

Generally, image theory is to convert an inhomogeneous problem to a homogeneous one by introducing an image source. Then the homogeneous space Green’s function could be used to solve the field distribution, which is much easier and faster than the full solution.

The conventional image theory is referring to one electrical dipole over the PEC interface, shown in Fig. 10.2. There is no field in the lower half-space. The field of upper half-space can be calculated by replacing the interface by introducing an image source at lower space and applying the homogeneous Green’s function. The field in upper space will be the summation of the fields generated by both sources. The two-layer inhomogeneous problem is then converted into a homogeneous problem.
More generally, the image sources of electrical dipoles (represented by *single arrows*) and magnetic dipoles (represented by *double arrows*) over PEC and PMC, respectively are shown in Fig. 10.3. Over the electric conductor, the image sources of the horizontal electrical dipole and vertical magnetic dipole have the opposite direction from the original sources. This agrees with the fact that the tangential current does not radiate along the PEC plane. Similarly, when the vertical electrical dipole and horizontal magnetic dipole (HMD) are over the magnetic conductor, there is no field radiation either.

10.2.2 Complex image theory in nonperfect medium

For deriving the complex image theory used into the application of geosteering, the transmitter of the directional resistivity logging tool is exacted to a horizontally placed magnetic dipole source. This assumption is consistent with the real implement of the transmitter antenna, which is a coil antenna around the tool body.
10.2.2.1 Horizontal dipole in half-space
Let a horizontal electrical dipole of moment $p$ be placed at $(0, 0, h)$ pointing at positive $x$ axis as shown in Fig. 10.4. It has been derived that the Hertz potential functions in these two regions satisfy the following two equations, respectively,

$$
\nabla^2 \Pi_1 + \gamma_1^2 \Pi_1 = p \delta(x) \delta(y) \delta(z - h), \quad z \geq 0 \quad \text{and} \quad \quad (10.1a)
$$

$$
\nabla^2 \Pi_2 + \gamma_2^2 \Pi_2 = 0, \quad z \leq 0 \quad \quad (10.1b)
$$

where $\gamma_1^2 = \omega^2 \mu_0 (\varepsilon_1 - j\sigma_1/\omega)$ and $\gamma_2^2 = \omega^2 \mu_0 (\varepsilon_2 - j\sigma_2/\omega)$.

The Hertz potential in region I could be decomposed into two directions

$$
\Pi_1 = x' \Pi_{1x} + z' \Pi_{1z} \quad \quad (10.2)
$$

10.2.2.2 Dipole in lossless half-space
If the horizontal dipole is within the air and above a conductive media, as shown in Fig. 10.4, $\gamma_1^2 = \gamma_0^2 = \omega^2 \mu_0 \varepsilon_0$ and $\gamma_2^2 = \gamma_0^2 = \omega^2 \mu_0 (\varepsilon - j\sigma/\omega)$. In spectral domain, the Hertz potential expressions for HMD are [8],

$$
\Pi_{1x} = \frac{p}{4\pi} \left[ e^{-\gamma_0 R_1} \frac{u_0}{R_1} + \int_0^\infty \frac{u_0 - u}{u_0 + u} J_0(\lambda \rho) e^{-u_0 (z + h)} \frac{\lambda}{u_0} d\lambda \right] \quad \text{and} \quad \quad (10.3a)
$$

$$
\Pi_{1z} = \frac{p}{2\pi} \left[ \int_0^\infty \frac{(u - u_0) e^{-u_0 (z + h)}}{(\gamma_0^2 u + \gamma_0^2 u_0) J_1(\lambda \rho) \lambda^2} \cos \phi \right] d\lambda \quad \quad (10.3b)
$$

Figure 10.4 A horizontal dipole placed in half-space model.
where
\[ R_1 = \left[ \rho^2 + (z-h)^2 \right]^{1/2}, \]
\[ u_0 = \left( \lambda^2 - \gamma_0^2 \right)^{1/2}, \]
and
\[ u = \left( \lambda^2 - \gamma^2 \right)^{1/2}. \]

For the application in well logging, most cases are low frequency and satisfy the quasistatic condition, where we can assume \( u_0 \approx \lambda \), then
\[ P_m = \int_0^\infty \frac{u_0 - u}{u_0 + u} J_0(\lambda \rho) e^{-u_0(z+h)} \frac{\lambda}{u_0} d\lambda \approx - \int_0^\infty \frac{u - \lambda}{u + \lambda} J_0(\lambda \rho) e^{-\lambda(z+h)} d\lambda \quad (10.4) \]

The Taylor series expansion of the function \( f(\lambda) \) can be written in the form
\[ f(\lambda) = e^{\lambda d_{\text{shift}}} \frac{u - \lambda}{u + \lambda} = \sum_{n=0}^{\infty} a_n \lambda^n \quad (10.5) \]

where \( d_{\text{shift}} = (1 - j)\delta \) and \( a_n = (1/n!)f^{(n)}(0) \).

Approximate using only the first term and consequently,
\[ \Pi_{1x} \approx \frac{p}{4\pi} \left( \frac{1}{R_1} - \frac{1}{R_a} \right) \quad (10.6) \]

\[ \Pi_{1z} \approx \frac{p \cos \phi}{4\pi \rho} \left[ \frac{(d_{\text{shift}} + z + h) - (z + h)}{R_a} \right] \quad (10.7) \]

where \( R_a = \left[ \rho^2 + (z + h + d_{\text{shift}})^2 \right]^{1/2} \) and \( R_2 = \left[ \rho^2 + (z + h)^2 \right]^{1/2} \). Because the boundary shift \( d_{\text{shift}} \) is very small, the difference between \( R_a \) and \( R_2 \) is almost zero, which means
\[ \frac{(d_{\text{shift}} + z + h)}{R_a} - \frac{(z + h)}{R_2} \approx 0 \quad (10.8) \]

Then, with the assumption of quasistatic, the Hertz potential of the horizontal dipole placed in a two-layer half-space media can be simplified to one component
\[ \Pi_{1x} \approx \frac{p}{4\pi} \left( \frac{1}{R_1} - \frac{1}{R_a} \right) \quad (10.9) \]

\[ \Pi_{1z} \approx 0 \quad (10.10) \]

Therefore the Sommerfeld integral is simplified to a summation of two terms. Both are in \( x \) direction and located at \( z = h \) and \( z = -(h + d_{\text{shift}}) \), respectively. The total field is the superposition of the fields radiated by the two discrete sources in the homogeneous medium.
We can extend this case when the boundary is not perfect conductive and the source region is within the relative low-conductive media. The nonperfect conductive boundary can be approximated as a perfect conductive boundary by introducing a complex depth shift $d_{\text{shift}}$. By shifting the boundary, the conventional image theory could be applied.

