Automatic high-precision measurements of the location and width of Kikuchi bands in electron backscatter diffraction patterns

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Summary
The demands for reliability and precision of crystal orientation data obtained through automatic analysis of electron backscattering patterns (EBSPs) in the SEM result in similar demands on the quality of the band position data which is provided by an image analysis procedure.

This paper describes a new image processing procedure which is capable of providing accurate measurements of the location and width of typically 10–15 bands in digitized EBSPs of average quality. The new procedure is based on the Hough transform (HT) for line detection, and employs a special backmapping technique for generating two simplified HTs which separately focus on bright and dark lines in the images. A coordinated search for peaks in the two HTs leads to precise estimates of both the position and the width of bands in the patterns.

A visual evaluation of the data produced by the new procedure shows that it performs significantly better than the conventional procedure with regard to both reliability and precision. Additionally, the measured band width data are fairly precise and can be used for obtaining a more robust and reliable indexing of the bands. Finally, the computational costs of the new procedure are smaller than for the conventional procedure.

Introduction
Electron backscattering patterns (EBSPs), also known as backscatter Kikuchi patterns (BKPs), have been applied for several years for measuring crystal orientations with high spatial resolution in the SEM (Venables & Harland, 1973; Dingley, 1981; Randle, 1992). In recent decades, several research groups have worked to further the EBSP technique and make it the important analytical tool it is today for researchers studying microstructure and its relation to texture. Some of the most important steps in the development of the technique were the introduction of online analysis of EBSPs (Dingley et al., 1987) and the development of user-friendly computer-aided indexing software (Schmidt et al., 1991).

A significant breakthrough for the EBSP technique took place in the early 1990s with the development of systems which were capable of performing a fully automated analysis of the patterns, thereby enabling the collection of huge amounts of crystal orientation data (Wright & Adams, 1992; Kunze et al., 1993; Krieger Lassen, 1994). An essential prerequisite for the development of these systems was the design of reliable pattern recognition procedures for detection and localization of Kikuchi bands in digitized EBSPs. Some early, but only moderately successful, attempts at designing such procedures are reported by Juul Jensen & Schmidt (1990) and Wright et al. (1991). In the first report on a fully functioning automatic system presented by Wright & Adams (1992), a line detection scheme know as the Burns algorithm (Burns et al., 1986) was employed. This algorithm applies local gradient information for grouping neighbouring pixels having similar gradient orientation and high-gradient magnitude into ‘line support regions’. A line is fitted to each of these regions and the output of the Burns algorithm is thus a number of straight-line segments of varying lengths, located at the straight edges of the image. When applied to EBSPs, the Burns algorithm attempts to locate the borders of the bands. A disadvantage of the algorithm in connection with its application to EBSPs is that it looks for line segments rather than for global lines extending across the entire image. Also the nontrivial problem of finding parallel pairs of lines corresponding to both sides of a band has to be solved. Consequently, a fairly complicated postprocessing is required if the output of the Burns algorithm is to be applied.
The performance of any hypothesis based on the sampled orientation data result in quite similar requirements to the ideal procedure. When applied as the backbone of an automatic EBSP analysis system, the new procedure should be capable of providing more reliable and precise orientation measurements.

The two most vital demands on a system for automatic crystal orientation measurements from EBSPs are reliability and precision of the collected data. If a significant fraction of the measured orientations is false or erroneous (several degrees away from the true orientation), such data must first be detected, which is not trivial, and secondly be discarded or, in some way, filtered out. In any case, it is obviously of fundamental importance to minimize the fraction of erroneously measured orientations. Another important requirement to the collected data is precision. The reliability and validity of any hypothesis based on the measured orientation data will thus always depend strongly on the accuracy of the individual data points. If the typical uncertainty of the measured orientations is too large, it may eventually be impossible to perform a statistically reasonable verification of some hypothesis based on the sampled orientation data. The demands for reliability and precision of the orientation data result in quite similar requirements to the performance of the procedure which measures the position of the bands in EBSPs. This procedure must be capable of detecting as many true bands (bands which correspond to reflecting crystal planes) as possible, while at the same time detect as few false or erroneous bands as possible. As the number of correctly detected bands increases, the reliability of the orientation measurements will increase, whereas the reverse is true for an increasing number of erroneously detected bands. Note also that an increasing number of correctly identified bands generally leads to a higher precision of the measured orientations (Krieger Lassen, 1999b). In addition to the number of true and false bands detected, the precision of the (true) bands has a strong influence on both the reliability and the precision of the calculated orientations. Finally, the ideal procedure for detection of EBSP bands should be capable of measuring also the width of the bands. When applied in combination with band position data, the measured width of the bands can be exploited by the indexing procedure (since Bragg angles can be calculated from the width and position of the bands) to increase the robustness and reliability of band indexing, and thereby of the estimated orientations. While band width data are normally not strictly required for obtaining a unique indexing of patterns from materials with simple crystal structures (e.g. fcc and bcc metals), it can be of great significance when analysing EBSPs from materials with a more complex crystal structure. The new band detection procedure which is described in this paper seeks to fulfill all of the above-mentioned requirements to the ideal procedure. When applied as the backbone of an automatic EBSP analysis system, the new procedure should be capable of providing more reliable and precise orientation measurements.

The paper starts with a description of the preprocessing steps which are typically required to transform the raw digitized EBSP into an image from which band positions may be extracted. This is followed by a section in which the Hough transform (HT) for line detection is described. Both the basic principles and the implementational aspects of the transform are considered, and a solution to the biasing problem is described. Those who are already familiar with automatic EBSP analysis and the HT may skip these two sections and proceed to the following section which describes the principle behind, and the calculation of, the maximum and minimum backmapped Hough transform. The following section describes a procedure which is capable of extracting the position and width of EBSP bands from the maximum and minimum HTs. This procedure is based partly on the results of an analysis of the peak structure which is generated in the HT by a simple model of an EBSP band. Following this section is a presentation of the results of applying the new procedure to the detection of 13 bands in five EBSPs. The performance of the new procedure is qualitatively evaluated and compared with that of the traditional HT-based procedure. In the final section, a general discussion of the band detection problem is presented along with an overview of the work that has been done so far in this area. The new procedure is viewed in this perspective and its distinctive advantages are highlighted.

