Optimal Trajectory for Swim-out Acoustic Decoy to Countermeasure Torpedo

Yingchun Chen, Huakui Wang, and Ye Zhao

Abstract—The optimal trajectory of a swim-out acoustic decoy defending against an acoustic homing torpedo is studied. First, the dynamic models of the torpedo and submarine, and several trajectory models of the decoy are established. The factors that affect the countermeasure effectiveness are analyzed. Then based on the assessment criterion, genetic algorithm is used to obtain the optimal parameters of the trajectory models under different cases. Simulation results show that the helical and circular models of decoy trajectory are dominated by three other models, i.e., straight line, second turn-angle and snake trajectory.

I. INTRODUCTION

In Anti-Submarine Warfare (ASW), torpedoes are one of the most effective weapons carried by surface ships, aircrafts or submarines to attack submarines. Acoustic decoy is deceitful acoustic countermeasure equipment, mainly used to countermeasure torpedo or sonar. It can not only emulate at a certain degree the characteristics of radiated noise, dynamics and echo reflection of submarines, surface ships and other targets of interest, but also respond to the active pulse signals transmitted by acoustic homing torpedoes. Consequently it can attract the torpedo to track and attack [1]. The objective of the submarine to use decoy is to misguide the torpedo so that the submarine has evaded away from the acoustic search sector of the torpedo when the torpedo recognizes that it is misguided by a false target. Thus, the submarine becomes safer if it launches a decoy before evasion.

The appropriate launch time of a decoy was obtained under different warning angles of a torpedo [2]. Based on mathematical simulation analysis, an admissible launch area and the most disadvantageous launch angles were obtained under different warning angles of a torpedo [3]. Linear programming method was used to obtain the optimal evasive course of the submarine and optimal parameters of the decoy [4].

There exist many trajectory models [5] for decoys, e.g., the straight line, the straight line plus turn-angle, straight line plus circular movement, straight line plus snake movement, and straight line plus helical movement. As the first stages of five trajectory models are all straight lines, the movements of the second stage are used to name the trajectory models, i.e., straight line, turn-angle, cycle, snake, helix. The optimal selection of trajectory models and their parameters are very complicated as the interactions among torpedo, submarine and countermeasure equipment vary with the time, positions and technical specifications of the submarine, acoustic countermeasure equipment and torpedo.

Any change of the tactical decisions by submarine commander may make the outcome of submarine survival different. There are several decision variables that will significantly affect the outcome of the submarine’s evasive tactics. In general, all the decision variables [6] are divided into two parts. The first part is about the countermeasure equipment and the second is about the submarine. The decision variable of the submarine is the evasive course. The decision variables of the countermeasure equipment include the time to release the decoy, the predetermined course of the decoy, the speed of the decoy, the time for the decoy to maneuver, the selection of trajectory models and the parameters of the selected trajectory model. All these variables have to be well calculated and some of them should be well coordinated in order to get the submarine out of the dangerous zone successfully [6].

In this paper, the interactions among the submarine, torpedo and swim-out decoy are simulated. The submarine would follow the instructions of the predetermined anti-torpedo tactics. The effectiveness of the tactics is measured by the distance between the submarine and torpedo after the exhaustion of torpedo voyage or the capture of the submarine or decoy. To optimize the tactics, genetic algorithm is used as an optimization tool.

The rest of this paper is organized as follows. Section II describes the simulation assumptions, simulation scenario, and the dynamic models of submarine, torpedo, and decoy. Section III introduces the advantages of genetic algorithm (GA), presents genetic operators and fitness evaluation function, and describes its optimization process. Section IV presents the simulation results and analysis. The evasive tactics are given for different trajectory models at different initial target angles. Finally, Section 5 gives a brief summary and future work.

II. MODEL DESCRIPTION

A. Basic Assumption

The combat process of a submarine using decoy to defend against torpedo with acoustic homing is very complicated. For simplicity, some basic assumptions are given as follows.

1) The submarine is warned of torpedo threat by its sonar system. After a certain reaction time of the underwater acoustic countermeasure system, the submarine launches a swim-out acoustic decoy to defend against torpedo threat. The submarine turns to its evasion course. During the turning process, it follows circular trajectory. After turning to the given evasion course, it goes in straight line. During the whole process, the submarine keeps constant speed.

