NUMERICAL ANALYSIS OF HIGH STRAIN RATE SPLITTING-TENSILE TESTS

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Abstract—Experimental splitting-tension tests were conducted on 2-in. diameter concrete specimens in a Split Hopkinson Pressure Bar at strain rates of 4.4, 10.6, and 14.7/sek. The specimens were instrumented with electrical resistance strain gages and break circuits to detect crack initiation and growth. Experimental results indicate that there is a shift of crack initiation time relative to the peak stress. Also, experimental strength vs strain rate data reveal that the dynamic tensile strength of concrete is significantly higher than the static tensile strength. A comprehensive numerical analysis was conducted on the splitting-tensile experiments to investigate the effects of varying the uniaxial tensile strength of the concrete on the crack initiation time, stress state, crack growth characteristics, and failure mode in the concrete specimens. The results of the numerical analyses are used to enhance the understanding of concrete tensile strength strain rate sensitivity.

1. INTRODUCTION

Shelters used by the military to protect personnel and equipment from conventional weaponry attacks are typically constructed of massive concrete slabs. Therefore, the investigation of concrete’s response to high-amplitude, short duration, impulse loads is an important consideration in protective construction design and analysis [1]. To experimentally model high strain rate loads similar to those generated in a weapons environment, a laboratory arrangement must accurately reflect the type of confinement, magnitude of stress change, and the time scale of loading that materials would undergo in the field [2]. The Split Hopkinson Pressure Bar (SHPB) apparatus is capable of replicating such an environment in the laboratory.

The strain rate region typically referred to as ‘high’ ranges from approximately 10 to 10⁶/sek. Such high strain rates may be imposed on structures by conventional weapons having loading pulses with rise times on the order of 1.0 msec. Therefore, it is important to ascertain the effects of these strain rates on structural materials, particularly concrete. Several researchers have shown that the SHPB technique is capable of producing these high stress and strain rates in soils [3–7] and concrete [8–11]. Studies have further revealed that most materials, including concrete, respond differently to different loading rates or strain rates. Evidence of this phenomenon is illustrated in Fig. 1.

Recently, in an effort to determine the effects of high strain rates on the tensile strength of concrete, a series of splitting-tensile tests were performed on 2-in. long, 2-in. diameter specimens in a SHPB. Each specimen was instrumented with several electrical resistance strain gages and break circuits to detect crack initiation and growth patterns. In addition, high-speed photography was employed to further monitor the formation of cracks in the specimens. The experimental results indicated that there was a shift of crack initiation time relative to the time at which the peak stress was reached. Also, experimental strength vs strain rate data revealed that the dynamic tensile strength of concrete is significantly higher than the static tensile strength.

Although the SHPB experimental technique is an extremely useful tool for studying dynamic material properties, it does have some inherent limitations. For instance, the data collected is insufficient for determining the stress condition of the specimen at failure. Additionally, it is not always possible to correctly assess the mode of failure of the specimen. Thus, a number of analytical studies have been carried out that numerically simulate direct compression, direct tension, and splitting-tensile SHPB experiments [12–17]. The results of these studies indicate that finite element modeling can effectively represent the stress and strain states present in specimens tested in a SHPB.

The purpose of this study is to lend insight into observed experimental results by conducting a comprehensive finite element method (FEM) analysis of the high strain rate splitting-tensile tests performed in the SHPB. The objective of the numerical simulation is to investigate the effects of varying the uniaxial cut-off tensile strength of the concrete on the crack initiation time, stress state, crack growth characteristics, and failure mode in the concrete specimens. The results of the FEM analyses are used to enhance the understanding of concrete tensile strength strain rate sensitivity.
2. SPLITTING-TENSILE TESTS

2.1. Static test methods

There are three generally accepted methods used to experimentally measure the tensile strength of concrete. These include the direct tension test, the modulus of rupture test, and the splitting-tensile test. The direct tension test consists of gripping a specimen at both ends and pulling it apart in tension; the tensile strength is calculated by dividing the failure load by the specimen area. For the modulus of rupture test, a rectangular beam is loaded at the center point or at its third points and failure occurs in bending; the tensile stress computed at failure is called the modulus of rupture and is considered to represent the tensile strength of the concrete. The splitting-tensile test involves applying a compressive load to either side of a cylinder along its length, causing the specimen to fail in tension on the plane between the loads. The tensile strength in this case is computed by a formula involving the maximum load applied and the specimen dimensions.