Preprocessing of digitized EBSP images

EBSPs are normally recorded on a fluorescent phosphor screen which converts the signal of electrons emitted from the sample into a very weak light signal. This signal is then recorded by a highly sensitive video camera (a camera of the silicon intensified target type is used in our set-up) mounted outside the SEM vacuum chamber. The video signal passes through a digital signal processor which is capable of performing temporal averaging of a variable number of
video frames, and in many cases also various types of background correction. Finally, the video signal is digitized into a number of pixels by a framegrabber which typically has a digitization depth of 8 bits (i.e. 256 levels of intensity). Figure 1 shows a raw, unprocessed EBSP image as it appears after temporal averaging of 64 video frames and digitization into 480×512 pixels. The first step of preprocessing consists then of a correction for the background signal. This signal is normally found experimentally by operating the microscope in scanning mode at a low magnification, so that the ‘average’ pattern from a large area of the sample is collected. Correction for background is done either by subtraction or division of the background signal, and since this signal is reasonably even, the difference in performance between the two methods is quite small. However, the subtraction method will produce a corrected image which shows a smaller contrast in areas of low intensity, and therefore the division method is generally preferred. In the present EBSP system, background correction is performed by division.

The next steps of preprocessing consist in the present case of an elimination of irrelevant pixels (pixels carrying no pattern information) and a correction for simple linear distortions which are introduced by the digitization process (illustrated in Fig. 1 by the elliptical appearance of the circular phosphor screen). After these corrections the resolution of the image is reduced from 400×400 to just 100×100 pixels. If done properly (see e.g. Krieger Lassen, 1996a) this process will not lead to any significant loss of relevant pattern information, and will greatly increase the speed of the subsequent processing. The reduced image obtained at this stage of the preprocessing procedure will normally show some variations in the background intensity level. These variations which are caused by the topography of the sample will inevitably have a negative effect on the performance of the Hough transform and should therefore be eliminated as effectively as possible. A standard method for removing a slowly varying (low-frequency) background signal is to subtract a low-pass filtered version of the original image. The calculation of this low-pass filtered image can be achieved efficiently through a simple averaging of pixel values over square regions of size N×N. This type of smoothing can be performed very rapidly but is not ideal (convolution with a Gaussian mask would be better) and has a slight tendency to generate block-like artefacts. To minimize the influence of these artefacts, it is better to apply averaging several times with a small value of N than just once with a large N; we apply averaging three times with N=5. Figure 2 shows the end result of the preprocessing procedure outlined above. Note that the image area of interest – in our set-up the circular region of the phosphor screen – has been marked by setting all pixels outside this region to zero.

The Hough transform for line detection

The Hough transform (HT) (Hough, 1962; Duda & Hart, 1972) has become one of the most popular tools in digital image processing, where it is used mainly for solving the problem of detecting simple geometrical objects such as lines, circles and ellipses. Over the last decades numerous researchers have contributed to the development of the HT and other closely related techniques, and comprehensive

Fig. 1. Electron backscattering pattern (EBSP) after temporal averaging, digitized to 480×512 pixels.

Fig. 2. Final EBSP image after complete preprocessing procedure (100×100 pixels).
The HT can be described as a parametric transformation that transforms a set of points into a discrete parameter space in which each point represents a unique instance of the sought-after shape. In the present case, as well as in most cases where the HT is being applied, the set of points are the pixels of a digital image, and the shape of interest is the 2D line. In order to perform the HT, a parameterization of the shape must first be introduced. The most popular choice for lines is by far the so-called normal parameterization, which describes the position of a line by its distance $r$ to the origin and the direction of its normal vector $\theta$. The equation of the line in its normal form is

$$\rho = x \cos \theta + y \sin \theta, \theta \in [0, \pi], \rho \in [-R, R].$$

It is customary to restrict $\theta$ to the interval $[0, \pi]$ and then let $\rho$ assume both positive and negative values in the interval $[-R, R]$, where $R$ is the distance from the origin to the corners of the image. If the foot of the perpendicular from the origin to the line, the normal point, is in the upper half of the image then $\rho > 0$, otherwise $\rho \leq 0$. Consider now a digital image consisting of $NR \times NC$ pixels and let $(c_0, r_0) = (NC - 1)/2, (NR - 1)/2$ define the centre of that image. A standard Cartesian coordinate system can then be associated with the digital image so that pixel number $(i, j)$ is given the coordinates $(x_i, y_j) = (c_i - c_0, r_i - r_0)$. The principle of the HT may then be illustrated as in Fig. 4. A pixel with coordinates $(x_i, y_j)$ is mapped to all points in $\rho - \theta$ parameter space that specify a possible line through that pixel. From Eq. (1) it is evident that this set of lines is represented by the sinusoid $x_i \cos \theta + y_j \sin \theta$ in parameter space. As illustrated in Fig. 4, collinear points will map to sinusoid curves that intersect in a common point, and it is evident that the $(\rho, \theta)$ values of that point give the parameters of the line. For the purpose of calculating the HT, the continuous but bounded parameter space $\{(\rho, \theta) \in \mathbb{R}^2 \mid 0 \leq \theta < \pi \land -R \leq \rho \leq R\}$ must be quantized to $P \times \Theta$, where $\rho$ assumes values in the set: $P = \{\rho_0, \rho_1, \ldots, \rho_{N-1}\} = \{\rho_k \in \mathbb{R} \mid \rho_k = -R + (k + 0.5)\Delta \rho \land k \in [0, 1, \ldots, N - 1]\}$ and $\theta$ assumes values in the set $\Theta = \{\theta_l \in \mathbb{R} \mid \theta_l = (l + 0.5)\Delta \theta \land l \in [0, 1, \ldots, M - 1]\}$. The size of the quantization interval $\Delta \rho$ for the $\rho$ parameter is $2R/N$ and for the $\theta$ parameter, $\Delta \theta = \pi/M$. The choice of quantization or resolution of the parameter space is of fundamental importance to the performance of the HT, and this topic has been the subject of many publications (see e.g. Lam et al., 1994). I shall not discuss this in any detail here but will simply state that practical experience with the EBSP images considered here shows that values for $N$ and $M$ in the range $100\text{--}160$ seem appropriate. Throughout this paper I shall use $N = M = 120$, and since $NR = NC = 100$ and $R = (2 \times 49.5^2)^{1/2}$, the sizes of the quantization intervals are $\Delta \rho = 1.167$ pixels and $\Delta \theta = 1.500\degree$. An accumulator array $HT(\rho_k, \theta_l)$ is now defined on $P \times \Theta$ and used for storing the result of the HT. The procedure for calculating the HT is given below in pseudo-code:

```plaintext
Set HT(\rho_0, \theta_1) to zero
for i = 0 to NR \times NC - 1 do
    begin
        for l = 0 to M - 1 do
            begin
                \rho = x_i \cos \theta_l + y_j \sin \theta_l
                k = [(\rho + R)/\Delta \rho]
                HT(\rho_k, \theta_l) = HT(\rho_k, \theta_l) + I(x_i, y_j)
            end for
        end for
    end for
```