The equivalent two-layer model is shown in Fig. 10.5, in which the remote bed (upper layer, where the image source is located) is much more conductive than the near bed (lower layer, where the original source is located). Bannister gave the shifts for horizontal and vertical magnetic dipoles, respectively. They are

$$d_{\text{VMD}_{\text{shift}}} = \frac{1}{\sqrt{k^2_b - k^2_n}} \quad \text{and} \quad d_{\text{HMD}_{\text{shift}}} = \frac{\sqrt{k^2_b - k^2_n}}{-k^2_n}$$

where $k^2_b = \omega^2 \mu \varepsilon_b - j\omega \mu \sigma_b$ and $k^2_n = \omega^2 \mu \varepsilon_n - j\omega \mu \sigma_n$ are the wave numbers of near bed and the remote bed. If we further assume that the remote bed is sufficiently more conductive than the near bed. Then the shift distance can be simplified to

$$d_{\text{VMD}_{\text{shift}}} = d_{\text{HMD}_{\text{shift}}} \approx \frac{1}{jk_n}$$

**Figure 10.5** Two-layer equivalent model by applying the image theory.
For the logging tool with spacing \( L \), the \( H \) field received by the receiver is

\[
H_{xz} = \frac{P}{4\pi} \frac{e^{-jk_b r}}{r^3} \left( k_b^2 r^2 - 3 j k_b r - 3 \right) \frac{(2d_{\text{shift}} + 2h)L}{r} \quad \text{and} \quad (10.13a)
\]

\[
H_{xx} = \frac{P}{4\pi} \left[ \frac{e^{-jk_b r}}{r^3} \left( k_b^2 r^2 - 3 j k_b r - 3 \right) \left( \frac{2d_{\text{shift}} + 2h}{r} \right)^2 + \frac{2e^{-jk_b r}}{r^3} (j k_b r + 1) \right] + \frac{P}{4\pi} \frac{e^{-jk_b L}}{L^3} (j k_b L + 1) \quad (10.13b)
\]

Consider a three-layer model as shown in Fig. 10.6. In this model, for each boundary, only the first image is considered. According to the application condition of the approximated image theory, the middle layer, where the drilling bit stays, has higher resistivity compared with the other two adjacent layers. Then the three-layer model is simplified into a homogeneous model with three sources.

### 10.2.2.3 Dipole in the dissipative media

Consider another case when region I is dissipative while that region II is nonconductive. The parameters of those two regions are \( \gamma_1^2 = \gamma^2 = \omega^2 \mu_0 (\varepsilon - j\sigma/\omega) \) and \( \gamma_2^2 = \gamma_0^2 = \omega^2 \mu_0 \varepsilon_0 \). Then, the Hertz potential in the two regions become

\[
\Pi_{1x} = \frac{p}{4\pi} \left[ e^{-\gamma R_1} R_1 + \int_0^\infty \frac{u - u_0}{u + u_0} J_0(\lambda \rho) e^{-u(z+h)} \frac{\lambda}{u} d\lambda \right] \quad \text{and} \quad (10.14a)
\]

\[
\Pi_{1z} = \frac{p}{2\pi} \left[ \int_0^\infty \frac{(u_0 - u) e^{-u(z+h)}}{(\gamma_0^2 u + \gamma^2 u_0)} J_1(\lambda \rho) \lambda^2 d\lambda \right] \cos \phi \quad (10.14b)
\]
where

\[ R_0 = \left[ \rho^2 + (z-h)^2 \right]^{1/2}, \]
\[ u_0 = (\lambda^2 - \gamma_0^2)^{1/2}, \]
\[ u = (\lambda^2 - \gamma^2)^{1/2}. \]

Define \( n = (\varepsilon, -j\sigma/\omega\varepsilon_0)^{1/2}, \gamma = n\gamma_0 \) and apply Sommerfeld identity (10.15) in Eq. (10.14),

\[
\frac{e^{-\gamma R_2}}{R_2} = \int_0^\infty f_j(\lambda\rho) \frac{e^{-u|z+h|}}{u} d\lambda
\]

The Hertz potential can be rewritten as

\[
\Pi_{1x} = \frac{p}{4\pi} \left[ \frac{e^{-\gamma R_1}}{R_1} - \frac{e^{-\gamma R_2}}{R_2} + 2 \int_0^\infty f_j(\lambda\rho) e^{-u(z+h)} \frac{\lambda^2}{u + u_0} d\lambda \right] \quad \text{and} \quad (10.16a)
\]

\[
\Pi_{1z} = -\frac{p}{2\pi} (1 - n^2) \left[ \int_0^\infty f_j(\lambda\rho) e^{-u(z+h)} \frac{\lambda^2}{(u + u_0)(n^2u_0 + u)} d\lambda \right] \cos \varphi
\]

where \( R_1 = \left[ \rho^2 + (z-h)^2 \right]^{1/2} \) and \( R_2 = \left[ \rho^2 + (z+h)^2 \right]^{1/2} \). To simplify these expressions, by using Lien's method, define the abbreviations,

\[ G_1 = \frac{e^{-\gamma R_1}}{R_1}, \quad G_2 = \frac{e^{-\gamma R_2}}{R_2}, \]
\[ U = 2 \int_0^\infty f_j(\lambda\rho) e^{-u(z+h)} \frac{\lambda^2}{u + u_0} d\lambda \quad \text{and} \]
\[ W = -2(1 - n^2) \left[ \int_0^\infty f_j(\lambda\rho) e^{-u(z+h)} \frac{\lambda^2}{(u + u_0)(n^2u_0 + u)} d\lambda \right]
\]
\[ = 2(1 - n^2) \frac{\partial}{\partial x} \left[ \int_0^\infty f_j(\lambda\rho) e^{-u(z+h)} \frac{\lambda d\lambda}{(u + u_0)(n^2u_0 + u)} \right] \]

The Hertz potential function \( \Pi_1 \) is then given by

\[
\Pi_1 = \frac{p}{4\pi} (G_1 - G_2 + U)x' + Wz'
\]

(10.17)
The field components in region I can be found by following Norton method in cylindrical coordinates

\[ \frac{\partial W}{\partial z} = -2 \frac{\partial}{\partial x} \int_{0}^{\infty} \left( \frac{1}{u_{0} + u} - \frac{n^{2}}{u_{0}^{2} + u} \right) J_{0}(\lambda \rho) e^{-\gamma(z+h)} \lambda d\lambda \tag{10.18} \]

Define \( V = 2n^{2} \int_{0}^{\infty} \frac{J_{0}(\lambda \rho) e^{-\gamma(z+h)}}{u_{0}^{2} + u} \lambda d\lambda \), then

\[ \frac{\partial W}{\partial z} = -\frac{\partial U}{\partial x} + \frac{\partial V}{\partial x} \tag{10.19} \]

The divergence of \( \Pi_{1} \), therefore, is given by

\[ \nabla \cdot \Pi_{1} = \frac{\partial}{\partial x}(G_{1} - G_{2} + V) \tag{10.20} \]

Because \( H_{1} = \nabla(\nabla \cdot \Pi_{1}) + \gamma^{2} \Pi_{1} \), the \( z \) component is

\[ H_{1z} = \gamma^{2} W + \frac{\partial^{2}}{\partial x \partial z} (G_{1} - G_{2} + V) \tag{10.21} \]

Transfer the solution into cylindrical coordinates and apply the Sommerfeld integral representation of spherical wave function, Eq. (10.15), the integral in Eq. (10.18) becomes

\[ -2 \frac{\partial}{\partial x} \int_{0}^{\infty} \left( 1 - \frac{u}{u_{0}^{2} + u} \right) J_{0}(\lambda \rho) e^{-\gamma(z+h)} \lambda d\lambda = 2 \frac{\partial^{2} G_{2}}{\partial z \partial \rho} - \frac{1}{n^{2}} \frac{\partial^{2} V}{\partial z \partial \rho} \tag{10.22} \]

Furthermore, in the low-frequency assumption, using the leading term of the asymptotic expression of the Bessel function, the approximate expression of \( V \) is given

\[ H_{1z} = \frac{\partial^{2}}{\partial z \partial \rho} \left( G_{1} + G_{2} - \frac{V}{n^{2}} \right) \cos \phi \tag{10.23} \]