Researchers have determined that, of the three test methods, the splitting-tensile test gives the most accurate measure of the true tensile strength of concrete [18–20]. Difficulties are encountered in the direct tension tests when trying to apply a pure tension force free of any eccentricity. Often, when grips are used to anchor the specimen, compression from the grips is combined with tension from the testing machine. This particular combination of forces has been shown to result in failure at stress levels below either the maximum tensile or compressive strength [21]. Although the modulus of rupture test is easier to conduct than the direct tension test, it tends to overestimate the tensile strength and gives a higher value than would be determined from the other two test methods [18]. This overestimation is the result of the assumption that there is always a linear distribution of stress beneath the neutral axis of the beam, whereas the true stress block actually takes on a parabolic shape when the failure load is approached.

The standard (static) splitting-tensile test arrangement is shown in Fig. 2. A cylindrical concrete specimen is positioned such that its longitudinal axis lies horizontally between the platens of a loading mechanism. Narrow bearing strips are placed between the specimen and the loading platens. The load is then applied and increased until failure occurs by splitting along the vertical axis of the specimen.

The stresses associated with this loading configuration are illustrated in Fig. 3. When the compressive load, \( P \), is applied to the specimen, elements located near the center of the cylinder along its vertical diameter are subjected to a vertical compressive stress equal to

\[
\sigma_z = \frac{2P}{\pi LD} \left[ \frac{D^2}{z(D - z)} - 1 \right],
\]  

\[(1)\]
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Fig. 3. Stresses associated with a splitting-tensile test.

where $P$ is the compressive load applied to the cylinder, $L$ is the specimen length, $D$ is its diameter, and $z$ is the distance from the element to the top of the cylinder. The element is subjected to a horizontal tensile stress, as well, whose magnitude is equal to

$$
\sigma_y = \frac{2P}{\pi LD}.
$$

The narrow bearing strips that are placed between the cylinder and the loading platens are used to carry a portion of the high compressive stress that is induced directly beneath the load. The tensile strength determined from tests conducted without the bearing strips is typically about 8% lower than that recorded by tests conducted with the bearing strips [18]. ASTM recommends that the width of the strips be approximately 1 in. [22]; however, in practice, it is convenient to make their width equal to $1/12$ of the diameter of the cylinder. The distribution of horizontal stress, $\sigma_y$, on a section through the vertical diameter, under these loading conditions (loaded over $1/12$ of its diameter), is given in Fig. 4 [18]. Although there is a rather high horizontal compressive stress immediately underneath the load, it is accompanied by a vertical compressive stress of comparable magnitude. Therefore, a state of biaxial stress is created, preventing failure in compression.

2.2. Dynamic tests on the SHPB

It is well documented that most structural materials, with the exception of some work-hardened aluminum alloys, exhibit an increase in strength with increasing strain rate [18–20, 23, 24]. Concrete is no exception. Its strength, both in tension and compression, has been shown to be dependent upon the strain rate to which it is subjected. Increases in dynamic compressive strength of up to 2.5 times that of the static strength have been reported, while dynamic tensile strengths of up to five times the static tensile strength have been observed. The data that have been collected in studies of this phenomenon show that there is a 'threshold' strain rate at which the strengthening effect begins to appear. For tension, this critical strain rate is approximately 5/sec, whereas the critical compressive strain rate has been shown to occur at approximately 60/sec [23, 25]. Figure 5 presents the results of several tension tests performed at various strain rates. It can be seen that some strengthening does occur at strain rates below 5/sec, however, a significant rise in the dynamic/static strength ratio is exhibited just beyond this threshold rate.