In the code above, $\lceil z \rceil$ refers to the largest integer strictly smaller than $z$ and $I(x_i, y_j)$ to the value of the pixel with coordinates $(x_i, y_j)$. Expressed in words, the procedure for calculating the HT proceeds as follows: for each pixel with coordinates $(x_i, y_j)$ and value $I(x_i, y_j)$, the pixel value is added to all cells $HT(\rho_k, \theta_l)$ in parameter space along the sinusoid $x_i \cos \theta + y_j \sin \theta$. Note that for practical reasons, it is customary to sample the $\theta$ parameter ($\theta_l$) and then perform a quantization of the $\rho$ parameter ($\rho_k$). To maximize the speed of calculation, it is useful to precalculate all values of $k$ (or rather just $k$) obtained for each pixel $(x_i, y_j)$ and for each value of $l$.

The HT is most often viewed as an evidence gathering procedure. Each pixel ‘votes’ for all the lines on which it could be located. The votes are summed in the accumulator array $HT(\rho_k, \theta_l)$, and the final totals indicate the relative

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Fig. 3. Illustration of the normal parameterization of lines: parameter $\rho$ expresses the distance of the line from the origin, and $\theta$ describes the direction of the normal vector pointing from the origin to the line.
probabilities of the different lines. From a slightly different perspective, however, the HT can also be viewed as a discrete version of the Radon transform (Radon, 1917; Deans, 1980). In this perspective, the focus is on the total counts obtained in the cells of the accumulator. Each cell \((\rho_k, \theta_l)\) in the array defines a line \(\rho = x \cos \theta + y \sin \theta\) in the image plane, and the votes accumulated in HT \((\rho_k, \theta_l)\) are simply a sum of pixel values \(I(x, y)\) along this line. HT\((\rho_k, \theta_l)\) may thus be viewed as the integrated intensity across the image along the line defined by \((\rho_k, \theta_l)\), i.e. essentially a discrete Radon transform. It must be noted, however, that an actual implementation of a discrete Radon transform (as integrated intensities along lines) generally would not produce exactly the same result as the HT.

As described above, the sum of votes \(\Sigma I(x, y)\) assigned to a particular cell \((\rho_k, \theta_l)\) in the discrete parameter space originates from pixels along the line defined by \((\rho_k, \theta_l)\). Because of the finite size of the input image, the number of pixels along a given line will depend on its position and thus on its parameters \((\rho_k, \theta_l)\). This implies that the HT of a uniform image, \(I(x, y) = K\) for all \((x, y)\), is nonuniform and exhibits ‘artificial’ maxima for the cells which correspond to the longest lines within the image frame. This effect of the finite image size or retina is commonly referred to as biasing, and in practice implies that lines close to the borders of the image have a smaller probability of being detected than lines located nearer to the centre. Since the location of short lines, made up of only a few pixels, is likely to be less well defined than the location of longer lines, the biasing effect is regarded as desirable in many applications of the HT. In the case of EBSP images, however, the primary objective is to detect the thinner and brighter bands of the patterns, since these correspond to the crystal planes which are considered in the subsequent indexing procedure. The brighter bands of the EBSPs can be located anywhere within the image retina, and for EBSP images it is therefore normally advantageous to eliminate the biasing effect. This can be achieved in a simple manner by first calculating the HT of an artificial image in which each pixel inside the retina (the image area of interest) assumes the value 1. Each cell \((\rho_k, \theta_l)\) in the HT of this image, denoted HTB\((\rho_k, \theta_l)\), will now contain the number of pixels which contributed votes to that particular cell. Then, by division of the total counts in each cell of the HT by the corresponding count in HTB,

\[
HT(\rho_k, \theta_l) = \begin{cases} 
\frac{HT(\rho_k, \theta_l)}{HTB(\rho_k, \theta_l)}, & \text{if } HTB(\rho_k, \theta_l) \geq \text{MIN} \\
0, & \text{if } HTB(\rho_k, \theta_l) < \text{MIN},
\end{cases}
\]

Fig. 4. Principle of the Hough transform and backmapping. (a) Three collinear points in image space. (b) Each of the three collinear points in image space is mapped to a sinusoid curve in parameter space. (c) Each of the three collinear points in image space is mapped to the common intersection point in parameter space of the three (dotted) sinusoid curves.
the HT is normalized and the biasing effect eliminated. The variable MIN in Eq. (2) defines the minimum number of pixels which should contribute votes to a cell in the HT. Parts of the HT to which only a few pixels contribute votes (corresponding to short lines) are thus ignored so that the risk of detecting ‘noisy’ or erroneous lines near the borders of the image is reduced. The results presented in this paper are all based on MIN = 20 pixels. Note that after normalization, each cell of the HT contains the average of pixel values \( <I(x,y)> \) along the corresponding line instead of simply the sum \( \Sigma I(x,y) \). Figure 5 shows the normalized HT of the preprocessed EBSP image in Fig. 2.

The maximum and minimum backmapped Hough transform

In the HT for line detection each pixel \((x_i,y_i)\) is mapped to, and votes for, all points in \( \rho - \theta \) parameter space that specify a possible line through that pixel. As described above, these points are defined by the sinusoid curve \( \rho = x_i \cos \theta + y_i \sin \theta \). However, with the exception of the rare pixels which are located at the intersection points of lines, a pixel can only be part of one line. Therefore, ideally, each pixel should vote for just one line: the line which it is most likely to be a part of. The problem is now how to find the parameters of this unique line, and thus establish a one-to-one mapping from pixels in the image plane to points in the discrete parameter space. The parameters of the line to which a pixel with coordinates \((x_i,y_i)\) shall be mapped will be written \((\rho_{li}, \theta_{li})\), and it is evident that this point must be located somewhere along the sinusoid \( \rho = x_i \cos \theta + y_i \sin \theta \) in the discrete parameter space.