From Eq. (10.23), the cross \( z \) component is generated by original source \( G_{1} \), image source \( G_{2} \), and a correction term related to \( V \). Roy Harold Lien gave the low-frequency approximation of the integral \( V \) under the assumption that \( |n^{2}| \gg 1 \) and \( |jnkR_{2}/2| \gg 1 \), the leading term approximation is given as

\[ V = 2k_{0} e^{-j\gamma(z+h)} \text{ and} \tag{10.24} \]

\[ H_{1z} = \frac{\partial^{2}}{\partial z \partial \rho} (G_{1} + G_{2}) \cos \phi - \frac{2k_{0}}{n^{2} \rho^{2}} \gamma e^{-j\gamma(z+h)} \cos \phi \tag{10.25} \]
Considering the tool is always located in $xz$ plane and parallel with $x$ axis, $\phi = 0$ degree, we will have

$$H_{xz} = \frac{\partial^2}{\partial z \partial \rho} (G_1 + G_2) + H_t$$  \hspace{1cm} (10.26)

$$H_t = -\frac{2k_0}{n^2 \rho^2} \gamma e^{-jr(z+h)}$$  \hspace{1cm} (10.27)

The final expressions for the magnetic field in region I, for the case $\sigma/\omega\varepsilon \ll 1$, are generated by the original source placed at $z = h$, an image place at $z = -h$ and a correction term expressed in Eq. (10.27).

Then, the $H$ field received by the receiver in cylindrical coordinator is

$$H_{xz} = \frac{P}{4\pi} \left\{ \frac{1}{2} \left[ -n^2 k^2 + j3nkr_1^{-1} + 3r_1^{-2} \right] \sin 2\theta_1 G_1 ight. \\
+ \left. \frac{1}{2} \left[ -n^2 k^2 + j3nkr_2^{-1} + 3r_2^{-2} \right] \sin 2\theta_2 G_2 - \frac{1}{n^2} \frac{\partial^2 V}{\partial z \partial \rho} \right\}$$  \hspace{1cm} (10.28a)

$$H_{xx} = \frac{P}{4\pi} \left\{ \left[ n^2 k^2 \cos^2 \theta_1 + jnk(2 - 3\cos^2 \theta_1)r_1^{-1} + (2 - 3\cos^2 \theta_1)r_1^{-2} \right] G_1 ight. \\
+ \left. \left[ n^2 k^2 - j3nkr_2^{-1} - 3r_2^{-2} \right] \cos^2 \theta_2 G_2 - \frac{1}{\rho} \frac{\partial V}{\partial \rho} \right\}$$  \hspace{1cm} (10.28b)

where $\theta_1$ and $\theta_2$ are the angles shown in Fig. 10.4.

### 10.3 SIMULATION RESULTS AND DISCUSSIONS

#### 10.3.1 One-dimensional formation model

If the borehole is neglected and only the depth variation is considered, an isotropic formation could be modeled as a layered medium, as shown in Fig. 10.7. In this model, the $z$ direction is the depth direction. In the application of geosteering, the tool is always kept in the production layer, which means in most cases, the tool is placed horizontally. This assumption applies to all following testing examples. The testing points will be along the $z$ direction. Then the received signal is a function of distance from the tool position to the boundary. According to the electrical properties of the near bed and remote bed, two cases are possible. One is when the tool is within the high-resistive bed.
The other is when the tool is in the high-conductive bed. For these two cases, different approximations must be used to apply the complex image method. For each case, the simulation results generated by the complex image method will be presented together with the results obtained by a full solution code, which is named INDTRI developed by the Well Logging Lab at the University of Houston. The Hankel integral is solved by 283 points fast Hankel transform. The relative permittivity and permeability of each layer are set to be 1. That is because, firstly, the tool is working at relatively low frequency, the effect of permittivity is not significant. Secondly, in the most cases, the earth is nonmagnetic. We can always neglect the permeability of the earth.

10.3.2 Tool configuration

Consider the Azitrack directional resistivity tool discussed in Chapter 4, Triaxial Induction and Logging-While-Drilling Resistivity Tool Response in Homogeneous Anisotropic Formations, by Baker Hughes as an example. Fig. 10.8 shows tool structure. As described in Chapter 4, Triaxial Induction and Logging-While-Drilling Resistivity Tool Response in Homogeneous Anisotropic Formations, this tool works at two frequencies, 2 MHz and 400 kHz. The configuration is symmetric, which is called a compensated LWD configuration. As shown in Fig. 10.8, there are several

![Figure 10.7 Three-layer model.](image)

![Figure 10.8 Azitrack tool configuration.](image)
different spacings. For convenience sake without losing generality, only two spacings are considered in the following discussions. The long spacing is 33.375 in. and the short spacing is 22.265 in.

In the following simulation, actually, we only consider the radiation of dipole source and neglect the effect of mandrel, the reason why we can use this assumption is, firstly, compared with the geological size of the formation, the size of the mandrel can be neglected. Secondly, the effect of mandrel is to enhance or reduce the magnitude of the field, but not change the distribution. The third one is the effect of the mandrel can be compensated by the symmetrical configuration. So, in the following simulation, we can neglect the effect of the mandrel.

### 10.3.3 Simulation results

#### 10.3.3.1 \( R_1 = R_3 = 1 \text{ ohm-m}, R_3 = 100 \text{ ohm-m} \)

Consider the three-layer model, as shown in Fig. 10.7. The high-resistivity layer is in the middle. The parameters of these three layers are \( \varepsilon_{r1} = \varepsilon_{r3} = 1, \mu_{r1} = \mu_{r3} = 1, \) and \( \sigma_1 = \sigma_3 = 1 \) for the upper and lower layer, \( \varepsilon_{r2} = 1, \mu_{r2} = 1, \) and \( \sigma_2 = 0.01 \) for the middle layer. The boundaries are at \( z = 10 \text{ ft} \) and \( z = -10 \text{ ft} \). This is the general case when the drilling bit is in the high-resistivity layer.

1. **Frequency = 2 MHz, spacing = 33.375 in.**

   Fig. 10.9 shows simulation results of the cross component \( H_{zx} \). The pink circle (light gray in print versions) indicates result calculated by approximated method and the blue dash line (dark gray in print versions) is the result of a full solution. From the results, we can see that the image method works pretty well. Even when the tool crosses the boundary, there is only small error between the approximation and full solution. Here, the results show that when the tool is working at 2 MHz, the long-spacing channel works well. The cross component could represent the boundary information. Compared with the full solution results, there is no much error.

   Fig. 10.10 shows the phase shift and attenuation of the compensated propagation tool. These two parameters will be used in inversing the apparent resistivity of the formation. Compared with the full solution results, there is noticeable error when the logging tool is close to the boundary. Because our three-layer model is symmetrical, the simulation results are also symmetrical.

2. **Frequency = 2 MHz, spacing = 22.265 in.**

   Figs. 10.11 and 10.12 give the simulation results when the logging tool is working at 2 MHz and the spacing is short. Compared with the full solution results, the approximation method also works well. Only small error appears around boundary. Compared with the long-spacing case, cross component \( H_{zx} \) has a little more error at the boundary. Phase shift and attenuation are a little bit better. Although in this case, the error of cross component is a little bit larger, it does not affect the boundary information.
Figure 10.9 Tool response $H_{zx}$ component ($\sigma_1 = \sigma_3 = 1$, $\sigma_2 = 0.01$, 2 MHz, long).

Figure 10.10 Phase difference and attenuation ($\sigma_1 = \sigma_3 = 1$, $\sigma_2 = 0.01$, 2 MHz, long).
Figure 10.11 Tool response $H_{zx}$ component ($\sigma_1 = \sigma_3 = 1, \sigma_2 = 0.01$, 2 MHz, short).

