Analysis of Torques in Single-Side Linear Induction Motor With Transverse Asymmetry for Linear Metro
Gang Lv, Zhiming Liu, and Shouguang Sun

Abstract—Due to the magnetic distortion and nonuniform force density of the single-side linear induction motor (SLIM) caused by the lateral movement, edge, and end effects, three types of torques, i.e., the side, normal, and horizontal torques, are produced. This paper investigates these special torques of the SLIM for metro and the variations of these torques when the primary is displaced sideways from a symmetrical position. It is of prime importance to take longitudinal end effects, edge effects, overhang of the primary windings, and lateral displacement into consideration when the air-gap flux density distributions and special torques are calculated via a 3-D analytical method. The air-gap flux density distributions along longitudinal and transverse directions, as well as the force density and torques with the different lateral displacements, are comprehensively investigated when the motor works in the motoring operation. The analytical results are experimentally validated by measurements on a reciprocating power-fed test rig. It shows that the horizontal and side torques, which increase the instability of the suspension system and the antirolling bar of bogie, increase almost linearly with the increase of the lateral displacement. Normal torque decreases with the increasing lateral displacement.

Index Terms—Edge effects, horizontal torque, linear induction motor, longitudinal end effects, normal torque, side torque, three-dimensional analysis, thrust, vertical force.

I. INTRODUCTION

THE SINGLE-SIDE linear induction motor (SLIM) has advantages for its thin rectangular body and nonadhesion drive when it is used in the linear metro system. There are more than 20 lines and 400 km commercial linear motors all over the world, such as the linear metro in Japan, the sky train in Canada, the Guangzhou subway lines 4–6, the Beijing airport rapid transport line in China, etc.

In linear metro system, the floor height above the rail is reduced by using the SLIM; hence, the cross-sectional area and construction cost of tunnels can be greatly reduced by about 40% and 20% than those of the conventional tunnels, respectively [1]. On the other hand, the linear metro car can travel on 60 m turning radius rails, while the limit is around 200 m for railcars driven by rotary induction motor. In addition, the nonadhesion drive system can make the car travel on grades as steep as around 8% while the maximum is only around 3.5% for rotary motor railcars. Thus, the maintenance on brake blocks can be reduced significantly. All of these contribute to an improvement of the railcars performance, but with a reduction of construction costs.

The SLIM and bogie are shown in Fig. 1, as well as the secondary side is installed in the center of the rails and along the whole track. The primary side is also called “linear motor,” which consists of iron cores and coppery coils and is hanged under bogie. The secondary side, which is also called as “reaction plate,” consists of magnetically conductive parts, such as iron plate, and an electrically conductive part, such as aluminum or copper plate.

For the magnetic distortion of the SLIM caused by the lateral movement, the end and edge effects, three types of torques, i.e., the side, normal, and horizontal torques, are produced. The calculation of thrust, vertical, and lateral force density is very important to obtain these torques and is investigated in a few papers by finite element method (FEM) or 3-D mathematical model.

The 1-D and 2-D mathematical models are investigated in many papers. In [2], a simple equation for the end effect factor which modifies the airgap electromotive force is obtained, and a simple equivalent circuit incorporating the end effect factor is established. In [3], a per-phase equivalent circuit of a linear induction motor with sheet secondary is developed. In [4], a simple mathematical model of SLIM is developed to consider the end effects. In [5], the state space-vector dynamic model of the linear induction motor by taking the dynamic end effects into consideration is established. In [6], the rotary-motor model is modified to account for the end effect and is used to predict output thrust, vertical forces, and couples. In [7] and [8], an improved equivalent circuit model of SLIM is presented to obtain the thrust. Analytical equations are derived in [10] for end effect braking force.
efficiency, power factor, and output thrust. A hybrid finite-element-boundary element method is adopted in [11] for the analysis of linear induction machines. In [12] and [13], thrust and vertical force are obtained by 2-D mathematical model. However, the lateral force, the side, and horizontal torque cannot be obtained by above methods with assumption of the symmetry.

The 3-D FEM or mathematical model is investigated in a few papers. In [14], by assuming that the deflection is within the overhang of the secondary, the thrust, vertical, and lateral forces of the double-side linear induction motor (DLIM) are calculated using 3-D analytical model. However, the secondary of the DLIM is only electrically conductive plate and simpler than composite secondary of SLIM. Only the thrust, vertical, and lateral forces of DLIM are investigated, but the side, normal, and horizontal torques are not obtained. In addition, the DLIM is seldom used in the metro. In [15], the coupling effect of DLIM is studied using a 3-D finite-element method and the forces are calculated. However, transient analysis of the 3-D full modeling is difficult for the insufficient computer memory and solving time. Nevertheless, the side, normal, and horizontal torques and the influence of the lateral displacement on the performance of SLIM have not yet gone into the study. Only in [16], normal torque caused by the dolphin effect is presented and the experimental results are given.