If we assume that the HT has been calculated, a natural choice for \((\rho_{li}, \theta_{li})\) would be the point \((\rho_{li}^*, \theta_{li}^*)\) along the sinusoid at which HT reaches its maximum value, since this point represents the line with the largest number of (normalized) votes. If this process of finding maximum points along sinusoids is repeated for each pixel of the input image, a one-to-one mapping from image space to parameter space is established and a new HT can be accumulated. Since each pixel now votes for just one cell in parameter space, calculation of this new HT is much faster than the standard HT (which, however, has to be calculated prior to the new HT). It also evident that the resulting accumulator array will be much more sparse than after the standard HT.

This idea of linking pixels in image space to specific points in parameter space was introduced by Gerig & Klein (1986) who applied the technique to circle detection in binary images. A more formal description of the technique can be found in Gerig (1987). Gerig and Klein refer to the technique as backtransformation or backmapping since it establishes a map from parameter space and back to image space. We shall refer to the new HT obtained after backmapping as the maximum backmapped HT and denote it BHT*. Figure 4(c) illustrates the result of backmapping. For each of the three collinear pixels, the maximum point along the corresponding sinusoid curves will be at their common point of intersection. The three pixels are thus mapped to the same point in parameter space, and the backmapped HT will therefore assume the value zero everywhere but at this common point. The procedure for establishing the mapping from pixels in image space to points in parameter space is presented in pseudo-code below:

```plaintext
for i = 0 to NR x NC - 1 do
    MAX = 0
    for l = 0 to M - 1 do
        begin
            \( \rho = x_i \cos \theta_l + y_i \sin \theta_l \)
            \( k = [(\rho + R)/\Delta \rho] \)
            if HT(\(\rho_l, \theta_l\)) > MAX do
                begin
                    MAX = HT(\(\rho_l, \theta_l\))
                    MAP(x_i, y_i) = (\(\rho_l, \theta_l\))
                end if
        end if
    end for
end for
```

Just like for the calculation of the HT, the speed of the procedure outlined above can be substantially increased if
through division by HTB as described by Eq. (2). Just like for the HT, this normalization can therefore be normalized if all lines, regardless of their location, are precalculated and stored in an array. Following the procedure outlined below:

Set $\text{BHT}^+(\rho_k, \theta_l)$ to zero

for $i = 0$ to $\text{NR} \times \text{NC} - 1$

begin

$(\rho_{k,i}^+, \theta_{l,i}^+) = \text{MAP}(x_i, y_i)$

$\text{BHT}^+(\rho_{k,i}^+, \theta_{l,i}^+) = \text{BHT}^+(\rho_{k,i}^+, \theta_{l,i}^+) + I(x_i, y_i)$

end for

The maximum backmapped HT exhibits the same type of biasing effect as the standard HT, and $\text{BHT}^+$ should therefore be normalized if all lines, regardless of their location, should be given the same probability of detection. Just like for the HT, this normalization can be accomplished through division by HTB as described by Eq. (2).

Figure 6 shows the normalized $\text{BHT}^+$ corresponding to the HT shown in Fig. 5. It is evident that the backmapping technique results in a significant simplification of the pattern of votes accumulated in parameter space: the majority of cells in the $\text{BHT}^+$ receive no votes at all, and only the more prominent peaks of the HT are also observed in $\text{BHT}^+$. An effect of the backmapping technique is thus a restriction of vote accumulation to areas of parameter space where the HT has significant local peaks. One may view $\text{BHT}^+$ as obtained from the HT by a vote focusing procedure: pixels located on the same bright line are forced to vote for just one line and are then likely to ‘agree’ on voting for the same point in parameter space. As described above, the total count accumulated in a specific cell $(\rho_k, \theta_l)$ of the normalized HT is simply the average of pixel values along the corresponding line in the image. The interpretation of the counts accumulated in the cells of the normalized $\text{BHT}^+$ is somewhat more complicated. Let $L(\rho_k, \theta_l)$ denote the set of pixels which vote for the cell $(\rho_k, \theta_l)$ during accumulation of the standard HT. Obviously, this set defines a digital line through the image. Let $L_1(\rho_k, \theta_l)$ be the subset of $L(\rho_k, \theta_l)$ which contains the pixels voting for the cell $(\rho_k, \theta_l)$ after backmapping. The number of elements (pixels) in the sets $L(\rho_k, \theta_l)$ and $L_1(\rho_k, \theta_l)$ will now be denoted $N(\rho_k, \theta_l)$ and $N_1(\rho_k, \theta_l)$, respectively. Note that $N(\rho_k, \theta_l) = \text{HTB}(\rho_k, \theta_l)$ and $N_1(\rho_k, \theta_l) = N(\rho_k, \theta_l)$. The counts accumulated in the cells of the normalized HT may now be written

$$\text{HT}(\rho_k, \theta_l) = \frac{\sum_{(x,y) \in L(\rho_k, \theta_l)} I(x,y)}{N(\rho_k, \theta_l)} , \quad (3)$$

which again shows that HT$(\rho_k, \theta_l)$ is the average value of pixels within the set $L(\rho_k, \theta_l)$. The result for the maximum backmapped HT can be written

$$\text{BHT}^+(\rho_k, \theta_l) = \frac{\sum_{(x,y) \in L_1(\rho_k, \theta_l)} I(x,y)}{N(\rho_k, \theta_l)}$$

$$= \frac{N_1(\rho_k, \theta_l)}{N(\rho_k, \theta_l)} \cdot \frac{\sum_{(x,y) \in L(\rho_k, \theta_l)} I(x,y)}{N(\rho_k, \theta_l)} . \quad (4)$$

Similar to the standard HT, the second factor of Eq. (4) is the average value of the pixels which vote for the line $(\rho_k, \theta_l)$. Note that this average alone is not a very good measure of evidence for the line $(\rho_k, \theta_l)$ if $N_1$ is small. To compensate for this, the first factor of Eq. (4), $N_1/N$, measures the fraction of pixels which vote for the line $(N_1)$ relative to the number of pixels which potentially could have voted for it ($N$). This factor could be said to measure the degree of vote focusing and assumes values in the interval $[0,1]$, where the value 1 corresponds to the situation where all pixels are mapped to the same point.