An illustration of the SHPB device used in the experimental study is shown in Fig. 6, and a schematic of the same is shown in Fig. 7. The device is owned and operated by the Air Force Civil Engineering Support Agency, Tyndall AFB, Florida. The pressure bars are constructed of PH 13-8 Mo stainless steel.
steel. Each pressure bar is 2.0 in. in diameter. The loading compressive stress wave is initiated by the impact of the striker bar (which is propelled by the gas gun) on the incident bar (Fig. 6). The amplitude of the incident stress pulse is determined by the impact velocity and material properties of the striker bar, while the duration of the pulse is dependent on the length and modulus of the striker bar.

The incident stress wave \( \sigma_i \) generated in the incident bar travels down the bar and is recorded at strain gage A (Fig. 7), is partially reflected at the incident bar/specimen interface, and partially reflected at the specimen/transmitter bar interface. Strain gage B (Fig. 7) on the transmitter bar records the portion of the wave that has transmitted the specimen \( \sigma_T \), while strain gage A on the incident bar records that portion of the wave reflected at the incident bar/specimen interface \( \sigma_R \). From these strain gage measurements, the stress and strain in the specimen, which is sandwiched between the two pressure bars, can be computed as a function of time using simple wave mechanics.

The splitting-tensile test arrangement in the SHPB is illustrated in Fig. 8. Each specimen was a right cylinder measuring 2.0 in. in length and 2.0 in. in diameter. The concrete specimen is placed between the incident bar and the transmitter bar such that its longitudinal axis is perpendicular to the longitudinal axis of the bars. One-quarter inch square steel bearing strips are placed between the bars and the specimen along its length. For such an arrangement, the dynamic splitting tensile stress, \( f_{st} \), is determined by eqn (2), and is restated here for convenience:

\[
 f_{st} = \frac{2P}{\pi LD} \tag{2'}
\]

where \( P \) represents the force that is transmitted through the specimen, which is determined from the transmitted stress as

\[
P = \pi R^2 \sigma_T, \tag{3}
\]

in which \( R \) is the radius of the SHPB. The loading rate, \( \dot{\sigma} \), and the strain rate, \( \dot{\varepsilon} \), may also be determined from the following expressions

\[
\dot{\sigma} = \frac{f_{st}}{\tau}, \tag{4}
\]

where \( \tau \) is the time delay between the start of the transmitted stress wave and the peak transmitted stress, and

\[
\dot{\varepsilon} = \dot{\sigma}/E, \tag{5}
\]

where \( E \) is Young's modulus of the concrete specimen.

The trace of the incident stress and splitting-tensile stress for a typical SHPB experiment are presented in Figs 9 and 10, respectively. It is convenient to note that since the SHPB bar diameter, specimen length,
and specimen diameter are each nominally 2 in., eqn (2) reduces to

$$f'_{id} = \frac{P}{2\Pi}$$

which can be combined with eqn (3) and further simplified to give

$$f'_{id} = \frac{\sigma_t}{2\Pi}.$$  

Therefore, the transmitted stress may be determined by doubling the splitting-tensile stress shown in Fig. 10. A summary of the results from the SHPB tests is given in Table 1.

In this particular series of splitting-tensile tests, an additional effort was made to detect the crack initiation time and crack growth patterns created as the stress waves traveled through the specimen. Each specimen was instrumented with several electrical resistance strain gages and break wire circuits in the pattern indicated in Fig. 11. Also, high-speed photography (100,000 frames/sec) was employed to further monitor crack formation in the specimen. The estimated time of crack initiation determined from the photographs and break wire circuits for each test are indicated on the splitting-tensile stress histories such as that given in Fig. 10. It is interesting to note that in each case, the crack initiation times estimated from the photographs exceed those estimated from the break wire circuits. This discrepancy is due to the camera's inability to detect the crack in its earliest stage.