Figure 10.12 Phase difference and attenuation ($\sigma_1 = \sigma_3 = 1, \sigma_2 = 0.01$, 2 MHz, short).
3. Frequency = 400 kHz, spacing = 33.375 in.
   When the tool is working at 400 kHz and the spacing is long, from Fig. 10.13, the real part of the cross component $H_{zx}$ still matches well with full solution results, even when the logging point is at the boundary. Image part of $H_{zx}$ has obvious error. Based on this property, we consider that the boundary inversion could be developed only in terms of the real part of $H_{zx}$.

   Fig. 10.14 shows the phase shift and attenuation of the logging tool, when it is working at 400 kHz and the spacing is long. The results show that, in most range, the simulation results of the approximation method agree with the full solution results. Only exception is around the boundaries, where noticeable error is witnessed.

4. Frequency = 400 kHz, spacing = 22.265 in.
   When the tool is working at 400 kHz with short spacing, the cross component $H_{zx}$ is not as good as before, as shown in Fig. 10.15. Not only imaginary part, but also real part of $H_{zx}$ deviates from full solution around the boundaries. However, this error only exists within the area 2 in. away from the boundaries. For the application of geosteering, this distance is relatively small. So, this error is acceptable.

   Fig. 10.16 shows the phase shift and attenuation when the tool is working at 400 kHz and with short spacing. Compared with other channels, the simulation results of the approximation method show enough agreement with the full solution results. Error only occurs near the boundaries. Based on the phase shift and attenuation, the apparent conductivities of the three layers could be inversed.
**Figure 10.14** Phase difference and attenuation ($\sigma_1 = \sigma_3 = 1, \sigma_2 = 0.01, 400$ kHz, long).

**Figure 10.15** Tool response $H_{zx}$ component ($\sigma_1 = \sigma_3 = 1, \sigma_2 = 0.01, 400$ kHz, short).
Consider the three-layer model, as shown in Fig. 10.7. The resistivity of the middle layer is less than case in Section 10.3.3.1. The parameters of these three layers are $\varepsilon_{r1} = \varepsilon_{r3} = 1$, $\mu_{r1} = \mu_{r3} = 1$, and $\sigma_{1} = \sigma_{3} = 1$ for the upper and lower layer, $\varepsilon_{r2} = 1$, $\mu_{r2} = 1$, and $\sigma_{2} = 0.1$ for the middle layer. The boundaries are at $z = 10$ ft and $z = -10$ ft. This is also the general case when the drilling bit is in the high-resistivity layer.

1. Frequency = 2 MHz, spacing = 33.375 in.

Figs. 10.17 and 10.18 give the simulation results when the tool is working at relative high frequency and long spacing, where the middle layer of the formation is relatively less resistive. The figure shows that the cross component $H_{2x}$ simulated by approximation method has a good agreement with the data given by full solution. This means this approximation method can be applied into the forward modeling of geosteering tool and the simulation results are good enough to be used to extract boundary information.

2. Frequency = 2 MHz, spacing = 22.265 in.

3. Frequency = 400 kHz, spacing = 33.375 in.

4. Frequency = 400 kHz, spacing = 22.265 in.

Figs. 10.19–10.24 show the simulation results when the tool is working in other three channels. As in the first case, the approximation method works well in all other three channels, 2 MHz with short spacing, 400 kHz with long spacing, and 400 kHz with short spacing. Although when the frequency is lower, the error

**Figure 10.16** Phase difference and attenuation ($\sigma_{1} = \sigma_{3} = 1$, $\sigma_{2} = 0.01$, 400 kHz, short).
around boundaries becomes larger, however, the accuracy is still within an acceptable range. From the results of phase shift and attenuation, the apparent resistivity of each layer can be inverted. Based on the inverted resistivity and the cross component $H_{zx}$ data, the boundary information can be extracted.

**Figure 10.17** Tool response $H_{zx}$ component ($\sigma_1 = \sigma_3 = 1, \sigma_2 = 0.1$, 2 MHz, long).

**Figure 10.18** Phase difference and attenuation ($\sigma_1 = \sigma_3 = 1, \sigma_2 = 0.1$, 2 MHz, long).
10.3.3.3 \( R_1 = R_3 = 100 \text{ ohm-m}, \ R_2 = 1 \text{ ohm-m} \)

In this case, the formation model is also three layer. The difference between this and previous two cases is that, in this case, the middle layer is of high conductive and two remote layers are of relatively high resistivity. In this case, the parameters of these three layers are \( \varepsilon_{r1} = \varepsilon_{r3} = 1, \ \mu_{r1} = \mu_{r3} = 1, \) and \( \sigma_1 = \sigma_3 = 0.01 \) for the upper and lower
Figure 10.21 Tool response $H_{zx}$ component ($\sigma_1 = \sigma_3 = 1$, $\sigma_2 = 0.1$, 400 kHz, long).

Figure 10.22 Phase difference and attenuation ($\sigma_1 = \sigma_3 = 1$, $\sigma_2 = 0.1$, 400 kHz, long).
Figure 10.23 Tool response $H_{zx}$ component ($\sigma_1 = \sigma_3 = 1$, $\sigma_2 = 0.1$, 400 kHz, short).

Figure 10.24 Phase difference and attenuation ($\sigma_1 = \sigma_3 = 1$, $\sigma_2 = 0.1$, 400 kHz, short).
layer, $\varepsilon_{r2} = 1$, $\mu_{r2} = 1$, and $\sigma_2 = 1$ for the middle layer. The boundaries are at $z = 10$ ft and $z = -10$ ft. This is also the general case when the drilling bit is in the high-resistivity layer.

1. Frequency = 2 MHz, spacing = 33.375 in.
2. Frequency = 2 MHz, spacing = 22.265 in.
3. Frequency = 400 kHz, spacing = 33.375 in.
4. Frequency = 400 kHz, spacing = 22.265 in.

Figs. 10.25–10.32 show the simulation results of full channels, when the middle layer of the formation is high conductive. Because the three-layer model is treated as the combination of two independent boundaries, the simulation results are similar to the ones where the middle layer is of high resistive. We find that the approximation method works well in this case. The cross component $H_{2x}$, phase shift and attenuation are all good enough to be used into boundary detection.

10.3.3.4 $R_1 = 1$ ohm-m, $R_1 = 20$ ohm-m, $R_1 = 0.5$ ohm-m

In all the previous three cases, the upper layer and lower layer have the same conductivity, which means the models are all symmetrical. The unsymmetrical case is also tested. The parameters of these three layers are $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1$, $\mu_{r1} = \mu_{r2} = \mu_{r3} = 1$, and $\sigma_1 = 1, \sigma_2 = 0.05, \sigma_3 = 2$. The boundaries are at $z = 10$ ft and $z = -10$ ft. This case is more general as in real application.

Similarly, four channels are all tested in this case. As is expected, the approximation method also works well in this unsymmetrical formation. The approximation method
Figure 10.26  Phase difference and attenuation ($\sigma_1 = \sigma_3 = 0.01$, $\sigma_2 = 1$, 2 MHz, long).

Figure 10.27  Tool response $H_{zx}$ component ($\sigma_1 = \sigma_3 = 0.01$, $\sigma_2 = 1$, 2 MHz, short).
Figure 10.28  Phase difference and attenuation ($\sigma_1 = \sigma_3 = 0.01, \sigma_2 = 1, \text{2 MHz, short}$).

Figure 10.29  Tool response $H_{zx}$ component ($\sigma_1 = \sigma_3 = 0.01, \sigma_2 = 1, \text{400 kHz, long}$).
Figure 10.30  Phase difference and attenuation ($\sigma_1 = \sigma_3 = 0.01, \sigma_2 = 1$, 400 kHz, long).