It has been an implicit assumption in the sections above that the aim was the detection of bright lines, and that the value of the pixels therefore could serve as a reasonable measure of evidence for the case that the pixels are part of a line. In this paper, the aim is not only the detection of the bright central part of the Kikuchi bands, but also the detection of the thinner dark lines observed just beyond the borders of the bands. One obvious way of detecting dark lines in a digital image is to apply the HT to an image in which all pixel values have first been inverted, $I'(x,y) = 255 - I(x,y)$. The normalized HT of the inverted
HT, denoted BHT mapping has been established, the minimum backmapped replaced by a search for minima. Once the minimum the search for maximum points along sinusoids is now the one presented above, with the only difference being that procedure for establishing this mapping is equivalent to space so that each pixel votes for just one line. The mapping from pixels in image space to points in parameter space, which shows that the result is simply an inversion of the (normalized) HT of the original image. As a consequence, the biasing effect is eliminated through a simple normalization which favours the detection of long lines. As before, this minimum backmapped HT is affected by a biasing effect observed in the patterns as two dark lines (strictly speaking, hyperbolas) on each side of the bright central region of the bands. The width of these dark border lines appears to be independent of the reflecting crystal plane, and is thus observed to be larger for the thinner bands (e.g. the {111} and {200} reflections in fcc crystals). In all cases, however, the width is small and of the order of 1–3 pixels in the preprocessed patterns.

Consider now the simple model of a backscattered Kikuchi band shown in Fig. 8. The bright central part of the band is assumed to have the shape of a rectangle of width \( W^* \), and the intensity within the area is assumed to be constant. Surrounding the central part of the band are two dark, parallel rectangles or strips of width \( W^* \). Again, the intensity reaches a minimum value which can be observed in the patterns as two dark lines (strictly speaking, hyperbolas) on each side of the bright central region of the bands. The width of these dark border lines appears to be independent of the reflecting crystal plane, and is thus observed to be larger for the thinner bands (e.g. the {111} and {200} reflections in fcc crystals). In all cases, however, the width is small and of the order of 1–3 pixels in the preprocessed patterns.

Fig. 7. Minimum backmapped Hough transform of the image in Fig. 2 after normalization. Corresponds to the standard Hough transform shown in Fig. 5.

Detection of EBSP bands from the maximum and minimum backmapped Hough transform

From an image analysis perspective, the backscattered Kikuchi bands of EBSPs can be recognized by an increased intensity between two almost straight, parallel and fairly sharp band borders (see Fig. 2). This increase in intensity, however, is relatively small (a few per cent over the average intensity level) and furthermore depends on the corresponding reflecting crystal plane. At the band borders, which are located one Bragg angle away from the trace of the corresponding crystal plane, the intensity drops off over a very short distance. These band edges are only about one pixel wide in the reduced EBSP images with a resolution of 100 × 100 pixels. Just beyond the two band borders/edges, the intensity reaches a minimum value which can be observed in the patterns as two dark lines (strictly speaking, hyperbolas) on each side of the bright central region of the bands. The width of these dark border lines appears to be dependent on the reflecting crystal plane, and is thus observed to be larger for the thinner bands (e.g. the {111} and {200} reflections in fcc crystals). In all cases, however, the width is small and of the order of 1–3 pixels in the preprocessed patterns.

In the code above, \((\rho_i^-, \theta_i^-)\) refers to that point along the sinusoid curve at which HT assumes its minimum value. Note that in this case, the value accumulated for each pixel is not the pixel value \(I(x_i, y_i)\), but the inverted value, \(255 - I(x_i, y_i)\). Obviously, this value should now be used as the natural measure of evidence for the case that a pixel is part of a dark line. Like both the HT and BHT\(^+\), the minimum backmapped HT is affected by a biasing effect which favours the detection of long lines. As before, this biasing effect is eliminated through a simple normalization as described by Eq. (2). Figure 7 shows the minimum backmapped HT corresponding to the HT shown in Fig. 5.

\[
\text{HT}^*(\rho, \theta) = \left( \sum_{(x,y) \in L(\rho, \theta)} [255 - I(x, y)] \right) / N(\rho, \theta) = 255 - \text{HT}(\rho, \theta),
\]

(5)

which shows that the result is simply an inversion of the (normalized) HT of the original image. As a consequence, calculation of HT\(^*\) is redundant and dark lines may be detected as local minima in the HT instead of as local maxima in HT\(^*\). Furthermore, it is evident that the principles of backmapping presented above can easily be applied also in the case where focus is on the detection of dark lines. Again the idea is to establish a one-to-one mapping from pixels in image space to points in parameter space so that each pixel votes for just one line. The procedure for establishing this mapping is equivalent to the one presented above, with the only difference being that the search for maximum points along sinusoids is now replaced by a search for minima. Once the minimum mapping has been established, the minimum backmapped HT, denoted BHT\(^-\), can be calculated as illustrated by the following piece of pseudo-code:

Set BHT\(^-\)\((\rho_i^-, \theta_i^-)\) to zero
for \(i = 0 \) to \(NR \times NC - 1\) do
  begin
    \((\rho_i^-, \theta_i^-) = \text{MAP}(x_i, y_i)\)
    BHT\(^-\)\((\rho_i^-, \theta_i^-) = \text{BHT}(\rho_i^-, \theta_i^-) + [255 - I(x_i, y_i)]\)
  end for

the intensity within the bordering rectangles is assumed to be constant. If the standard HT of this idealized EBSP band is calculated, the parameter space will after accumulation contain one maximum and two neighbouring minima as sketched in Fig. 9. The peak generated in parameter space by a line of finite width has a specific shape which is often compared to that of a butterfly (Leavers & Boyce, 1987). The body of the butterfly has a shape like the one sketched in Fig. 9, and its wings (not shown in Fig. 9) are extending along the \( v \)-axis in both directions away from the body. The central part of the peak, the butterfly body, corresponds to the collection of lines which are located completely within the corresponding rectangle and thus intersect the rectangle at both of its two short borders. Recalling that the votes accumulated in parameter space may be interpreted as the values of line integrals, it is evident that the value of the HT within this central peak region must be virtually constant.