A typical set of traces for a selected few of the strain gages and break wire circuits is shown in Fig. 12. For this test, the crack initiation site was on the side of the specimen nearest the transmitter bar, as shown by the increase in voltage of the break circuits near that point at approximately 60 µsec. The corresponding strain in this area of the specimen at that time is 400–500 microstrains. As illustrated in the figure, the strain increases almost linearly up to the point of crack initiation, and the time to fracture in the specimen is approximately 40 µsec (measured from the time the strain begins increasing to the time of crack initiation). This time to fracture is in accordance with previous splitting-tensile and direct tension tests performed in the SHPB.

3. FEM ANALYSIS

In an attempt to more fully comprehend the response of concrete specimens to the impact loading conditions produced in the SHPB, a FEM simulation, using the ADINA [26] finite element computer programs, of the splitting-tensile test was performed for the three different load cases. An illustration of the FEM model employed in the study is presented in Fig. 13. The concrete splitting-tensile specimen was modeled with 1200 eight-node, two-dimensional finite elements. To avoid the occurrence of artificially reflected stresses developed by the imposition of a rigid boundary at the specimen/transmitter bar interface, a 10-in. length of the steel transmitter bar was also included in the model. This section of the model consisted of 200 eight-node, two-dimensional finite elements. The load was applied to the specimen at the incident bar/specimen interface, which was designated as the 'top' of the model.

The load applied to the specimen for each load case was determined from the traces of the experimental splitting-tensile stresses such as that shown in Fig. 10. The maximum stress transmitted to the specimen was calculated from the maximum splitting-tensile stress
according to eqn (7). Past experience in numerical modeling of splitting-tensile tests [17] indicated that a ramp loading function of the type illustrated in Fig. 14 should be applied in the simulation. Table 2 summarizes the pertinent ramp loading parameters employed for each of the three load cases. The strain rates imposed on the specimen for load cases 1, 2, and 3, respectively, were 4.4, 10.6, and 14.7/sec. For each load case, three separate analyses were performed, each employing a different value of uniaxial tension cut-off stress as input to the FEM computer program. The cut-off values that were examined were the static value of 560 psi (analysed for comparison purposes), 1.5 times the static value (840 psi), and twice the static value (1120 psi).

The concrete specimen was characterized by a nonlinear material model. The nonlinear concrete model is a hypoelastic model based upon a uniaxial stress-strain relationship that is appropriately modified to account for multiaxial stress conditions. There are three main features that can be used to describe the model's material behavior: (1) a nonlinear stress–strain relation that includes strain-softening to allow for the weakening of the material under increasing compressive stress, (2) failure envelopes that define cracking in tension and crushing in compression, and (3) a strategy to model the post-cracking and crushing behavior of the material [27]. An illustration of the uniaxial stress–strain relation...
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Fig. 12. Selected strain gage and break circuit traces for the instrumentation shown in Fig. 11.

Fig. 13. FEM model of concrete specimen and portion of transmitter bar.

employed in the concrete model is presented in Fig. 15. Variable notations included in the figure are described as follows:

\[
\begin{align*}
\epsilon &= \text{uniaxial strain} \\
\sigma &= \text{uniaxial stress} \\
\sigma_t &= \text{uniaxial cut-off tensile strength} \\
E_0 &= \text{uniaxial initial tangent modulus} \\
\sigma_c &= \text{maximum uniaxial compressive stress} \\
\epsilon_c &= \text{uniaxial strain corresponding to } \sigma_c \\
\sigma_u &= \text{ultimate uniaxial compressive stress} \\
\epsilon_u &= \text{ultimate uniaxial compressive strain}
\end{align*}
\]

As seen in the figure, the stress–strain curve is linear in the tensile region (the primary region of interest in this study) until tensile failure at the uniaxial tensile cut-off, \( \sigma_u \), and a constant Young’s modulus is employed [27], where

\[\sigma = E_0\epsilon\] (8)

and

\[E_0 = d\sigma/d\epsilon.\] (9)

The three-dimensional failure envelope that is utilized in the model to establish the uniaxial stress–