Figure 10.31  Tool response $H_{zx}$ component ($\sigma_1 = \sigma_3 = 0.01, \sigma_2 = 1$, 400 kHz, short).
can be also used into the layer with relatively high conductivity. One additional term was introduced to correct the image results. The real part of cross components $H_{zx}$ has more accuracy than the imaginary part, which indicates that it is better to extract the boundary information only from the real part of the signal. Compared with the cross component, although there is a little bit more error of phase shift and attenuation, the logging values away from boundaries are good enough to invert the apparent resistivity of the formation. So, the approximation method can be used into the application of geosteering. The boundary information can be extracted from the cross component.

1. Frequency = 2 MHz, spacing = 33.375 in. (Figs. 10.33 and 10.34)
2. Frequency = 2 MHz, spacing = 22.265 in. (Figs. 10.35 and 10.36)
3. Frequency = 400 kHz, spacing = 33.375 in. (Figs. 10.37 and 10.38)
4. Frequency = 400 kHz, spacing = 22.265 in. (Figs. 10.39 and 10.40)

10.3.4 Discussion

According to the simulation results, the real part of cross component $H_{zx}$ shows that nonzero values only exist near boundary. In the area far away from boundary, the values of $H_{zx}$ are zero. This is the advantage of the orthogonal configuration tool. The cross component is only sensitive to the boundary. When the tool is approaching the boundary, the cross component will increase. Then when the distance from the
Figure 10.33 Tool response $H_{zx}$ component ($\sigma_1 = 1, \sigma_2 = 0.05, \sigma_3 = 2$, 2 MHz, long).

Figure 10.34 Phase difference and attenuation ($\sigma_1 = 1, \sigma_2 = 0.05, \sigma_3 = 2$, 2 MHz, long).
Figure 10.35  Tool response $H_{zx}$ component ($\sigma_1 = 1, \sigma_2 = 0.05, \sigma_3 = 2$, 2 MHz, short).

Figure 10.36  Phase difference and attenuation ($\sigma_1 = 1, \sigma_2 = 0.05, \sigma_3 = 2$, 2 MHz, short).
Figure 10.37 Tool response $H_{zx}$ component ($\sigma_1 = 1, \sigma_2 = 0.05, \sigma_3 = 2$, 400 kHz, long).

Figure 10.38 Phase difference and attenuation ($\sigma_1 = 1, \sigma_2 = 0.05, \sigma_3 = 2$, 400 kHz, long).
Figure 10.39 Tool response $H_{zx}$ component ($\sigma_1 = 1, \sigma_2 = 0.05, \sigma_3 = 2, 400$ kHz, short).

Figure 10.40 Phase difference and attenuation ($\sigma_1 = 1, \sigma_2 = 0.05, \sigma_3 = 2, 400$ kHz, short).
drilling bit to the boundary is larger than a specific value, the tool cannot detect the boundary any more. Based on the simulation results we have, tool’s sensitivity of boundary is affected by the combination of frequency and spacing. Besides, the conductivity of formation also affects the tool response.

10.3.4.1 Effects of conductivity contrast

In this section, the tolerance of the image method at different conductivity contrast will be investigated for the two-layer model, as shown in Fig. 10.41. The upper layer of the model is a low-conductivity layer. The resistivity of the lower layer varies from 10 ohm-m to 1 kohm-m. The tolerance of the image method is tested at 2 MHz and antenna spacing is 34 in.

Fig. 10.42 shows the absolute error and relative error between the approximation results and full solution results at the observation point 1 ft and 2 ft away from the boundary, respectively. The results show that when the resistivity ratio between the upper layer and the lower layer is increased, the error between the approximation method and the full solution converges. The relative error of the $xz$ component is smaller when the resistivity ratio of the upper layer and the lower layer is larger. At the observation point 1 ft away from the boundary, when the resistivity ratio between the upper layer and the lower layer is more than 100, the absolute error is less than 0.0123; the relative error between the approximation method and the full solution is about 15%. When the observation point is at the area 2 ft away from the boundary, the error will be less.

10.3.4.2 Frequency

In terms of practical application, assume that the current excited into the transmitter is 200 mA. The area of antenna is 2.5 in.$^2$. Then, the moment of single-turn antenna is about 3.2e-4 A m$^2$. Then, the $H_{zx}$ data in Figs. 10.8 and 10.12 can be converted to the received voltage signal, shown in Fig. 10.43. For evaluating the sensitivity of the boundary detection, the detectable minimum signal power should be considered. Currently, the minimum detectable voltage is about 100 nV.
In Fig. 10.43, the $H_{zx}$ is converted into voltage by considering that the transmitter has only single turn and its moment is $3.2e^{-4}$. The parameters of the formation is $\sigma_1 = \sigma_3 = 1$ and $\sigma_2 = 0.01$. The yellow line (gray in print versions) shows the minimum voltage value that can be detected by the sensor.

As shown in Fig. 10.43, in high-resistive area, tool responses at two working frequencies have similar sensitivities. The signal of cross component fades to zero at the position about 5 ft away from the boundary. In the high-conductive range, the signal decays even further. The detectable distances are around $2-4$ ft. In this range, the high frequency signal decays faster, so the relatively low-frequency working channel has better sensitivity. Tool can detect further at the relatively low frequency.
Similarly, by comparing the simulation results in other formations, we can always get at least 5 ft detectable distance in high-resistive region. The detectable distances in high-conductive region are different caused by the different working frequencies. Low-frequency channel has larger detectable distance.

10.3.4.3 Spacing
To investigate the effect of spacing, two-layer model with only one boundary is considered, as shown in Fig. 10.44. The parameters of the two layers are \( \sigma_1 = 1 \) and \( \sigma_2 = 0.01 \). The boundary is located at \( z = 0 \). For testing the effect of spacing, frequency should be fixed. The fixed frequency is chosen to be 400 kHz, simply because the detectable distance is larger at low frequency. The spacing range is from 33 to 55 in.

**Figure 10.43** Voltage signal generated by cross component \( H_{zx} \) of single-turn transmitter \((\sigma_1 = \sigma_3 = 1, \sigma_2 = 0.01)\).

**Figure 10.44** Two-layer model with boundary at \( z = 0 \) ft.

Similarly, by comparing the simulation results in other formations, we can always get at least 5 ft detectable distance in high-resistive region. The detectable distances in high-conductive region are different caused by the different working frequencies. Low-frequency channel has larger detectable distance.
Fig. 10.45 shows the cross-component simulation results with different spacing. The results are all divided by 100 nV, which is the minimum detectable voltage in application. When the spacing is larger, the peak at the boundary is lower. In the high-resistive region, the detectable distance is larger. On the contrary, in the high-conductive region, the detectable distance becomes smaller. Replot the results in log scale in Fig. 10.46. It is easy to find that when spacing is 55 in., the
detectable distance is about 7 ft. It is also noticed that the detectable distance is not sensitive to the spacing. That probably because the wavelength effect, which is compared with the wavelength, the spacing is relatively small. The property is good for tool design, which means the tool does not need to be too long.

Figs. 10.47 and 10.48 show the same results when the tool is working at 2 MHz. Compared with the results at 400 kHz, when both transmitter and receiver have one turn, with spacing 55 in., the detectable distance of both frequencies are around 7 ft.

![Figure 10.47 Cross-component response versus 100 nV in different spacing (2 MHz).](image1)

![Figure 10.48 Cross-component response versus 100 nV in different spacing in log scale (2 MHz).](image2)
However, when transmitter has 10 turns, the receiving signal will be enlarged 10 times. In this situation, the tool working at 400 kHz has larger detectable distance than the tool working at 2 MHz.