Furthermore, it is easily shown that the size \( S_r \) of the region in the \( \rho \)-direction, and the size \( S_\theta \) in the \( \theta \)-direction are given by

\[
S_\rho = W, \quad S_\theta = 2 \arctan \left( \frac{W}{L(\rho_0, \theta_0)} \right),
\]

where \( W \) denotes the width of the rectangle, and \( L(\rho_0, \theta_0) \) denotes the length of the line with parameters \((\rho_0, \theta_0)\) located in the centre of the rectangle. Note that to a good approximation \( L(\rho_0, \theta_0) \approx \text{HTB}(\rho_0, \theta_0) \). The approximate size of the peak region measured in cells of the discretized parameter space can be found as \( n_\rho = \lfloor S_\rho / \Delta \rho \rfloor + 1 \) and \( n_\theta = \lfloor S_\theta / \Delta \theta \rfloor + 1 \), where \( \lfloor z \rfloor \) refers to the largest integer strictly smaller than \( z \).

The parameters of the centre line of the idealized EBSP band shown in Fig. 8 can be found as the centre point of the bright peak shown in Fig. 9. As described above, the counts accumulated in the HT within the central bright peak region will be practically constant, and the maximum point within this area is therefore unlikely to correspond exactly to the centre line. One solution to this problem is to convolve the HT with a butterfly mask which has the shape and size of the expected peak. Thereafter, the centre line may simply be found as the maximum point of the convoluted HT. This is the approach which was first suggested by Krieger Lassen et al. (1992) and which is now widely employed. A drawback of the method is that the size of the butterfly mask needs to be fixed so that it only performs optimally for bands of a certain width. Furthermore, the peaks generated by actual EBSP bands are often quite far from having the shape of the idealized butterfly, which obviously degrades the performance of the convolution method. As an alternative, we propose here to use the two neighbouring minima shown in Fig. 9 for estimating the position of the centre line. The primary motivation for this is that the size of the central peak region of the black peaks (minima) is much smaller than for the corresponding bright peaks, and they can therefore be located with greater ease and a higher degree of precision. In this new procedure we shall, however, also apply the maximum found within the bright central peak region for two important purposes: (1) for indicating the possible presence of a true peak (generated by a genuine Kikuchi band), and (2) as the starting point for a search for the two neighbouring

\begin{align*}
S_\rho^+ & = (S_\rho^+, S_\theta^+) \\
S_\rho^- & = (S_\rho^-, S_\theta^-)
\end{align*}
For the value of $W$, use the maximum expected width of two neighbouring peaks $W_{\text{max}}^+$ and for the centre point $(p_0, \theta_0)$, use $(p_1^+, \theta_1^+)$. The calculated measures for the peak size are denoted $S_{p^+}$ and $S_{q^+}$.

3. Estimate the size of the nearby dark peaks using Eq. (6). For the value of $W$, use the maximum expected width of the dark border lines $W_{\text{max}}^-$, and for the centre point $(p_0, \theta_0)$, use $(p_2^-, \theta_2^-)$. The calculated measures for the peak size are denoted $S_{p^-}$ and $S_{q^-}$.

4. Find the maximum point $(p_{k1}^+, \theta_{11}^+)$ in BHT in the region of parameter space defined by $\theta \in [\theta_1^+ - \Delta \theta, \theta_1^+ + \Delta \theta]$ and $\rho \in [\rho_k^+ - \Delta \rho, \rho_k^+]$. Find also the maximum point $(p_{k2}^-, \theta_{12}^-)$ in BHT in the region of parameter space defined by $\theta \in [\theta_1^- - \Delta \theta, \theta_1^- + \Delta \theta]$ and $\rho \in [\rho_k^- + \Delta \rho, \rho_k^-]$. The size of the regions above is defined by $\Delta \theta = (S_{p^+} + S_{q^+})/2$ and $\Delta \rho = S_{p^-} + S_{q^-}$.

5. If the value of BHT at $(p_{k1}^+, \theta_{11}^+)$ or $(p_{k2}^-, \theta_{12}^-)$ is smaller than MIN then go to step 6. If $|\theta_{11}^- - \theta_{12}^-| > \Delta \theta_{\text{max}}$, then go to step 6. If $p_{k2}^- - p_{k1}^+ > W_{\text{max}}^+ + 2W_{\text{max}}^-$, go to step 6. Go to step 7.

6. Discard the detected peak triplet. Mark the detected maxima, $(p_0^+, \theta_1^+)$ and $(p_{k1}^+, \theta_{11}^+)$ in BHT, and $(p_{k2}^-, \theta_{12}^-)$ in BHT, so that they no longer can be considered as part of a peak triplet. Go to step 1.

7. A new peak triplet has been found. Estimate the parameters $(p_0^+, \theta_0)$ of the centre line using $p_0^+ = (p_{k1}^+ + p_{k2}^-)/2$ and $\theta_0 = (\theta_{11}^+ + \theta_{12}^-)/(\theta_1^+ + \theta_2^-)$, where $w_1 = \text{BHT}^{-}(p_{k1}^+, \theta_{11}^+)$ and $w_2 = \text{BHT}^{-}(p_{k2}^-, \theta_{12}^-)$. Estimate also the width of the band as $W = (p_{k2}^- - p_{k1}^+) - W_{\text{avg}}^-$, where $W_{\text{avg}}^-$ denotes the average width of the dark border lines.

8. Calculate the size of the bright peak from Eq. (6) using the estimates of centre line position $(p_0^+, \theta_0)$ and band width $W$ found in step 7. Mark a rectangular region of parameter space in BHT, $\theta \in [\theta_0 - S_{p0} \theta_0 + S_{\theta0}]$ and $\rho \in [\rho_0 - S_{\rho0} \rho_0 + S_{\rho0}]$.

9. Calculate the size of the two dark peaks from Eq. (6) using $(p_{k1}^+, \theta_{11}^+)$ and $(p_{k2}^-, \theta_{12}^-)$ for the centre point $(p_0^+, \theta_0)$ and $W_{\text{max}}^-$ for $W$. Mark the two corresponding rectangular regions of parameter space in BHT as described under step 8. Go to step 1.