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2) After being launched from the submarine, the swim-out decoy moves in straight line according to its initial course. After a certain time, it begins to maneuver according to its predetermined trajectory models. As for the turn-angle model, it means here that it turns clockwise or counter-clockwise for some degrees and then follows the new course in a straight line. During the whole process, the decoy keeps constant speed. At the tail of the decoy, there exists a blind area of about ±30° [7].

3) Before target detection, the torpedo moves with constant speed in straight line trying to detect the submarine, with a fixed lead angle between its voyage course and the line of sight from the torpedo to the assumed position of submarine. If both the submarine and decoy enter the homing range of torpedo, the torpedo prefers to track the target that is close to the beam center according to their bearing differences. However, once a target is tracked by the torpedo, it will keep tracking that target by adopting proportional guidance law. It is assumed that the torpedo has certain capability of decoy recognition. That is to say if its distance to the decoy is less than a fixed value, e.g. 200m, the decoy is recognized. Once recognition. That is to say if its distance to the decoy is less than a fixed value, e.g. 200m, the decoy is recognized. Once recognition.

The scenario between the torpedo and target is shown in Fig. 1.

\[ \omega = \frac{v_t}{2} \] (5)

where \( r \) is the distance between torpedo and target (submarine or decoy); \( q \) is the relative bearing; \( \varphi_r \) is the yaw angle of torpedo; \( v_M \) is the speed of target; \( v_M \) is the speed of target; \( \varphi_M \) is the course angle of target; \( a \) is the coefficient of proportional guidance. Given the initial parameters in (1) and the launch time, initial course, speed and trajectory parameters of decoy, and the speed and evasion course of submarine, the combat scenario and outcome can be obtained.

C. Trajectory Model of Decoy

1) Trajectory of straight line: Trajectory of straight line is the basis of all trajectories. After launch, the decoy adopts trajectory of straight line. Assuming that \((x_0, y_0)\) is the position of decoy at launch time and that it moves with speed \(v_D\) along direction \(\theta_D\) in straight line, then its coordinate at any time \(t\) is as follows [9]

\[ x = x_0 + v_D(t - t_0) \cos \theta_D \]
\[ y = y_0 + v_D(t - t_0) \sin \theta_D \] (2)

Equation (2) can cancel the accumulative error. As for the trajectory of straight line plus turn-angle, the decoy first moves according to (2). Then at the maneuver time it turns clockwise or counter-clockwise for a certain degree and moves along the new course in straight line for the rest of time.

2) Snake trajectory: In order to establish the mathematical model of snake trajectory, a convenient method is to build a new but simple coordinate system and get the position and course under this coordinate system first. Then its original position and course can be obtained through coordinate transformation. The method is described as follows [9]:

Establish a new coordinate system by taking the initial point \((x_0, y_0)\) of snake trajectory at time \(t_0\) as original point and the base course of snake movement as \(Y_1\) axis. Let \(v_D\) be the decoy velocity and \(\omega_D\) be angular velocity, and then the turn-radius is \(R_D = v_D / \omega_D\); let \(2\alpha\) be viewing angle of snake movement. Then after each time interval \(\Delta t = 2\alpha / \omega_D\), the center of the circle will change once. At any time \(t\), the number of semi-cycles traversed by decoy is

\[ i = \frac{\omega_D(t - t_0)}{2\alpha} \] (3)

The traversed angle by the decoy in the current cycle is

\[ \beta = \omega_D(t - t_0) - i \cdot 2\alpha \] (4)

Let \((x_i, y_i)\) be the starting point of the \(i^{th}\) semi-cycle and \((x_{0i}, y_{0i})\) be the center of the current cycle. Then

\[ x_i = 0 \]
\[ y_i = 2R_D \sin \alpha \]
\[ x_{0i} = x_i - (i - 1) \cdot LR \cdot R_D \sin \alpha \]
\[ y_{0i} = y_i + R_D \sin \alpha \] (5)

Where \(LR\) stands for the direction of the decoy turning left or right in the first semi-cycle, with “1” for left and “-1” for right.