<table>
<thead>
<tr>
<th>Load case No.</th>
<th>Incident stress ( \sigma_I ) (psi)</th>
<th>Transmitted stress ( \sigma_T ) (psi)</th>
<th>Dynamic splitting tensile stress ( f_u ) (psi)</th>
<th>Loading rate ( \dot{\epsilon} ) (psi)</th>
<th>Strain rate ( \dot{\epsilon} ) (psi)</th>
<th>Experimental Dynamic increase factor ( f_d / f_u / f_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11,519</td>
<td>2263</td>
<td>1132</td>
<td>( 2.29 \times 10^7 )</td>
<td>4.4</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>17,766</td>
<td>2843</td>
<td>1422</td>
<td>( 5.53 \times 10^7 )</td>
<td>10.6</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>26,759</td>
<td>3858</td>
<td>1929</td>
<td>( 7.67 \times 10^7 )</td>
<td>14.7</td>
<td>3.4</td>
</tr>
</tbody>
</table>

\( f_u \) is the static tensile strength = 560 psi.
strain law accounting for multiaxial stress conditions is depicted in Fig. 16. In this figure

\[ \sigma_1^t = \text{uniaxial cut-off tensile stress under multiaxial conditions} \]

\[ \sigma_0 = \text{uniaxial compressive failure stress under multiaxial conditions} \]

\[ \sigma_{11}, \sigma_{22}, \sigma_{33} = \text{principal stresses in directions 1, 2, and 3 at time } t. \]

To determine whether or not the material has failed, the principal stresses are used to locate the current stress state in the failure envelope. When the tensile stress in a principal direction exceeds the tensile failure stress, a failure has occurred. It is interesting to note that although the tensile strength of the material in a principal direction is not affected by the introduction of tensile stresses in other principal directions, compressive stresses in other directions do affect this tensile strength. Table 3 lists the concrete model parameters used in the FEM analyses.

In a FEM analysis, as in any method of system analysis, two major features characterize the difference between the prediction of a material's response to intense, short-duration (impact) loading and its response to static or quasi-static loading. The first is that for a dynamic analysis, inertia effects must be considered in formulating all of the governing equations, which are based upon the fundamental laws of conservation (conservation of mass, conservation of momentum, and conservation of energy). The second feature is the consideration of stress wave propagation in a dynamic analysis, where steady-state conditions do not exist [24]. Since modal analysis techniques (in which the equations of motion of a system are uncoupled through a transformation to principal coordinates) do not generally produce cost-effective, accurate results for wave propagation problems [17], a direct numerical integration of the
coupled equations of motion is required. For the current study, the Newmark method of direct time integration with a consistent mass formulation was employed.

For a nonlinear dynamic analysis, an iterative procedure must be employed at each time step to achieve an accurate solution. This is due to the fact that an approximation of the increment in internal

![Fig. 16. Three-dimensional tensile failure envelope of concrete model.](image)

![Fig. 17. Time history for horizontal stress of z = 0.3 D, σ_t = 560 psi.](image)
Fig. 18. Time history for horizontal stress of \( z = 0.5 \) D, \( \sigma_z = 560 \) psi.

Fig. 19. Time history for horizontal stress of \( y = 0.3 \) D, \( \sigma_y = 560 \) psi.
Fig. 20. Time history for horizontal stress of $z = 0.3 \, D$, $\sigma_x = 840 \, \text{psi}$.

Fig. 21. Time history for horizontal stress of $z = 0.5 \, D$, $\sigma_x = 840 \, \text{psi}$.
Fig. 22. Time history for horizontal stress of \( y = 0.3 \, D \), \( \sigma_y = 840 \, \text{psi} \).

Fig. 23. Time history for horizontal stress of \( x = 0.3 \, D \), \( \sigma_y = 1120 \, \text{psi} \).
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Fig. 24. Time history for horizontal stress of $x = 0.5 \, \text{D}$, $\sigma_x = 1120 \, \text{psi}$.

Fig. 25. Time history for horizontal stress of $y = 0.3 \, \text{D}$, $\sigma_y = 1120 \, \text{psi}$. 
Fig. 26. Profile for horizontal stress along the vertical diameter at $t = 62 \mu$sec, $\sigma_1 = 560$ psi.