The procedure above terminates when a predefined number of peak triplets have been detected or when the maximum of BHT becomes smaller than MIN. All the results presented in this paper are based on MIN$^+ = \text{MIN}^- = 50$. Larger values of MIN$^+$ and MIN$^-$ (the maximum value is 255) will reduce the number of detected bands, but will also reduce the chance of detecting false bands. For the maximum expected band width $W_{\text{max}}^+$ we are using 14-0 pixels, the maximum expected width of the dark border lines $W_{\text{max}}^-$ is set to 3-0 pixels, and the average width of the border lines $W_{\text{avg}}^-$ is set to 2-0 pixels. The performance of the procedure outlined above is strongly affected by the value of $W_{\text{max}}^+$; smaller values will obviously exclude the broader bands from possible detection, whereas larger values will increase the chance of detecting erroneous bands. The expressions given for $\Delta \theta$ and $\Delta \rho$ (not to be confused with the parameters describing the resolution of parameter space) in step 4 above follows from a simple consideration of the worst-case positions (leading to the largest possible distances in the $\rho$- and $\theta$-directions, respectively) of peaks located within the three regions shown in Fig. 9. Note that the three maxima $(p_{k1}^+, \theta_{11}^+)$ and $(p_{k2}^-, \theta_{12}^-)$ can be expected to be located anywhere within the three corresponding peak regions. The maximum allowed angular distance between the two detected black border lines is determined by the parameter $\Delta \theta_{\text{max}}$; we use 4-0$^\circ$. In general terms, smaller values for $\Delta \theta_{\text{max}}$ will reduce the number of detected bands, but should also decrease the probability of detecting erroneous bands.

A practical difficulty which is often encountered during the peak detection process is the ‘wrap-around’ effect of parameter space at $\theta = 0$ and $\pi$. An important consequence of this effect is that the peak generated by a nearby vertical band will split into two parts: one part at the $\theta = 0$ border, and one at the $\theta = \pi$ border. The difficulty arises when we are considering a region of parameter space which crosses the two borders; this may happen during steps 4, 5, 8 and 9 above. A simple solution to the problem is to initially allow $\theta$ to assume values below 0 and above $\pi$. If we then seek the value of BHT (or HT) at a point $(p_{0}^+, \theta_0)$ with $\theta_0 < 0$, it should be measured at the point $(-p_{0}, \theta_{0} + \pi)$. Similarly, a point $(p_{0}^+, \theta_0)$ with $\theta_0 = \pi$ is mapped to the point $(-p_{0}, \theta_{0} - \pi)$ located inside the range of parameter space. The validity of these results is easily verified using Eq. (1). Experience has shown that the precision of the detected lines may be slightly improved by calculating the ‘centre of

mass in a small neighbourhood around the detected peaks. Let \((\rho_k, \theta_l)\) represent a local maximum in either \(\text{BHT}^+\) or \(\text{BHT}^–\), and let \(N\) define the size of a square region around \((\rho_k, \theta_l)\) in the discrete parameter space. An estimate \((\rho^*_k, \theta^*_l)\) of the position of the peak may then be found from

\[
(\rho^*_k, \theta^*_l) = \frac{\sum_{l=-N/2}^{N/2} \sum_{k=-N/2}^{N/2} (\rho_{k+l}, \theta_{k+l}) H(\rho_{k+l}, \theta_{k+l})}{\sum_{l=-N/2}^{N/2} \sum_{k=-N/2}^{N/2} H(\rho_{k+l}, \theta_{k+l})},
\]

where \(H\) refers to either \(\text{BHT}^+\) or \(\text{BHT}^–\). For the peaks found in both \(\text{BHT}^+\) and \(\text{BHT}^–\) we use \(N = 3\).

The peak triplets which were detected in \(\text{BHT}^+\) and \(\text{BHT}^–\) (shown in Figs. 6 and 7) with the procedure above are marked with small circles in Figs. 10 and 11. For illustration of the relative positions of the peaks found in \(\text{BHT}^+\) and \(\text{BHT}^–\), Fig. 12 shows the location of the peak triplets in the original HT. The peak detection procedure was set to search for 13 peak triplets and terminated when it reached this number.

**Results and evaluation**

The performance of the new procedure is demonstrated on five EBSPs obtained from a sample of partly recrystallized pure copper. The patterns are of varying quality and were chosen so as to constitute a reasonable representation for the set of patterns which can be collected from the given specimen. A qualitative evaluation of the procedure is performed by visually comparing the detected Kikuchi bands with those which are detected by the traditional procedure (Krieger Lassen, 1994, 1996a). The \((\rho, \theta)\)-parameter space is quantized into 120×120 cells for both

![Fig. 10. The white circles illustrate the positions of 13 peaks detected in the maximum backmapped Hough transform of Fig. 6. Each peak is associated with one of the peak pairs shown in Fig. 11.](image)

![Fig. 11. The white circles illustrate the positions of 13 pairs of peaks detected in the minimum backmapped Hough transform of Fig. 7. Each peak pair is associated with one of the peaks shown in Fig. 10.](image)

![Fig. 12. The normalized Hough transform (Fig. 5) overlayed with black circles which illustrate the positions of 13 peak triplets detected in the maximum and minimum backmapped Hough transforms (Figs. 10 and 11, respectively).](image)
procedures so that a fair comparison of the performance of the two procedures can be made. Furthermore, both procedures are set to locate exactly 13 bands in each pattern. This is done to test the performance of the procedures around the limit where the probability of detecting erroneous bands increases rapidly. Note, however, that when these procedures are to be used for reliable measurements of crystal orientations it is hardly recommendable to let them search for more than 7–11 bands. The 13 bands detected by the new procedure in each of the five patterns are shown in Figs. 13(a) to 17(a). The solid black lines show the location of the detected centre lines and the parallel pairs of dotted lines illustrate the estimated band widths. Note that the dotted lines are exactly parallel to the corresponding centre lines and that they are not the lines which correspond to the pairs of peaks detected in BHT¹⁰. Figures 13(b) to 17(b) show the positions of the 13 bands which were detected by the traditional procedure as solid black lines.