In [17], the levitation force and thrust in a current-controlled SLIM which propels electrodynamically levitated vehicles are discussed and calculated by 2-D mathematical model. However, lateral force and transverse asymmetry are not considered, and the side, normal, and horizontal torques cannot be obtained. In [18], two 1-D mathematical models, i.e., the longitudinal and transverse direction models, are presented for the thrust and lateral force when the conductor secondary of DLIM is displaced sideways from a symmetrical position. However, the modulation of the flux wave due to slotting, the longitudinal end effects and the overhang of primary windings are negligible, as well as the overhang, which is important to obtain the lateral force. In addition, the DLIM so far is seldom used in the metro.

The same as [18], the transverse edge effect and thrust of DLIM are presented in [19]. In [20], the coupling effect of a double-stator linear induction motor is studied with the 3-D FEM and only thrust is obtained. In [21] and [22], an equivalent circuit method used to obtain the thrust and influence of design parameters on linear induction motor end effect is presented in [23]. In [24] and [25], 2-D mathematical model is built, and the thrust and vertical forces are calculated for magnetic levitation vehicles.

In this paper, the side, normal, and horizontal torques of SLIM for the metro are analyzed by the 3-D analytical method when the primary is displaced laterally sideways from the symmetrical position. The variations of torques with different lateral displacements are predicted, and the redistribution of the thrust, vertical, and lateral forces along the primary is investigated. The end effects and overhang of the primary windings, which are important to obtain the torques and force distribution but usually neglected, are taken into account. In order to validate the analyses, experiments are carried out with a reciprocal power-fed test rig as well.

II. TORQUES OF SLIM WITH LATERAL DISPLACEMENT

In this section, torques of SLIM with lateral displacements are introduced. The coordinates are defined as in Fig. 2, $x$ is the traveling direction of the magnetic field and called the longitudinal, $y$ is the direction of lamination of the iron core and called the transverse, and $z$ is the normal direction of the pole face and called the vertical.

In Fig. 2(a), the overhang of secondary $c_2$ is generally adopted to reduce edge effects. While the lateral displacement is $\Delta y$ in Fig. 2(b), the two hangouts change from $c_2$ to $c_2 - \Delta y$ and $c_2 + \Delta y$ which are called the narrower and wider hangouts, respectively. Side torque which is caused by the lateral movement has a great influence on the lateral stability of the railcar. The vertical force distribution along $y$-axis is distorted and asymmetrical for the lateral displacement, then the side torque occurred, as shown in Fig. 2(c).
where \( d_s \) and \( d \) are the skin depth and secondary conductor thickness, respectively. Equivalent primary surface currents \( j_1 = (j_{1x}, j_{1y}, 0) \) (A/m) [14] in the directions of \( x \) and \( y \)-axes are investigated to consider the end effects, edge effects, lateral displacements, modulation of the flux wave due to slotting and overhang of the windings.

With the air-gap flux density, which can be represented by the equivalent primary surface currents, the thrust, vertical, and lateral force distributions can be derived by using Maxwell’s stress tensor. Finally, three special toques are obtained.

Components of air gap flux density are given by

\[
B_x = \mu_0 j_{1y} \tag{1}
\]

\[
B_y = -\mu_0 j_{1x} \tag{2}
\]

\[
B_z = -j\mu_0 \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{\tau_{1n} k_{n}^2}{\pi} \sin \frac{\pi}{h_{2k}} \left[ y + \left( \frac{h_2}{2} - \Delta y \right) \right]
\cdot \left[ -J_{1f,nk} H_{f,nk} e^{j(\pi/\tau_{1n}) (v_{1n} t + x)} 
\pm J_{1b,nk} H_{b,nk} e^{j(\pi/\tau_{1n}) (v_{1n} t + x)} \right] \tag{3}
\]

where “+” and “−” are taken when the number of poles is even and odd, respectively. \( \tau_{1n} = \tau/n \) is the equivalent pole pitch for \( n \)th component in the longitudinal direction. \( \tau \) is the fundamental pole pitch. \( v_{1n} = 2\tau_{1n} f_1 \) is the moving velocity of the \( n \)th harmonic component. \( f_1 \) is the inverter’s output frequency. \( J_{1f,nk} \) and \( J_{1b,nk} \) are the peak amplitude of the \( n \)th and \( k \)th forward and backward components of primary surface current, respectively. \( h_{2k} = h_2 / k \) is the equivalent half-wavelength of the transverse \( k \)th harmonic component.

