Abstract. In this paper, we present some results of computer simulations oriented on frequency and noise properties of an active RC notch filter (ARCN filter), which is intended to provide high reference spur suppression in the phase detector output voltage of PLL. In this paper also has been presented the substitution of complicated active ARC structure with the simple passive RLC filter. On the base numerical simulations results it is possible to design good ARCN filters with small phase shift within active PLL bandwidth.

Keywords
PLL, linear model of PLL, active RC notch filter, computer simulation model, spurious signal in PLL.

1. Introduction
Phase-locked loops are one of the basic building blocks in modern electronic systems. PLL are used for frequency synthesis, signal regeneration and in many other applications [1].

Switching noise in the frequency dividers and the phase detector at the reference rate may cause unwanted FM sidebands at the RF output - spurious. The phase detector comparison frequency is generally a multiple of the RF channel spacing for Integer-N PLL (in this presentation \( f_{ref} = 10 \text{ kHz} \)).

Using of an active RC notch filter for suppressing of frequencies in narrow band around a rejection frequency is known and it is often used.

Modern digital three-state phase-frequency detectors used in PLL ICs for frequency synthesis generate short parasitic spikes, which are unwanted in tuning voltage, under steady state condition.

On the other side high demands on suppression of unwanted short spikes at the output of PD is complicated by stability problem of PLL as a feedback system. From this point of view there is a primary need on LPF in PLL circuitry - great attenuation and small phase shift, and this is serious problem. One way to solve it is to design notch type active RC filter structures which exhibits small overall phase shift in active PLL bandwidth.

The Linear PLL frequency synthesizer block model is in Fig.1.

![Fig. 1. Linear model of the one loop frequency synthesizer.](image)

A resultant transfer function of this loop is:

\[
H(p) = \frac{\Phi_{\text{out}}}{\Phi_{\text{ref}}} = N \frac{K_D \frac{K_O}{N} F(p)}{K_D \frac{K_O}{N} F(p) + K_{PLL} F(p)} = \frac{N}{p + K_{PLL} F(p)},
\]

\[
K_{PLL} = \frac{K_D K_O}{N}
\]

where \( K_D \) is phase detector constant, \( K_O \) is the tuning voltage constant of VCO and \( K_{PLL} \) is open loop gain.

This work extends results in [6], [7] in sense of optimal frequency and noise properties of the ARCN filter according to Fig. 2a).

In this paper also has been presented extends results of the substitution of complicated active ARC structure with the simple passive RLC filter [4],[5].

2. Additional Active RC Notch Filter for Reference Spurs Attenuation
ARCN filters are used for additional damping of the unwanted harmonics (1, 2 and 4-th) at the phase detector output voltage.
The transfer function of ARCN filters (see Fig. 2a), with more than one section has high order unsuitable for analytical evaluation. The transfer function of ARCN filter with one section is given by Eq. 3 where \( K \) is voltage gain of output amplifier with zero, or very small output resistance comparable to \( R \), \( m \) is the components value scaling parameter.

Voltage gain constant \( K \) is from interval 0 to 1 for stable operation of ARCN. Numerical analysis indicate, that for stable PLL operation with ARCN filter are needed higher values of \( K \). Additional phase shift in active PLL bandwidth caused by ARCN is lower, so phase margin of PLL will increase [6].

Origin transfer function \( H(p) \) of the linear model of PLL of the 1st order with \( F(p)=1 \) and PLL BW = 1 kHz is in Fig. 3a). Transfer functions of the same PLL with additional triple ARCN filter and with LP equivalent RLC filter are in Fig. 3b,c [4]. From the simulation results could be seen that ARCN filter in PLL effectively suppress unwanted 1st, 2nd and 4th harmonics of the phase detector output.

### 2.1 The Passive RLC Filter Equivalent to ARCN Filter for Low Frequencies

Transient response of the passive RLC low pass filter was fitted with response of the triple ARCN filter by least-mean-square algorithm. In contrast to [4],[5] approximation in this contribution considerate value of DC resistance of ARCN filter.