A close examination of the positions of the solid lines shown in Figs. 13–17 clearly reveals that the new procedure in general provides significantly more accurate estimates of the positions the Kikuchi bands. The advantage in accuracy of the new procedure is most pronounced for the wider bands and generally seems to increase with increasing band width. It is also observed that the most significant improvement in localization corresponds to changes of the ρ parameter (translation of the line) and to a lesser extent to changes of the θ parameter (rotation of the line). For the thinnest bands, {111} reflections, the difference in precision is very small, and it is difficult to say which one of the procedures provides the best estimates. It may also be concluded that with the present, very limited, collection of patterns, there appears to be no noticeable influence of pattern quality on the advantage of the new procedure. Besides the clear advantage in accuracy, the new procedure is also capable of detecting more correct and fewer erroneous bands in the patterns. Thus, the only nonexistent band which was detected in all of the five patterns by the new procedure is observed in Fig. 17(a): the pattern of the lowest quality. The traditional procedure did, on the other hand, detect one false band in Fig. 13(b), Fig. 14(b) and Fig. 17(b). In addition to these clearly erroneous bands, the traditional procedure detected several bands which are, although near to an observable Kikuchi band, so poorly localized that they are difficult to index, and – if they can be correctly indexed and are used for calculating the crystal orientation – would lead to significant errors in the measured data. There is one such band in each of Figs. 13–16(b), and two in Fig. 17(b). Although it is sometimes difficult to make a precise judgement of whether a detected band is strictly correct or false, the presented results clearly show that the new procedure provides both more precise and reliable measurements of the positions of the EBSP bands.

The pairs of dotted lines in Figs. 13–16(a) illustrate that the new procedure is capable of providing fairly precise estimates of the width of the bands. There appears to be a slight tendency for the procedure to overestimate the width of the thinnest bands which is probably due to the fact that black borders of these bands are wider than for the bands of greater width. One may compensate for this by replacing

the constant value $W_{\text{avg}}$ (used in step 7 of the peak detection procedure presented in the preceding section) by a function $W(d)$ which depends on the distance $d = r_2 - r_1$ between the detected pair of peaks in BHT and assumes decreasing values as the value of $d$ increases. Otherwise, the accuracy of the measured band width data seems to compare well with the data which possibly could have been provided by a human operator. It is also evident that the quality of the data is such that they can provide important extra information for the indexing routines.

However, it is important to realize that such measurements of the band widths will always be approximate, owing to the fact that the black borders of the bands are not really straight lines but hyperbolas.

**Discussion**

The backscattered Kikuchi bands of EBSPs are in most cases fairly easy to recognize by the human eye. Despite this, the task of developing reliable image processing techniques for
automatic detection and localization of these bands is by no means trivial. There are several reasons for this: even after temporal averaging and reduction of the resolution, the patterns have a low signal-to-noise ratio. Furthermore, the contrast of the features of interest, the Kikuchi bands, is very low in proportion to the noisy background signal. Finally, the detection of the bands is severely complicated by the fact that their appearance is strongly varying with respect to both size and intensity. All EBSP bands, however, have some common features which can be used when they are to be recognized by a computerized algorithm: to a good approximation the bands appear as linear strips of slightly increased intensity. Furthermore, just outside the two borders of the strip, a fairly sharp drop in intensity is observed, and two thin and approximately straight dark lines can be observed.

The fact that EBSP bands are characterized by approximately linear features, and that these features are overlaid by a strong noise signal suggests that the Hough transform for line detection may serve as a very useful tool in the

The HT is convolved with a small butterfly mask, and the edges of the bands can then be found as local maxima in the filtered HT. With this procedure it is possible to detect 5–8 bands with high precision in patterns of good quality, but the performance of the procedure was found to degrade rapidly with decreasing pattern quality. A comparison of this procedure with the standard HT- and convolution-based procedure showed that the latter performed significantly better (Krieger Lassen et al., 1992). Instead of aiming at the detection of the band edges, one may instead try to locate the dark and relatively thin lines observed just beyond the borders of the bands. These dark lines are clearly visible for the thinner bands but are more difficult to recognize by the human eye for the bands of greater width. That these dark lines are indeed also characteristic for the wider bands is, however, clearly shown by the HT of the patterns, in which they are seen as pairs of dark peaks on each side of the bright peaks (see Fig. 5). This characteristic of backscattered Kikuchi bands was exploited by Dorkel et al. (1993) who used an EBSP image for demonstrating the performance of a special variant of the HT. While the focus in Dorkel et al. (1993) is on the development of this modified HT, the presented example involving an EBSP image clearly demonstrates the potential of using the dark lines for localizing the bands. In the example presented by Dorkel et al. (1993), seven bands were detected with high accuracy. It should be noted, however, that the applied EBSP image was obtained through digitization of a film exposed inside the SEM vacuum chamber.

A limitation of the procedures presented immediately above is that they utilize either the edges of the bands, or the dark lines observed just beyond the edges for recognizing and localizing the bands. They do not in any way exploit the increase in intensity which is observed between the edges of the bands, and which is a distinct feature of all backscattered Kikuchi bands. The standard HT-based procedure, on the other hand, is only exploiting the bright part of the bands for their detection and not the presence of either the band edges or the dark band borders. An HT-based procedure which seeks to exploit both the presence of the bright band region and the band edges was presented in Krieger Lassen (1996a). Following the detection of a bright maximum in the HT, this procedure tries to locate the centre of the peak by considering the output of a simple 1D edge detector applied to the columns of the HT (i.e. in the \( \rho \)-direction) in a small region around the detected maximum. When this 1D edge detector is applied in the \( \rho \)-direction of the HT, it will produce a high output at points in parameter space which represent the edges of the bands. With reference to Fig. 9, these points are located where the two dark peaks touch the central bright peak. This procedure was found to be capable of providing high-precision measurements of both the location and width of typically 10–12 bands in patterns of high quality, but its performance was also found to degrade quite significantly with decreasing quality of the patterns. The main problem of the procedure appeared to be the presence of spurious maxima

The recent development of fully automated, computerized systems for analysing EBSPs obtained in the SEM has made it possible to rapidly collect huge amounts of local crystal orientation data over large areas on the surface of crystalline samples. While the speed of data collection and thereby the amount of data are of great importance for obtaining a good statistical description of the material under investigation, it is obviously also of fundamental importance that the collected orientation data are both reliable and accurate. It is my belief that application of the presented procedure can result in significant improvements in the quality of crystal orientation data obtained through automated analysis of EBSPs.

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References


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