The stator iron has infinite permeability to flux passing through. In the back iron of the secondary, the permeability is assumed to be constant. The conductivity of the secondary conductor is corrected for skin effect by

\[
K_{ss} = \frac{d_s \sinh (2d/d_s) + \sin (2d/d_s)}{d_s \cosh (2d/d_s) - \cos (2d/d_s)}
\]

III. Force Distributions and Torques of SLIM

In actual SLIM, longitudinal length of primary is limited and there are longitudinal end effects at the entry and exit ends, then the primary magnetomotive (MMF) is distorted in the longitudinal directions. When the primary moves sideways from a symmetrical position, the amplitude and center of the normal torque will be changed.

The induced currents in secondary and the changed center are shown in Fig. 4 when the primary is displaced sideways from a symmetrical position. Horizontal torque is caused by the lateral movement \( \Delta y \) and movement of the thrust center. It appears to rotate primary in \( xoy \). This feature is called pitch asymmetry.

In Fig. 3(a), distribution forces of the suspension system become uneven for the longitudinal and edge effects, and then normal torque is produced as shown in Fig. 3(b). When the primary is displaced laterally sideways from the symmetrical position, the amplitude and center of the normal torque will be changed.

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The stator iron has infinite permeability to flux passing through. In the back iron of the secondary, the permeability is assumed to be constant. The conductivity of the secondary conductor is corrected for skin effect by

\[
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\cdot \left[ -J_{1f,nk} H_{f,nk} e^{j(\pi/\tau_{1n}) (v_{1n} t + x)} 
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\cdot \left[ -J_{1f,nk} H_{f,nk} e^{j(\pi/\tau_{1n}) (v_{1n} t + x)} 
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The stator iron has infinite permeability to flux passing through. In the back iron of the secondary, the permeability is assumed to be constant. The conductivity of the secondary conductor is corrected for skin effect by

\[
K_{ss} = \frac{d \sinh (2d/d_s) + \sin (2d/d_s)}{d \cosh (2d/d_s) - \cos (2d/d_s)}
\]
where $s_{fn}$ and $s_{bn}$ are the slips of the forward and backward components of the nth harmonic, and $g$ is the air gap.

The vertical force densities along the $y$- and $x$-directions, which is shown in Fig. 2(c) and Fig. 3(b), respectively, are calculated by flux density using Maxwell’s stress tensor

$$f_z(y) = \int_{-L/2}^{L/2} \frac{1}{2\mu_0} \mathop{Re} \left( B_x B_y^* - B_y B_x^* - B_z B_x^* \right) dx$$

and trust destiny shown in Fig. 4(b) can be obtained

$$f_x(y) = \int_{-L/2}^{L/2} \frac{1}{2\mu_0} \mathop{Re} \left( B_z B_y^* - B_y B_z^* - B_x B_z^* \right) dx. \quad (10)$$

and trust destiny shown in Fig. 4(b) can be obtained

$$f_x(y) = \int_{-L/2}^{L/2} \frac{1}{2\mu_0} \mathop{Re} \left( B_z B_y^* - B_y B_z^* - B_x B_z^* \right) dx. \quad (11)$$

Not only the amplitudes but also the centers of forces change when the primary is laterally away from the symmetrical position. The center variations produce special torques of SLIM.

From (11), the side torque shown in Fig. 2(c) can be obtained

$$T_{yz} = \int_{-(h_z/2 - \Delta y)}^{h_z/2 + \Delta y} \int_{-L/2}^{L/2} y \cdot f_z(y) dy dx$$

$$= \int_{-(h_z/2 - \Delta y)}^{h_z/2 + \Delta y} \int_{-L/2}^{L/2} \frac{1}{2\mu_0} \mathop{Re} \left( B_z B_x^* - B_y B_z^* - B_x B_z^* \right) dx dy. \quad (12)$$

As shown in Fig. 2, the side torque can be observed due to nonuniform transverse vertical force density with lateral displacement, and increase the lateral instability of the railcar.

