SINS/ANS/GPS Integration using Federated Kalman Filter
Based on Optimized Information-Sharing Coefficients

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The elemental notion of the multisensor integrated navigation system is the utilization of a medium exactitude INS in conjunction with one or more auxiliary sensors which perform as error confining sources. The SINS/ANS/GPS integration is an operative way of providing an accurate and reliable system for midcourse navigation and guidance of medium and long range weapon systems. Contained by the INS, a Kalman filter is employed which accepts the auxiliary source inputs and provide a best estimate of the navigation information as well as estimates of the modeled error sources. This paper presents a theory and implementation of Federated Kalman Filter realized with an optimized information sharing algorithm, and designs the integrated navigation system. Using the proposed method, there is no upshot on the global optimality of the filter. Simulation results of the integrated navigation system for a ballistic missile application demonstrate the validity of this method on improving the navigation system’s reliability, robustness and accuracy with the estimation and compensation for inertial sensors errors.

I. Introduction

Multisensor integrated navigation systems have the potential to endow with high levels of accuracy and fault tolerance. The presence of multiple data sources provides functional redundancy as well as greater observability of the desired navigation states4. Integration of strapdown inertial navigation system (SINS) with global positioning system (GPS) has received much attention for providing this augmentation but the effects of jamming on GPS receiver range from complete loss of data to reduced tracking performance and degraded navigational accuracy. Nevertheless, the mission’s performance objectives need not be made vulnerable by a dependence on the GPS data availability. Consequently, astronavigation system (ANS) is integrated with SINS/GPS to yield high performance autonomous mission capability.

Therefore, in order to achieve long duration mission accuracy and reliability requirements, GPS and ANS measurements are fused with SINS to provide a best estimate of the navigation information. This integration offers more reliable, robust and possibly more accurate navigation solution than is provided with any standalone system. In ANS, stars observations are made by charge coupled device (CCD) electro optical star sensor. Stored in the onboard computer memory is a star catalog with the celestial coordinates of each star, in some reference system, which will be used for star field recognition through some identification techniques. Once stars in the current field of view have been identified from the reference star catalog, attitude of the vehicle is estimated through an attitude algorithm.

Multisensor integration techniques are exercised in many tracking and surveillance systems as well as in applications where fidelity is a foremost apprehension25. One method for design of such systems is to make use of a number of sensors (may be different type) and to blend the information obtained from all these sensors in a central processor. The Kalman filter, with the centralized structure in multi-sensor situations, is desirable in many applications such as military surveillance, air traffic control, and mobile robots and other systems. Target tracking using multiple sensors can offer improved functioning than using a single sensor. Prior efforts in developing multisensor integration algorithms for a centralized architecture, where measurements from all sensors are sent to the central processor, have shown that momentous gains in performance are possible with using multiple sensors. If multi-sensor systems are to be able to process their data in real-time, however, the performance of the centralized Kalman filter will be mortified25.

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In a distributed system, data handling is performed at local sensors and results are passed on to the data fusion center for processing to get the ultimate outcomes\(^{16}\). The algorithms based on parallelization of the Kalman filter equations, as suggested in\(^{8}\), extend the previous results permitting one to acquire the global estimation using only local estimates devoid of information diffusion amongst sensors. An utterly decentralized system is mediated in\(^{15}\) which is based on the inter-nodal interactions between local processor units without the necessity of any central processor. Into the bargain, linear sensor fusion algorithms are exploited in\(^{2}\) with a feedback from the central processor to local processing units and lacking such a feedback. It is shown in\(^{17}\) that the track fusion formulae with feedback are, like the track fusion with no feedback, unerringly comparable to the corresponding centralized track fusion formulae. What's more, it is revealed that the feedback does trim down the covariance of apiece local tracking error. A suboptimal filtering equation with sundry categories of observations has been derived in\(^{13}\).

One more approach to enhance the fusion problem is the information filter, an algebraic-equivalent to the Kalman filter\(^{9}\), was developed\(^{1,3,7,10-12}\). The information filter is basically a Kalman filter articulated in terms of measures of information about state estimates and their related covariance. This filter has been called the inverse covariance form of the Kalman filter. Regardless of its prospective application, however, it was not extensively employed and it was sparsely comprised in literature. The brainchild of information estimation has been concisely reflected in\(^{10}\), but did not unambiguously derive the algorithm in terms of information as done in\(^{11}\), nor did they use it as a principal filtering method. Information filter algorithm given in\(^{11}\) is a Kalman filter uttered in terms of information analytic variables, which are measures of the amount of information about the parameter (state) of concern. Furthermore, a decentralized information filter was developed by\(^{5,11}\). Performance appraisal of the information matrix form of state vector fusion is imparted by\(^{5}\). Closed-form analytical solution of steady state fused covariance has been derived as a measure of performance employing this methodology.

The federated Kalman filter (FKF) configuration was contemplated as another approach for data fusion by\(^{14,4}\). It is recognized that the FKF has advantages of simplicity and fault-tolerant competence over other decentralized filter techniques. The federated filter method based on meticulous information-sharing principles makes available the globally optimal or near-optimal estimation accuracy with a high degree of fault tolerance. The federated filter structure employs sensor-dedicated local filters (LFs), and a master filter (MF) to combine or fuse the LFs outputs.