**10.3.4.4 Calculation speed**

Table 10.1 shows the CPU time comparison between the image method and the full solution for different numbers of iterations. The results show that the image method is much faster than the full solution. When iteration time is 1000, the image method is 160 times faster than the full solution. In addition, when iterative times increase, the image method will have a greater advantage in computation speed.

<table>
<thead>
<tr>
<th>Logging points</th>
<th>6000</th>
<th>60,000</th>
<th>600,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image method(s)</td>
<td>0.12</td>
<td>0.59</td>
<td>5.37</td>
</tr>
<tr>
<td>Full solution(s)</td>
<td>8.67</td>
<td>86.17</td>
<td>859.88</td>
</tr>
<tr>
<td>Speed ratio</td>
<td>72</td>
<td>150</td>
<td>160</td>
</tr>
</tbody>
</table>

The values in italics indicates comparison for the logging points of 6000, 60,000, and 600,000.

**10.3.4.5 Logging with high deviated angle**

Until now, all simulation cases assume a horizontal well. However, in the real application, most cases are not in exactly horizontal situation. To further understand the effectiveness of the image theory method, the well with high deviated angle is investigated. The schematic of the well with high deviated angle is shown in Fig. 10.49,

![Figure 10.49](image-url)
where the tool is not exactly horizontally placed. The dipping angle of the simulated tool is from 60 to 85 degrees. For convenience, phase shift and amplitude ratio is not shown. Only the cross-component response is shown below.

The three-layer 1D model is shown in Fig. 10.49. The parameters of this formation are $\varepsilon_{r1} = \varepsilon_{r3} = 1$, $\mu_{r1} = \mu_{r3} = 1$, and $\sigma_{1} = \sigma_{3} = 1$ for the upper and lower layer, $\varepsilon_{r2} = 1$, $\mu_{r2} = 1$, and $\sigma_{2} = 0.01$ for the middle layer. The boundaries are at $z = 10$ ft and $z = -10$ ft. The logging is working at 2 MHz and the spacing is 34 in.

1. Dipping angle = 85 degrees

Fig. 10.50 shows the cross component of the simulation results when the dipping angle is 85 degrees. Because in this case, the tool is almost horizontally placed, the simulation results look similar as the case when the logging tool is placed exactly horizontally. Comparing the image theory method with the full solution, there is not much difference. The complex image theory works pretty well when the logging tool is placed almost horizontally.

2. Dipping = 75 degrees

Fig. 10.51 shows the simulation results when the dipping angle is reduced to 75 degrees. In this case, the complex image method works well too. The fast solution shows enough agreement with the full solution. In addition, because the tool is not horizontal with the bed boundaries, although the formation is symmetry, the simulation results become asymmetry. The cross component shows stronger response at the lower boundary.

Figure 10.50 Tool response $H_{zx}$ component (dipping = 85 degrees).
3. Dipping angle = 65 degrees
When the dipping angle goes to 65 degrees, the asymmetry of the cross component is more obvious, as shown in Fig. 10.52. In this case, the complex image method gives the large peak when the tool is across the lower boundary. But, in the area near the upper boundary, the simulation results from complex image method can follow the full solution results closely. The reason is that, when the dipping angle is less than 90 degrees, for the upper boundary, the transmitter is closer to the boundary and the receiver is relatively further away from the boundary. In opposite, for the lower boundary, the receiver is closer to the boundary than the transmitter. As the instruction shown before, the complex image method has larger error in the area near the boundary. So, in Fig. 10.52, there is larger error appearing near the lower boundary. But, as shown in Fig. 10.42, the error is still within 2–3 ft from the boundary. This is acceptable in the application of geosteering system.

4. Dipping angle = 60 degrees
Similarly, Fig. 10.53 shows the cross component of the simulation results when the dipping angle is 60 degrees. Because the dipping angle is much less than horizontal case, the simulation results show more obvious asymmetric and the complex image method gives larger error near the lower boundary. It is already shown
in Fig. 10.49 that the middle layer is relative high-resistive layer. Zoom in the Fig. 10.53 and show the middle layer only in Fig. 10.54. The simulation results show that the complex image method works well in the relative high-resistive layer, even when the dipping angle is 60 degrees. The simulation results of
complex image method can follow the variation of the results from full solution in the most area. Error only occurs near the lower boundary and the error area is within 2 ft away from the boundary. Based on the discussion above, it can be concluded that the complex image method can work in the highly deviated well with dipping angle varying from 60 to 90 degrees.

10.4 BOUNDARY DISTANCE INVERSION

10.4.1 Theory of inversion

The inversion methods have been extensively discussed in Chapter 9, Theory of Inversion for Triaxial Induction and Logging-While-Drilling Logging Data in One- and Two-Dimensional Formations. In Chapter 9, Theory of Inversion for Triaxial Induction and Logging-While-Drilling Logging Data in One- and Two-Dimensional Formations, our interests are the formation conductivities. In geosteering, the most important parameters are the distance of the boundaries from the tool. Due to the real-time control requirements, the inversion must be fast enough and can be done in downhole. The downhole processors are usually not as powerful as the one on the surface, the inversion process must be simple and fast. In this chapter, we discuss a special inversion method, which is similar to the methods discussed in Chapter 9, Theory of Inversion for Triaxial Induction and Logging-While-Drilling Logging Data in One- and Two-Dimensional Formations, but with significant smaller number of variables.

Figure 10.54 Tool response $H_{zx}$ component in the middle layer (dipping = 60 degrees).
10.4.2 Workflow of inversion problem

For real-time drilling direction adjustment, geosteering system is a negative feedback control system, which adjusts the direction of the drilling bit based on the real-time data collected from downhole. Such data includes the real-time position of the drilling bit and its distance away from the boundary. Boundary Detection is thus the key part of this system. A fast and accurate method is essential for real-time control.

Boundary detection is usually modeled as an inversion problem. In an iterative manner, we are to minimize the difference between the data collected from the receiving antenna and the simulation results from the forward modeling in certain tolerance. The value of parameters, e.g., the distance to boundary, is calculated as a by-product in the minimization process. Fig. 10.55 shows the flowchart of the inversion process used in this chapter, which generally includes forward modeling and model correction. Because of such an iterative procedure, real-time system requires that the forward modeling, which calculates the field distribution of dipole in multilayered media, to be fast and accurate.

10.4.3 Processing flow of boundary detection in geosteering

In the geosteering system, there are three steps to process the measurement before going to the boundary distance inversion. Those steps help the system to get the basic information of the formation and initialize the simulation model used in the boundary distance inversion. Fig. 10.56 gives the general flow of such process. Firstly, logging data is collected by the receiver. Secondly, a brief geological model of the
formation is generated from the logging data. This step mainly focuses on finding the positions of all boundaries. Thirdly, based on the phase shift and attenuation of each layer, the apparent resistivity of each layer can be inverted out.

After those three steps, the depth of the boundary, the apparent conductivities of both layers, and the logging curves are ready. The only unknown is the distance from the drilling bit to the boundary. Then follow the flowchart shown in Fig. 10.55, by iterating the forward modeling, the optimized boundary distance can be inverted out. Because the boundary inversion in last step is supposed to be finished downhole, the fast forward modeling is required. The complex image method discussed in this chapter is used to speed up the last step, which is the boundary distance inversion.

10.4.4 Bolzano bisection method

The bisection method is a root-finding method that repeatedly bisects an interval and then selects a subinterval in which a root must lie for further processing.