From (12), the normal torque shown in Fig. 3(b) can be presented by

$$T_{xz} = \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} x \cdot f_x(x) dx dy$$

$$= \int_{-(h_z/2 - \Delta y)}^{h_z/2 + \Delta y} \int_{-L/2}^{L/2} \frac{1}{2\mu_0} \mathop{Re} \left( B_z B_x^* - B_y B_z^* - B_x B_z^* \right) dx dy. \quad (13)$$

As shown in Fig. 3, the normal torque along the longitudinal direction can be observed due to nonuniform transverse vertical force density with lateral displacement, and increase the lateral instability of the railcar.

From (13), the horizontal torque shown in Fig. 4 can be obtained

$$T_{xy} = \int_{-(h_z/2 - \Delta y)}^{h_z/2 + \Delta y} \int_{-L/2}^{L/2} y \cdot f_x(y) dx dy$$

$$= \int_{-(h_z/2 - \Delta y)}^{h_z/2 + \Delta y} \int_{-L/2}^{L/2} \frac{1}{2\mu_0} \mathop{Re} \left( B_z B_x^* \right) dx dy. \quad (14)$$

Due to the longitudinal end effect, the distribution of vertical force destiny along the longitudinal direction, as shown in Fig. 3. The center of the vertical force destiny varies with the variations of the longitudinal end effect. Thus, normal torque can be observed. It produces the “bow wave” effect, which lead to that the trailing end is lower than the leading end, and makes the loads of the front and back wheels uneven.

From (13), the horizontal torque shown in Fig. 4 can be obtained

$$T_{yz} = (F_{z3} + F_{z4}) \cdot \frac{W_1}{2} - (F_{z1} + F_{z2}) \cdot \frac{W_1}{2}. \quad (15)$$

Horizontal torque is caused by the variation of the thrust destiny along the transverse direction and increases the lateral asymmetry.

IV. SIMULATION AND EXPERIMENTS

To validate the 3-D analytical method developed in the previous section, the analytically predicted results are experimentally verified by the measurements on a SLIM installed on a reciprocating power-fed test rig, as shown in Fig. 5(a). Table I lists the main parameters of the prototype SLIM. In order to measure the torques, several pressure and tension sensors are installed on the bogie, as shown in Fig. 5(b) and (c). The specifications of the pressure and tension sensors are shown in Table II.

The side torque can be obtained by

$$T_{yz} = (F_{z3} + F_{z4}) \cdot \frac{W_1}{2} - (F_{z1} + F_{z2}) \cdot \frac{W_1}{2}. \quad (17)$$
TABLE I
MAIN PARAMETERS OF PROTOTYPE SLIM

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>190 A</td>
<td>RMS phase current</td>
</tr>
<tr>
<td>$f$</td>
<td>40 Hz</td>
<td>Frequency</td>
</tr>
<tr>
<td>$p$</td>
<td>6</td>
<td>Number of poles</td>
</tr>
<tr>
<td>$\tau$</td>
<td>33 cm</td>
<td>Pole pitch</td>
</tr>
<tr>
<td>$m$</td>
<td>3</td>
<td>Number of phase</td>
</tr>
<tr>
<td>$q$</td>
<td>2</td>
<td>Number of slots/phase/pole</td>
</tr>
<tr>
<td>$h_p$</td>
<td>18.92 cm</td>
<td>Thickness of primary core</td>
</tr>
<tr>
<td>$\beta$</td>
<td>79 cm</td>
<td>Coil pitch</td>
</tr>
<tr>
<td>$N_p$</td>
<td>90</td>
<td>Number of turns per phase</td>
</tr>
<tr>
<td>$c_1$</td>
<td>4.71 cm</td>
<td>Overhang of primary winding</td>
</tr>
<tr>
<td>$g$</td>
<td>1 cm</td>
<td>Air gap</td>
</tr>
<tr>
<td>$d$</td>
<td>0.5 cm</td>
<td>Secondary conductor thickness</td>
</tr>
<tr>
<td>$h_c$</td>
<td>28.34 cm</td>
<td>Secondary conductor width</td>
</tr>
<tr>
<td>$c_2$</td>
<td>4.71 cm</td>
<td>Overhang</td>
</tr>
<tr>
<td>$L$</td>
<td>219 cm</td>
<td>Primary length</td>
</tr>
<tr>
<td>$V$</td>
<td>1100 V</td>
<td>Input primary voltage</td>
</tr>
</tbody>
</table>