Then the coordinate of the decoy at any time \(t\) is
\[ x_t = x_0 - (-1)^t \cdot LR \cdot R_D \cos(\beta - \alpha), \]
\[ y_t = y_0 + R_D \sin(\beta - \alpha). \]  

(7)

As the direction of \( Y_1 \) axis in the new coordinate system is the base course angle \( \alpha_0 \) of the snake movement in the original coordinate system. Then point \((x',y')\) in the new space is:
\[ x' = x_0 + x \sin(\chi_0) + y \cos(\chi_0), \]
\[ y' = y_0 + y \sin(\chi_0) - x \cos(\chi_0). \]  

(8)

3) Circle trajectory: The circle trajectory can be dealt with in a similar way to snake trajectory. Establish a new coordinate system, taking the initial point \((x_0, y_0)\) in the new one
\[ x_t = x_0 + R \cos(\beta \cdot t), \]
\[ y_t = y_0 + R \sin(\beta \cdot t). \]  

(9)

Then the coordinate of decoy at time \( t_0 \) is
\[ x_t = x_0 + R \cos(\beta \cdot t_0), \]
\[ y_t = y_0 + R \sin(\beta). \]  

(10)

Using (8) it can be transformed into the original coordinate system.

4) Helix trajectory: At time \( t_0 \), the decoy at \((x_0, y_0)\) begins helical movement. Establish a new coordinate system, taking the initial point as original point and the course \( \alpha_0 \) at that time as \( Y_1 \) axis [9]. Then any time \( t \), the traversal angle by decoy is
\[ \beta = \omega_0 (t - t_0). \]  

(11)

Where \( \omega_0 = \nu_0 / R \), \( \omega_0 = \nu_0 / (R \beta). \)

Then the coordinate of the decoy is
\[ x_t = R \beta \cos(\beta \cdot t), \]
\[ y_t = R \beta \sin(\beta \cdot t). \]  

(12)

Using (8) it can be transformed into the original coordinate system.

III. PARAMETER OPTIMIZATION BASED ON GA

A. Virtue of GA

Inspired by the principles of natural selection and “survival of the fittest” in biological organisms, John Holland proposed Genetic Algorithm (GA) in 1975. GA is not guaranteed to find the global optimum solution to a problem, but it is generally good at finding “acceptably good” solutions to problems “acceptably quickly”. It does not need any knowledge about the evaluation function, e.g. its derivative, to guide its search. GA is therefore particularly well suited to problems with discontinuous or poorly behaved evaluation functions. Consequently, GA finds wide applications in combinatorial optimization, such as traveling salesman problem, job shop scheduling and numerical optimization [10].

B. Design of Genetic Operators

The chromosome is divided into several gene segments to represent the launch time of decoy \( t_1 \), initial course of decoy \( \varphi_0 \), maneuver time of decoy \( t_2 \), decoy speed \( \nu_d \), evasion course of submarine \( \varphi_s \) and maneuver parameter for the second stage of the trajectory. The maneuver parameter for straight line trajectory is null for it does not maneuver at all. As for the turn-angle, circle, snake and helix model, the maneuver parameter represents the new course \( \varphi_{D2} \), circle radius \( R_1 \), viewing angle \( 2\alpha \) of snake movement, helical polar radius \( R_3 \), respectively.

Take the central line of the submarine as the reference direction and its left direction as positive and right direction as negative, and then both the range of decoy course and submarine evasive course [7] are \([-\pi, \pi]\). A negative circle radius or polar radius means that the decoy turns right at the very beginning of maneuver movement. Their absolute values should be less than the minimum turn-radius of the decoy \( R_D \). Only after the reaction time of the underwater acoustic countermeasure system, can the decoy be launched. So the launch time of decoy \( t_1 \) is bigger than the reaction time of the underwater acoustic countermeasure system. The maneuver time \( t_2 \) of decoy is defined as the time instant when the decoy ends movement of straight line and begins maneuver.