![Fig. 2. Simulation model of: a) Active RC notch (ARCN) filter for effective suppressing 1st, 2nd and 4th harmonics b) the low pass fitting passive RLC filter for K=0.9. Reference frequency is 10 kHz in this example.](image)

<table>
<thead>
<tr>
<th>IN</th>
<th>OUT</th>
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<tr>
<td>( \text{Rs} ) 11851</td>
<td>( \text{Ls} ) 713.529mH</td>
</tr>
<tr>
<td>( \text{Cs} ) 3.474nF</td>
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\[ F(p) = \frac{V_o}{V_i} = \frac{mR^2C^2p^2 + (2CR - mCR)p + m}{mR^2C^2p^2 + (2CR + mCR - 2mKCR + m^2CR - m^2KCR)p + m} \]  

\[ f_{\text{Notch}} = 1/(2\pi RC) \]

![Fig. 3. Transfer functions of the PLL linear model: a) with \( F(p)=1 \) - PLL of the 1st order, b) with additional triple ARCN filter \( K=0.9 \) c) with simple passive RLC substitution filter \( K=0.9 \).](image)

In Fig. 3 b) and c) we can see good agreement between PLL transfer functions with ARCN filter (with \( K=0.9 \)) and RLC low pass equivalent circuit respectively for low frequencies.

The values of inductance and capacitance of simple passive RLC substitution filter obtained by approximations for different value of amplification \( K \) is in Fig. 4. The practical realization of inductor with these values of
inductance is space-consuming. The main meaning of such RLC filter is of course not realization, but analysis simplification of PLL properties in active PLL band.

3. Frequency and Noise Analysis of the Single ARCN Filter

Typical responses of single ARCN filter in frequency domain for amplification $K=0.9$ and different value of components value scaling parameter, $m=1.5$ to 2.5 are displayed in Fig. 5.

It can be seen that variation of parameter $m$, with respect to typically used value of $m=2$, has considerable influence to attenuation and phase shift of ARCN filter.

The 3D single ARCN filter attenuation plot as function of frequency and components value scaling parameter $m$ can be seen on Fig. 6.

Dependence of ARCN filter maximum attenuation value at notch frequency and phase shift value at $f/f_{\text{Notch}}=10^{-0.1}$ as function of components value scaling parameter $m$ for $K=0.9$ is in Fig. 7.

The attenuation of unwanted harmonics in PLL application don’t need to be maximum. So we can chose somewhat lower attenuation (maximum value is at $m=2$) by selecting $m < 2$ for the better phase margin of the PLL.

Additional phase shift in active PLL bandwidth caused by ARCN is lower, so phase margin of PLL will be higher.
The 3D single ARCN filter attenuation plot as function of components value scaling parameter \( m \) and amplification of output amplifier \( K \) is on Fig. 8.

![Fig. 8. The 3D single ARCN filter attenuation plot as function of components value scaling parameter \( m \) and amplification of output amplifier \( K \).](image)

The noise voltage spectral density \( u_n [V/\sqrt{Hz}] \) of single ARCN filter decrease when the components value scaling parameter \( m \) is chose smaller then optimal value for maximal attenuation ( \( m=2 \) ) as it can be seen in Fig. 9.

![Fig. 9. The ARCN filter noise voltage spectral density \( u_n [V/\sqrt{Hz}] \) versus components value scaling parameter \( m \) for \( K=0.9 \): a) single ARCN filter, b) triple ARCN filter.](image)

4. Conclusions

The modified active RC notch filter ( ARCN ) with components value scaling parameter \( m \) which is proper for investigating of attenuation, phase shift and noise analysis properties in the frequency domain has been presented.

From numerical simulation results we could see that the attenuation of unwanted harmonics of references signal can by chose lower for the better phase margin of the filter and for lower noise voltage spectral density. Based on numerical simulations results which are presented here, it is possible to design good ARCN filter for PLL.

We also have been presented the substitution of complicated active ARC structure with the simple passive RLC filter which is not suitable for practical realization.

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References