TABLE II
MAIN PARAMETERS OF THE PRESSURE AND TENSION SENSORS

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Capacity</th>
<th>Rated output</th>
<th>Excitation</th>
<th>Nonlinearity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{z1}$</td>
<td>-20 to 20 kN</td>
<td>-5 to 5 V</td>
<td>10 V</td>
<td>3% F.S.</td>
</tr>
<tr>
<td>$F_{z2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{z3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{z4}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{x1}$</td>
<td>-10 to 10 kN</td>
<td>-5 to 5 V</td>
<td>10 V</td>
<td>3% F.S.</td>
</tr>
<tr>
<td>$F_{x2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where $F_{z1}, F_{z2}, F_{z3}$ and $F_{z4}$ can be obtained by the sensors $F_{x1}, F_{x2}, F_{z3},$ and $F_{z4}$.

The normal torque can be obtained by

$$T_{xz} = (F_{z2} + F_{z4}) \cdot \frac{L_1}{2} - (F_{z1} + F_{z3}) \cdot \frac{L_1}{2}$$

and the horizontal torque can be obtained by

$$T_{xy} = F_{x2} \cdot \frac{W_1}{2} - F_{x1} \cdot \frac{W_1}{2}$$

where $F_{x1}$ and $F_{x2}$ can be obtained by the sensors $F_{x1}$ and $F_{x2}$.

The simulation and experiments are organized as follows. The flux density $B_z$ is given in Section A. The analytically forces density is predicted in Section B. Finally, in Section C, torques in experiments are completed.

A. Flux Density

The analytically predicted flux density $B_z$ is given in Fig. 6, in which the lateral displacement is 3 cm, the input primary voltage is 1100 V, the frequency is 35 Hz, and the slip is 0.25. The slip of 0.25 is a typical working point of the SLIM for metro, and is higher than rotary motors.

The 3-D air-gap flux distribution with $\omega t = \pi/2$ and $\Delta y = 3$ cm is shown in Fig. 6. It can be seen clearly that the flux density magnitude increases in the narrower hangout side.

Fig. 6. Air gap flux density $B_z$ when $V = 1100$ V, $\Delta y = 3$ cm, $f = 35$ Hz, and $s = 0.25$.

Fig. 7. Thrust distribution and center with different lateral displacements when $V = 1100$ V, $f = 35$ Hz, and $s = 0.25$.

Fig. 8. Lateral force distribution and center with different lateral displacement when $V = 1100$ V, $f = 35$ Hz, and $s = 0.25$. 
Fig. 9. Vertical force distribution and center with different lateral displacements when $V = 1100\, V$, $f = 35\, \text{Hz}$, and $s = 0.25$.

Fig. 10. Vertical force distribution in the longitudinal direction with different lateral displacements when $V = 1100\, V$, $f = 35\, \text{Hz}$, and $s = 0.25$.

B. Forces Density and Torques

Transverses distribution of the thrust is presented in Fig. 7, with different lateral displacements of 0, 1, 2, and 3 cm. More serious edge effects can be observed as the increase of the lateral displacement in Fig. 6, and $y$-axis component of the induced currents in the secondary, which will produce the thrust, decreases at the same time. Hence, the peak amplitudes of the thrust decrease with the increasing lateral displacement. The thrust vector was changed from the center of the primary to the wider overhang which is expressed by $c_2 + \Delta y$ in Fig. 2(b), and the horizontal torque is produced. This will cause destabilization of SLIM and increases the pitch asymmetry [11].

The lateral force includes attractive and repulsive components. The attractive force, which is restoring, acts between the primary and the back iron of the secondary along the $y$-axis, and the repulsive force, which is decentralizing, acts between the primary and the aluminum plate of the secondary. Hence, the net lateral force changes with ratio of the attractive and repulsive forces. If the attractive component force accounts for a large proportion, the net lateral force is restoring, vice versa, decentralizing.

In Fig. 8, the density of the net lateral force in the region of the narrower overhang is much lower than the one in the wider region, and lateral attractive force in the wider side is larger than the repulsive one in the narrower side.

Hence, the net lateral force is attractive and pulls the primary toward the symmetry position in the given conditions. This restoring force is helpful to improve the guide performance of the bogie for SLIM.

The influence of lateral displacement on the vertical force density distributions along transverse direction is shown in Fig. 9. In the middle region of the secondary, the vertical force is attractive. However, near the two overhangs of the secondary, the vertical force is repulsive.