Fig. 27. Profile for horizontal stress along the vertical diameter at $t = 160 \mu$sec, $\sigma_1 = 840$ psi.
Fig. 28. Profile for horizontal stress along the vertical diameter at $t = 100 \, \mu\text{sec}$, $\sigma_r = 1120 \, \text{psi}$.

Fig. 29. Profile for horizontal stress along the horizontal diameter at $t = 70 \, \mu\text{sec}$, $\sigma_r = 560 \, \text{psi}$. 
Fig. 30. Profile for horizontal stress along the horizontal diameter at $t = 160 \mu$sec, $\sigma_r = 840$ psi.

Fig. 31. Profile for horizontal stress along the horizontal diameter at $t = 100 \mu$sec, $\sigma_r = 1120$ psi.
forces corresponding to the increment in the displacements from time \( t \) to time \( t + \Delta t \) must be made using a tangent stiffness matrix, \([k]_t\), which relates to the geometric and material conditions at time \( t \). In the present study, the Newton–Raphson iteration scheme was employed to solve the nonlinear equations.

An important parameter that stems from consideration of the numerical integration and iteration procedures described above is the selection of an appropriate time step, \( \Delta t \), for the dynamic analysis. The time step chosen directly affects the accuracy and stability of the solutions produced. However, since

\[ \text{forces corresponding to the increment in the displacements from time } t \text{ to time } t + \Delta t \text{ must be made using a tangent stiffness matrix, } [k]_t, \text{ which relates to the geometric and material conditions at time } t. \]

In the present study, the Newton–Raphson iteration scheme was employed to solve the nonlinear equations.
the Newmark direct integration method is uncondi-
tionally stable, selection of a time step can be based
solely upon accuracy considerations when using
this method. For a wave propagation problem in
which an implicit unconditionally stable time inte-
gration method is used, the maximum time step
recommended is related to the maximum wave speed,
c, that can be reached in the material under consid-
eration, and the element size employed in the finite
element model [28]. The maximum time step, selected
such that the stress wave can propagate between
element integration points within that time incre-
ment, is defined as

$$(\Delta t)_{\text{max}} = \frac{(L_e/2)}{c},$$

(10)

where $L_e$ is the distance between element integration
points in the direction of wave propagation, and $c$
is the wave velocity, defined as $(E/\rho)^{1/2}$, in which $E$
is Young's modulus of the material and $\rho$ is its mass
density. Tedesco et al. [29] have determined that a
time step of

$$\Delta t < \frac{1}{6}(\Delta t)_{\text{max}}$$

(11)

produces accurate results in a dynamic analysis. In
the present study, a time step of 100 nsec was used for
the analyses.

4. RESULTS OF FEM ANALYSIS

The results for load case 1 are presented. Time
histories for the horizontal stress, $\sigma_{xy}$, at three lo-
cations in the specimen are presented in Figs 17–19
for a uniaxial tensile cutoff stress $\sigma_t = 560$ psi, Figs
20–22 for $\sigma_t = 840$ psi, and Figs 23–25 for $\sigma_t = 1120$ psi.

Profiles for the horizontal stress, $\sigma_{xy}$, along the
vertical diameter at selected times are presented in
Figs 26–28 for a uniaxial cut-off tensile stress of 560,
840, and 1120 psi, respectively. Similar profiles for the
horizontal stress along the horizontal diameter are
presented in Figs 29–31, for a uniaxial cut-off tensile
stress of 560, 840, and 1120 psi, respectively. The
propagation of a crack along the vertical diameter is
indicated in Figs 29 and 30. In the simulation for
which $\sigma_t = 1120$ psi, cracking in the specimen did not
occur, as indicated in Fig. 31.