Floating point numbers in \([0, 1]\) are used to represent genes, which greatly simplifies the crossover, mutation and other operations. As the ranges of gene values are not the same, proportional and translation transformations are used for each gene before fitness evaluation.

\[ t_1 = t + 100 f_1 \]
\[ t_2 = 100 f_2 \]
\[ \varphi_D = (f_3 - 0.5)2\pi \]
\[ \nu_D = f_4 \cdot \nu_{\text{max}} \]
\[ \varphi_s = (f_4 - 0.5)2\pi \]
\[ \varphi_{D2} = (f_5 - 0.5)2\pi \]
\[ \alpha = (f_5 - 0.5) / 2 \]
\[ R_1 = 20(f_6 - 0.5)R_D \]
\[ R_2 = 20(f_6 - 0.5)R_D \]

Where, \( t \) is the reaction time of the underwater acoustic countermeasure system; \( f_i, i = 1, \cdots, 9 \) is floating numbers in interval \([0, 1]\); \( \nu_{\text{max}} \) is the maximum speed of decoy; \( R_D \) is the minimum turn-radius of decoy.

C. Fitness of GA

The first objective for a submarine to use swim-out decoy is to make the coming torpedo detect the decoy first, and the second is to make the decoy move away from the submarine as far as possible. Consequently, the submarine should have been far away from the search area of torpedo when the torpedo finds out that it is misguided by the decoy. Thus, the fitness function is defined as

\[ f = \begin{cases} \sqrt{(x_t - x_d)^2 + (y_t - y_d)^2}, & \text{decoy not detected} \\ \sqrt{(x_t - x_d)^2 + (y_t - y_d)^2} / d_o, & \text{decoy detected} \end{cases} \]  

(14)

\[ d_o \]
models of straight line, turn-angle, circle, snake and helix, decoy are listed in Table I to Table V for the trajectory initial target angles, the optimal strategies of submarine and torpedo respectively. The torpedo moves in straight line and given setup parameters, the decoy adopts five trajectory models alternatively. The torpedo moves in straight line and given setup parameters, the decoy adopts five trajectory models alternatively. When the submarine enters the kill-radius of the torpedo or the torpedo recognizes that it has been misguided by the decoy, Equation (14) means that the bigger the distance between submarine and torpedo, and the less the distance between decoy and torpedo, the better.

D. Process of GA

Ratio selection operator, elite keeping strategy, and two-point mutation are adopted in GA. The optimization process of GA is described as follows [10].

Step 1: The evolutionary generation and setup parameters in (1) are initialized and the initial population is generated randomly.

Step 2: Once the setup parameters and genes of a chromosome are given, the submarine, torpedo and decoy move according to their trajectory models respectively. When the submarine enters the kill-radius of the torpedo or the torpedo recognizes that it has been misguided by the decoy, the game is over. Then (14) is used to calculate fitness.

Step 3: Evolutionary process:

a) Crossover operation of individuals.
b) Mutation operation of individuals.
c) The fitness of the population is evaluated based on the outcome of the play between submarine, torpedo and decoy.
d) Selection and copy operation of individual.

Step 4: Increase the number of generation. If it reaches the maximum evolutionary generation, the algorithm terminates after outputting the optimal individual; otherwise goes back to Step 3 for further evolution.

IV. Optimization Result and Analysis

A. Initial Conditions

The initial conditions of simulation are set up as follows [4][11]: warning distance of torpedo, flare angle of torpedo self-guidance system, effective homing range of torpedo to submarine, torpedo speed, range for a torpedo to recognize a decoy, submarine speed, minimum turn-radius of submarine, reaction time of underwater acoustic system, maximum speed of swim-out decoy, and minimum turn-radius of decoy.

Uniform mutation, proportional selection with elite replacement strategy and single point crossover operators are adopted. The size of population is 100 and evolutionary generation is 100. The crossover probability is 0.85 and mutation probability is 0.10.

B. Simulation Result

According to the above-mentioned mathematical model and given setup parameters, the decoy adopts five trajectory models alternatively. The torpedo moves in straight line before detecting any targets. Once it detects a target, it moves according to proportional guidance law. Under different initial target angles, the optimal strategies of submarine and decoy are listed in Table I to Table V for the trajectory models of straight line, turn-angle, circle, snake and helix, respectively.