The vertical force density near the wider overhang is larger than the one near the narrower overhang. This difference becomes larger when the lateral displacement increases. The vertical force vector changes from the center to the wider overhang.
of the secondary, which makes SLIM unstable in the y-axis direction, as shown in Fig. 9. Hence, the side torque of SLIM, which is expressed by $T_{xz}$ in Fig. 3(a), is produced by the nonuniform vertical force distribution, and must be considered for the antirolling bar of the bogie.

In Fig. 10, vertical force density in the longitudinal direction decreases with the increasing lateral displacement when $V=1100 \text{ V}$, $f=35 \text{ Hz}$, and $s=0.25$. Due to longitudinal end effect, vertical force at the entry end is very low and the center only shifts along the $z$-axis for same conditions. The normal torque is produced by the uneven vertical force distribution and has important influence on the suspension system.

C. Experiments

In order to validate analytical analyses, experiments are carried out with SLIM operated by the vector control inverter. The analytically predicted and measured side, normal, and horizontal torques are compared in Fig. 11, when input primary rated voltage is $1100 \text{ V}$, slip is 0.25, as well as frequencies are 5, 15, 25, and 35 Hz. The actual measurement values by sensors are listed in Table III and the sensor installation is shown in Fig. 5(c) and (d).

It can be seen that these two sets of results agree very well for various lateral displacements. Hence, the developed analytical 3-D method is able to predict the magnetic field distribution, the torques accurately with the end and edge effects as well as influence of lateral displacement being considered.

The normal torques with different frequencies are shown in Fig. 11(a) when the lateral displacements are 0, 1, 2, and 3 cm. The normal torques increase with the increasing frequencies, since the displacement of the center in the longitudinal direction will increase with the increasing velocity. When the frequency is same, the normal torques decrease with increasing lateral displacement for the decreasing the normal force. It must be considered for the design of the suspension bar of the bogie.

In Fig. 11(b), the horizontal and side torques increase with the variation of the lateral displacement from 0 to 3 cm, and have important influence on the suspension system and antirolling bar of the bogie, respectively. Horizontal torques decrease with the increasing frequencies due to the decrease of the normal force.

C. Experiments

In order to validate analytical analyses, experiments are carried out with SLIM operated by the vector control inverter.

The analytically predicted and measured side, normal, and horizontal torques are compared in Fig. 11, when input primary rated voltage is $1100 \text{ V}$, slip is 0.25, as well as frequencies are 5, 15, 25, and 35 Hz. The actual measurement values by sensors are listed in Table III and the sensor installation is shown in Fig. 5(c) and (d).

It can be seen that these two sets of results agree very well for various lateral displacements. Hence, the developed analytical 3-D method is able to predict the magnetic field distribution, the torques accurately with the end and edge effects as well as influence of lateral displacement being considered.

The normal torques with different frequencies are shown in Fig. 11(a) when the lateral displacements are 0, 1, 2, and 3 cm.
the maximum side torque occurs is 15 Hz, and side torques decrease with the increase of the frequency between 15 and 35 Hz.

The thrust, vertical force, and currents with different displacements are shown in Fig. 12. The thrust and vertical force decrease with the increasing lateral displacement, especially in constant power region. The currents, which are produced by the primary voltage, decrease with the increasing lateral displacement, since the leakage inductance and secondary resistance increase with the increasing lateral displacement and edge effects.

V. CONCLUSION

This paper develops a 3-D analytical method based on the space harmonic to predict special torques and force density in SLIM when the primary is displaced sideways from the symmetrical position. The longitudinal end effects, edge effects, and overhang of the primary winding are comprehensively considered. The analytical analyses are validated by the measurement on a SLIM prototype. Based on the analysis and investigation, it can be concluded as following to aid SLIM designs.

1) The peak amplitudes of the thrust density decrease with the increasing lateral displacement. The thrust vector was changed from the center of the primary to the wider overhang. Horizontal torques are caused by the effect of the redistribution of the thrust along the primary and increase almost linearly with the increasing lateral displacement.

2) The vertical force density near the wider overhang is larger than the one near the narrower overhang when the primary is displaced sideways from the symmetrical position. This difference becomes larger when the lateral displacement increases. The side torques are caused by the transversal asymmetry of the vertical force density along the primary and increase almost linearly with the increasing lateral displacement. It has important influence on the antirolling bar of the bogie.

3) Due to longitudinal end effect, vertical force at the entry end is very low and the center shifts along the longitudinal direction. The normal torque is produced by this uneven vertical force distribution and decreases with the increasing lateral displacement. It must be considered for the design of the suspension bar of the bogie.

REFERENCES


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