The cracking sequences predicted by the numerical
analyses, from crack initiation until failure, are illus-
trated in Fig. 32 for $\sigma_t = 560$ psi and in Fig. 33 for $\sigma_t = 840$ psi. For the condition of $\sigma_t = 560$ psi, the
initial crack appears along the vertical diameter at a
location approximately 0.75 in. from the top of the
cylinder, at a time $t = 59.0$ usec (Fig. 32). At time
$t = 62.0$ usec, the crack has propagated in both direc-
tions along the vertical diameter (Fig. 33b) to points
approximately 0.25 in. from the top and bottom of the
cylinder. At time $t = 78$ usec (Fig. 33c), the crack
has propagated to a point nearly 0.075 in. from the
top of the cylinder and the same distance from the
bottom of the cylinder. Additionally, flexural tensile

cracks have developed on either side of the cylinder
along the horizontal diameter. Finally, at time
$t = 93.5$ usec (Fig. 33d), the crack has propagated the
full depth of the cylinder in both directions along
the vertical diameter, and flexural tensile cracks
have developed more extensively at the sides of the
cylinder.

The cracking sequence simulated in the numerical
analysis, from the first crack until failure, for
$\sigma_t = 840$ psi is illustrated in Fig. 33. The initial crack
appears along the vertical diameter at a location
approximately 0.625 in. from the bottom of the cylin-
der, at time $t = 158$ usec (Fig. 33a). At time
$t = 160$ usec, the crack propagates in both directions
along the vertical diameter, moving downward to a
point 0.4 in. from the bottom of the cylinder, and
upward to a point approximately 0.675 in. from the
top of the cylinder (Fig. 33b). At time $t = 200$ usec,
the crack has propagated nearly to the top and
bottom edges of the cylinder, and flexural tensile

cracks have begun to form at the sides of the cylinder
along the horizontal diameter (Fig. 33c). Finally, at
time $t = 214.5$ usec, failure occurs along the vertical
diameter of the cylinder (Fig. 33d).

During the cracking sequence for this particular
simulation, a bifurcation of the crack occurs along
the vertical diameter as it grows toward the top of the
cylinder, beginning at time $t = 160$ usec (Fig. 33b).
The bifurcation begins at the center of the cylinder,
and the crack segments continue to grow apart until
they reach a distance of approximately 0.45 in. from
the top of the cylinder. At this point, the segments of
the bifurcation begin to move back towards each
other and eventually join at a distance of 0.05 in.
from the top of the cylinder.

5. CONCLUSIONS

The results of the numerical simulation for load
case 1 indicate that varying the tension cutoff value
in the concrete material model does greatly affect the
 cracking pattern and mode of failure experienced by
the specimen at high strain rates. At the static tension
cutoff value of 560 psi, cracking for the first loading
condition began at a location on the incident bar side
of the specimen (Fig. 32). Cracking of the specimen
was initiated at time $t = 58$ usec and propagated
along the vertical diameter of the cylinder without

bifurcation until it traversed the full depth of the
specimen.

When the tension cutoff value was raised to 1.5
times the static value (840 psi) for the same loading
condition, the crack initiation time was delayed until
time $t = 158$ usec. Cracking first occurred at a point
on the transmitter bar side of the specimen (Fig. 33a),
and grew towards both the top and bottom of the
cylinder, with a bifurcation occurring along the verti-
cal diameter on the incident bar side of the specimen.

This failure pattern much more closely resembles
experimental results obtained using the SHPB.
Results of high-speed photography show that cracks form first on the transmitter bar side of the specimen, and subsequently grow in both directions along the vertical diameter, with some bifurcation occurring near the center of the specimen.

The fact that cracks failed to form when the tension cutoff was raised to twice the static value (1120 psi) near the center of the specimen.

As stated previously, the strengthening effects of concrete begin to become more pronounced at strain rates above previously, the strengthening effects of concrete begin to become more pronounced at strain rates above approximately 5/sec, whereas the strain rate for load case 1 is 4.4/sec, slightly below the threshold rate.

Raising the tension cutoff value of the material model in a numerical simulation of a high strain rate test does more closely represent the actual dynamic response of a specimen subjected to dynamic loading. Care should be exercised, though, in choosing an appropriate factor by which to scale the static strength value to accurately model the specimen under consideration.

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