For a real variable \( x \), where \( f \) is a continuous function \( f(x) = 0 \) defined on an interval \([a, b]\) and \( f(a)f(b) < 0 \). Then, \( f(x) \) has at least one root in \([a, b]\). The procedure of the bisection is shown below.

Firstly, let \([a, b] = [a_1, b_1]\), denote the middle point of \([a, b]\) as \( p_1 \),

\[
p_1 = \frac{a_1 + b_1}{2} \tag{10.29}
\]

Give a threshold of length (TOL) (small enough). Plug \( p_1 \) back into the equation. If \(|f(p_1)| < TOL\), then \( p_1 \) is the approximate root of the equation \( f(x) = 0 \). If \(|f(p_1)| > TOL\), we will search the root in the interval \([a_1, p_1]\) or \([p_1, b_1]\).

Secondly, if \( f(p_1)f(b_1) > 0 \), the root will be in the interval \([a_1, p_1]\). Else, the root will be in the interval \([p_1, b_1]\). Then the searching region is reduced by half. Repeat the previous steps, the approximate root with acceptable error will be found.

10.4.5 Simulation results

The parameters of logging tool used in following cases:

Frequency = 2 MHz, spacing = 36.375 in., and the dipping angle is 90 degrees.

10.4.5.1 Sensitivity of depth

Sensitivity of depth is a parameter defined by \( \Delta H/\Delta d \), which represents the variation speed of \( H \) field along with varying of depth. Higher sensitivity of depth contributes higher convergence speed of boundary distance inversion. It is an important parameter to choose the component used into boundary distance inversion.

One two-layer model was used to test the inversion process. As shown in Fig. 10.57, the two-layer model has one boundary at \( z = 0 \) ft. The resistivities of the
two layers are 1 and 10 ohm-m, respectively. The right-hand side of Fig. 10.57 shows the simulation results of $H_{zx}$.

Fig. 10.58 shows the depth sensitivity of the real part and amplitude of cross component, respectively. Because the real part and imaginary part of $H_{zx}$ almost follow the same trend, there is not much difference shown in Fig. 10.58. From 0 to 4 ft, the absolute value of sensitivity is all larger than 0. This indicates that, in this range of depth, both the real part and the amplitude of the cross component can be used to inverse the boundary distance. Replot the depth sensitivity in Fig. 10.59, which only shows the depth from 4 to 10 ft. From this figure, it is easy to see that, the depth sensitivity of the amplitude of cross component decreases as the depth increases. But the absolute value of amplitude sensitivity is always larger than zero. However the sensitivity of the real part of cross component moves closer to zero when the depth is larger than 6 ft. That means, with a depth larger than 6 ft, the real part of the cross component is nonsensitive to the boundary. With a depth from 6 to 10 ft, the amplitude of cross component gives better performance.

10.4.5.2 $R_1 = 1$ ohm-m, $R_2 = 10$ ohm-m

Based on the processing flow shown in Fig. 10.56, the last step is to calculate the distance from the drilling bit to the boundary. In this processing, the apparent resistivity of the formation and the depth of the boundary are already known. The distance is the only unknown. For each distance, there will be a received $H_{zx}$ corresponding.
Then based on this model, by combining the received signal, the distance away from the boundary can be calculated. Generally, it can be calculated by solving one unknown equation. The problem is that this equation is of high order. The unknown cannot be solved explicitly. In this part, bisection method is used to calculate the distance.

Figure 10.58 Depth sensitivity of $H_{x}$ (0–10 ft).

Figure 10.59 Depth sensitivity of $H_{x}$ (4–10 ft).
Assume the unknown distance is \( d \). We thus have a high-order equation,

\[
f(d) = V_r
\]

where \( V_r \) is the measurement of received \( H_{zx} \). Rewrite the equation as

\[
f(d) - V_r = 0
\]  

Then the problem is to find the root of Eq. (10.31). Here, the \( H_{zx} \) measurement is obtained from the analytical full solution. The inversion process is running the forward modeling with complex image theory. Because we already know the position of the boundary, in this case, the boundary is at \( z = 0 \); and in the most case, the logging tool is working in the high-resistivity side, in this case, the high-resistivity side is located in the area \( z > 0 \). In this part, only lower half-space was tested. In the most cases, the tool is working in the high-resistive layer and approaching to boundary.

1. Distance inversion from the real part of the \( H_{zx} \)

Table 10.2 shows the inversion results, when the logging points are located at 5.0, 4.0, 3.0, and 2.0 ft away from the boundary. The real parts of the \( H_{zx} \) are all within the detectable range. The results show that when the distance from the drilling bit to the boundary is within 4 ft, the inversion method can find the distance accurately. However, when the distance between the drilling bit and the boundary is close to 5 ft or higher, the inversion process gives higher error. Fig. 10.60 shows the real and imaginary parts of the cross component \( H_{zx} \) in the range from 4 ft to 10 ft. It’s clear that, starting from 5 ft, the real part of \( H_{zx} \) is not monotonic. There exist two roots of Eq. (47). This is why the error becomes larger around this depth.

2. Distance inversion from the amplitude of the \( H_{zx} \)

To solve the problems appearing in case (1), the amplitude of \( H_{zx} \) is used in the distance inversion. Fig. 10.61 gives the amplitude of the cross component \( H_{zx} \). It shows that the amplitude of \( H_{zx} \) has a single value in each side of the boundary. The curve is monotonic. Fig. 10.62 is the zoom in figure of the amplitude in the range from 4 to 10 ft. It is obvious that the curve is monotonic and is always larger than zero. Then, the boundary distance is inversed based on the amplitude of the cross component \( H_{zx} \).

| Table 10.2 Distance inversion table \((H_{zx}\_real, R_1:R_2 = 1:10)\) |
|----------------|----------------|-----------|----------------|--------|
| Distance (ft) | \( H_{zx}\_real \_abs \) | Voltage (nV) | Inversion results (ft) | Error (%) |
| 5.0           | 0.0006          | 192       | 4.1414          | 17.17  |
| 4.0           | 0.0007          | 224       | 4.0518          | 1.30   |
| 3.0           | 0.0031          | 992       | 3.0444          | 1.48   |
| 2.0           | 0.0111          | 3552      | 2.0489          | 2.45   |
Table 10.3 shows the inversion results based on the amplitude of $H_{zx}$. Compared with the results in Table 10.2, the inversion method based on amplitude of $H_{zx}$ is faster and more accurate. The algorithm can even handle the case when the logging point is 10 ft away from the boundary and keeps the relative error within 1%.

Figure 10.60 Zoom in cross component $H_{zx}$ of tool response.

Figure 10.61 Amplitude of the cross component $H_{zx}$.

Table 10.3 shows the inversion results based on the amplitude of $H_{zx}$. Compared with the results in Table 10.2, the inversion method based on amplitude of $H_{zx}$ is faster and more accurate. The algorithm can even handle the case when the logging point is 10 ft away from the boundary and keeps the relative error within 1%.
Fig. 10.63 shows the relative error of the inversion results in Table 10.3, which shows that, when the drilling bit is located in the range 4 to 10 ft away from the boundary, the relative error of the boundary distance inversion is within 1%. When the drilling bit is close to the boundary, within the area 2 ft away from the boundary, the relative error is larger, but still within 3%. When the drilling bit moves to the area 1 ft away from the boundary, the boundary distance inversion is not as accurate. The relative error goes up to 10%. That is because the complex image theory does not work well around boundary. For the two-layer formation, shown in Fig. 10.57, considering the minimum detectable voltage 100 nV, for the transmitter antenna with $3.2 \times 10^5$ A-m, the detectable distance is about 6 ft. Neglect the limitation of
minimum detectable voltage, the inversion process can work even when the logging point is 10 ft far away to the boundary.