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**TABLE I**

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<thead>
<tr>
<th>Target angle (°)</th>
<th>Launch time (s)</th>
<th>Initial course (°)</th>
<th>Sub. course (°)</th>
<th>Decoy speed (kn)</th>
<th>Maneu. time (s)</th>
<th>Turn angle (°)</th>
<th>Dis. (m)</th>
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<td>13.9</td>
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<td>-85.7</td>
<td>2162.5</td>
<td>90 50</td>
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**TABLE II**

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<th>Initial course (°)</th>
<th>Sub. course (°)</th>
<th>Decoy speed (kn)</th>
<th>Maneu. time (s)</th>
<th>Circle radius (°)</th>
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<td>90 50</td>
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**TABLE III**

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<th>Initial course (°)</th>
<th>Sub. course (°)</th>
<th>Decoy speed (kn)</th>
<th>Maneu. time (s)</th>
<th>Circle radius (°)</th>
<th>Dis. (m)</th>
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<td>-85.7</td>
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TABLE V
OPTIMAL STRATEGY FOR DECOY WITH HELIX MODEL

<table>
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<th>Target angle (°)</th>
<th>Launch time (s)</th>
<th>Initial course (°)</th>
<th>Sub. course (°)</th>
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</table>

When the target angle of torpedo is 60 degrees and decoy adopts the trajectory models of straight line, turn-angle, circle, helix and snake, the play outcome are shown in Fig. 2 and Fig. 3 respectively.

**C. Result Analysis**

It is seen from Table I to Table V that the average evasive distance of submarine is 2027.64m and 2374.13m respectively for the decoy with the trajectory models of circle and helix, which is much less than those of the decoy with straight line, turn-angle and snake models, i.e., 4390.90m, 4099.31m and 4318.98m. In order to determine if there is significant difference among the evasive distances under different target angles for five trajectory models, non-parametric variance analysis is used for hypothesis test. These evasive distances are sorted and the orders of the evasive distances of each trajectory models are summed up. The summed result is represented by $R_k$. The static $H$ can be obtained [12],

$$ H = 12 \cdot \frac{\sum_{i=1}^{k} R_i^2 / n_i}{N(N+1)} - 3(N+1) $$ (15)

Where $k$ stands for the number of trajectory models; $n_i$ stands for the number of evasive distances belonging to trajectory model $i$; $N$ stands for the total number of evasive distances. For the evasive distances of five trajectory models, through sorting and calculation, we obtain

$$ H_3 = 38.50 > \chi^2_{0.01} (5-1) = 13.28 $$

Therefore, it is reasonable to conclude that the outcomes of five models are quite different. Eliminate the evasive distances of circle and helix models, and then obtain

$$ H_3 = -3.59 < \chi^2_{0.01} (3-1) = 9.21 $$

It is concluded that the outcomes of the trajectory models of straight line, turn-angle and snake are not obviously different.

It was concluded in [4] and [6] that in order to move far away from the self-guidance range of torpedo, the submarine should turn to the course having a bearing difference of 150°~180° with the decoy. It is obviously seen from data of column 3 and column 4 in five tables that the result almost agrees with that conclusion.

The optimization algorithm is implemented using C# under DotNet 4.0 framework. The running environment is Windows 7 operational system, Intel 2.13GHz CPU and 2G memory. The average running time for the optimization of a single trajectory model is 3.0s. It should be noted that it doesn’t use multi-threading technologies in programming.

**V. CONCLUSION**

The optimal trajectory for a submarine with a swim-out acoustic decoy to countermeasure acoustic homing torpedo is studied. First, the dynamic models of the torpedo, submarine and several trajectory models of the decoy are established and the factors that affect the countermeasure effectiveness are analyzed. Then based on the assessment criterion, genetic algorithm is used to obtain the optimal parameters of the trajectory models under different cases. Simulation results show that the helix and circle models of decoy trajectory are dominated by three other models, i.e., straight line, turn-angle and snake.
In future research, the style of torpedo re-search on decoy strategy will be studied. I.e., when the torpedo recognizes that it has been misguided by the decoy, the torpedo will discard the false target and re-search for the submarine. In other words, the game will continue until the torpedo kills the submarine or gets out of fuel or battery.

REFERENCES