**10.4.5.3 \( R_1 = 1 \text{ ohm-m}, R_2 = 100 \text{ ohm-m} \)**

Similar as Section 10.4.5.1, retest the inversion method in the case when the lower layer is 100 ohm-m and the upper layer is 1 ohm-m. The boundary is still at \( z = 0 \). Fig. 10.64 shows the formation model and the simulation results.
Table 10.4 shows the comparison between the inverted distance and actual distance. The results show that the inversion results are pretty close to the actual distance. Except the logging point at 1 ft away from the boundary, which is very close to the boundary, in the distance range from 2 ft to 10 ft, the relative error stays within 2% as is shown in Fig. 10.65.

Replot the relative error curves of the two cases in Fig. 10.66. It shows that when the conductivity contrast of the two layers is larger, the relative error of the inversion results is smaller.

10.4.6 Simulation results with noise added
The behavior of the inversion method in the presence of noise and error is also evaluated. To simulate the noise, an array of random number between −1 and +1 was
generated using a white-noise generator. This array was scaled to 1–10% of the minimum detectable voltage 100 nV. Convert the voltage to the $H$ field. The array was scaled to 1–10% of 3.125e-04. Use the scaled array as a noise. Add this noise to the data simulated from the analytical full solution as a measured data. Then plug this data into the inversion processing. By iterating the forward modeling developed based on the complex image theory, calculate the distance away from the boundary.

### 10.4.6.1 $R_1 = 1 \text{ ohm-m}$, $R_2 = 10 \text{ ohm-m}$

Considering the effect of noise, reprocess the two-layer model in Fig. 10.57. Table 10.5 shows the cross component with 1% noise and the inversion results.
generated using the noised data. The inversion results show that, less than 1% noise, the inversion method still gives reliable result. The relative error between the inversed distance and the accurate distance stays within 3%.

Table 10.6 shows processing results when the noise is increased to 5%. In this case, because the added noise is at the same order of the ideal data far from the boundary, the noise causes higher error to the inversion results. The relative errors of the logging points within 6 ft away from boundary still remain within 3%. For logging points away from boundary for more than 6 ft, the relative error can be as high as 20%.

Fig. 10.67 shows the curves of relative error when different percentages of noise are added to the ideal data. It is easy to see that when the added noise is increasing, the relative error is larger. The noise effects more in the area relatively further away from boundary than the area close to the boundary. That is because, in noise study,
the noise level is fixed, but for a fixed level of transmitter power, the received signal reduces a lot as the distance from the boundary is enlarged. The noise has more effect in the area further away from the boundary.

10.4.6.2 $R_1 = 1 \text{ ohm-m}, R_2 = 100 \text{ ohm-m}$

When the two-layer model shown in Fig. 10.64, where the conductivity of low medium is 100 ohm-m, compared with the two-layer model in Fig. 10.57, where the conductivity of low medium is 10 ohm-m, the received cross component has larger amplitude at the relative far area. For example, at the observation point 10 ft away from the boundary, in the case with 100 ohm-m lower medium, the amplitude of the cross component is 8.9276e-05. In the case with 10 ohm-m lower medium, the amplitude of the cross component is only 7.7888e-06 which is one magnitude lower. That means, in the same noisy environment, the logging tool has better performance in the case with larger conductivity contrast. This conclusion agrees with the results shown in the Section 10.3.4.

Since in the high-conductivity contrast formation, the cross component is stronger. The amount of noise in this formation is started to be added from 5%.

Table 10.7 shows the ideal data of the cross component, data with 5% noise of the cross component, inversion distance generated from the noised data as measurements and the relative errors between the inversed distance and real positions of the logging points.

Fig. 10.68 shows the curves of relative error when the added noise is increased to 10%, 20%, and 50%. It shows that, when the noise is increased to 20%, in the most area, the relative error of the inversion distance still remains within 5%. When the noise is increased to 50%, there is huge error for the testing logging points 6 ft or further away from boundary. Within 6 ft, the relative errors are always less than 5%.

<table>
<thead>
<tr>
<th>Distance (ft)</th>
<th>$H_{zx}$ (abs)</th>
<th>Voltage (nV)</th>
<th>$H_{zx\text{-noise}}$ (5%)</th>
<th>Inversion results (ft)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>8.9276e-05</td>
<td>28.57</td>
<td>8.5212e-05</td>
<td>9.8007</td>
<td>1.99</td>
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<tr>
<td>9.0</td>
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<td>44.20</td>
<td>1.4027e-04</td>
<td>9.1521</td>
<td>1.69</td>
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<tr>
<td>8.0</td>
<td>2.2105e-04</td>
<td>70.74</td>
<td>2.2331e-04</td>
<td>8.0140</td>
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<td>7.0</td>
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<td>118.09</td>
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<td>6.9739</td>
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<tr>
<td>6.0</td>
<td>6.4986e-04</td>
<td>207.96</td>
<td>6.5107e-04</td>
<td>5.9904</td>
<td>0.16</td>
</tr>
<tr>
<td>5.0</td>
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<td>392.29</td>
<td>1.2238e-03</td>
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</table>
10.5 CONCLUSION

Image theory, as a method used to simplify the inhomogeneous media, can be applied in geosteering to speed up the simulation. The advantage of this theory is the simplicity in formulation and fast in computation.

The complex image approximation method was tested at 2 MHz and 400 kHz, respectively. Compared with the full solution results, the complex image method has very good agreement at both frequencies. Error only occurs near boundary. However, in the application of geosteering, the error is acceptable. It works better at higher frequencies than lower frequencies. This is because when the frequency is higher, the skin depth of the formation is shorter, which is closer to a perfect conductor, and therefore the image theory is more accurate.

The accuracy of the complex image theory also depends on the conductivities of both layers. When the conductivity difference between the upper layer and the lower layer increases, the error decreases. The absolute error and relative error are collected at different observation points. The error is larger when the drilling bit is closer to the boundaries. For the 2 MHz tool, when the logging point is more than 2 ft away from the boundary, the relative error is less than 10%. For the 400 kHz tool, this distance is increased to 3 ft. Compared with the full solution method, the complex image approximation method can significantly speed up the simulation. In the testing with 1000 iteration and 600,000 logging points in total, the complex image method is more than 100 times faster than the full solution. This difference in efficiency is also enlarged along with the increase of the logging point. This method can be used in real-time data inversion of the distance to boundary computation in a geosteering system.
Effects of frequency and spacing are investigated. For one-turn antenna, with area 2.5 in.² and excited by 200 mA, the general detectable distance in high-resistive layer is about 5 ft. When tool is working at 400 kHz, longer spacing gives larger detectable distance. The simulation shows that when the spacing is 55 in., the detectable distance is about 7 ft.

Inversion process is given in last part. Two-layer model with boundary at $z = 0$ is tested. Boundary distance is inversed based on the amplitude curve of the cross component $H_{zx}$. The inversion results show that the inversion code works well in the distance range from 2 ft to 10 ft. The relative error is kept in 2%. By comparing the relative error from different formation combination, it can be concluded that larger conductivity contrast of the formation contributes more accurate inversion results.

The effect of noise was discussed in inversion processing. A random white noise with amplitude 100 nV, scaled from 1% to 50%, was added into the analytical full solution data to test the antinoise capacity. The relative errors of inversion results generated using ideal data with different amount noise added in two different formations are calculated and plotted. The results show that the proposed method is more robust in formations with higher conductivity contrast. Compared with the area relatively further away from the boundary, the relative error can be kept in a lower range in the area within 6 ft away from the boundary.

REFERENCES