The main contribution of this paper is the implementation of the multisensor navigation system using FKF based on optimized selection of information sharing coefficients. This paper is organized in four sections. Section II entails FKF that provides a great variety of possibilities for improving the computational efficiency as well as the fault tolerant performance. In this section, an algorithm is presented for an optimized selection of information sharing coefficients of the FKF. Simulation of the integrated navigation system using a medium accuracy SINS is a subject of the Section III. Some conclusions are drawn in Section IV.

### II. Federated Kalman Filtering

The federated Kalman filter is a partitioned estimation method that employs a two stage data processing architecture, in which the outputs of sensor related LFs are subsequently combined by a large MF\(^{4}\), as illustrated in Fig 1. As indicated, each LF is dedicated to a separate sensor subsystem, and also uses data from the common reference SINS. The SINS acts as a fundamental sensor in the system, and its data is the measurement input for the MF. The data from the GPS and ANS is dedicated to the corresponding LFs, after implementation, supply their resulting solutions to the MF for the master update, yielding a global solution\(^{8}\).

The federated filtering method described herein avoids the theoretical and practical difficulties of standard Kalman filtering. This is done by means of a simple, yet effective, information sharing methodology. The advantages of information sharing implemented within the federated filter are the increased data throughput by parallel operation of the LFs, enhanced system fault tolerance by retaining multiple component solutions, and improved accuracy and stability of cascaded filter operations\(^{10}\). The basic concept of the information distribution approach in the federated filter is to divide the total system information among LFs. Next, perform the local time propagation and measurement processing, and then recombine the updated local information into a new total sum.
A. Description of Multisensor Integration System

The dynamic system and observation equations of the Kalman filter for a discrete linear stochastic system with multiple sensors are given as

\[
x(k) = F(k-1)x(k-1) + Gw(k-1)
\]

(1)

\[
z_i(k) = H_i x(k) + v_i(k), \quad i = 1, 2, \ldots, l
\]

(2)

where \(x(k) \in \mathbb{R}^n\) is the state vector, \(w(k) \in \mathbb{R}^n\) is the system noise, \(v_i(k) \in \mathbb{R}^{Pl}\) are the measurement vectors and \(v_i(k) \in \mathbb{R}^{Pm}\) are the measurement noises. \(F(k), G\) and \(H_i\) are the system, perturbation-noise, and the measurement sensitivity matrices respectively. Here, it is assumed that noises are uncorrelated Gaussian white noise sequences with mean and covariances as follows:

\[
E[v_i(k)\nu_j^T(t)] = 0, \quad i \neq j; \quad \forall k,t
\]

\[
E[w(k)v_i(t)] = 0, \quad \forall k,t
\]

\[
E[w(k)] = 0; \quad E[v_i(k)] = 0
\]

(3)

where \(E[\cdot]\) denotes the expectation and \(\delta_{kl}\) is the Kronecker delta function. \(Q\) and \(R\) are bounded positive definite matrices (\(Q>0, R>0\)). The initial state \(x(0)\) is independent of \(w(k)\) and \(v_i(k)\) and

\[
E[x(0)] = \hat{x}(0), \quad E[(x(0) - \hat{x}(0))(x(0) - \hat{x}(0))^T] = P(0)
\]

(4)

The estimation error is defined as the difference between the unknown vector and the estimate at the present time i.e. \(e_x(k) = x - \hat{x}(k)\). This estimation can be characterized as unbiased since the expected value of the estimate can be demonstrated to be equal to the expected value of the actual unknown quantity. This can be proved with couple of postulations i.e. \(E[e_x(k)] = 0\) and \(E[v_i(k)] = 0\), which are realistic assumptions that the average estimation error would be zero and the noise is also of zero mean. Then it is straightforward that \(E[\hat{x}(k)] = E[x]\), which proves the estimator to be unbiased.

B. Federated Kalman Filter Equations

Time update equations for the LFs and MF are:

\[
\hat{x}_i(k|k-1) = F_i(k)\hat{x}_i(k-1), \quad i = 1, 2, \ldots, l, m
\]

(5)

\[
P_{l^{-1}}(k|k-1) = \left[ R_i(k) P_{l}(k-1) F_i^T(k) + Q_i(k) \right]^{-1}, \quad i = 1, 2, \ldots, l, m
\]

(6)

Measurement update equations for the LFs are:

\[
P_{l^{-1}}(k|k-1) = P_{l^{-1}}(k|k-1) + H_i^T R_i^{-1}(k) H_i, \quad i = 1, 2, \ldots, l
\]

(7)

\[
P_{l^{-1}}(k|k) = P_{l^{-1}}(k|k-1) + H_i^T R_i^{-1}(k) z_i(k), \quad i = 1, 2, \ldots, l
\]

(8)

where \(\hat{x}_i(k|k-1) \in \mathbb{R}^{Pl}\) is the priori estimate of \(x(k)\), \(Q_i \in \mathbb{R}^{Pl \times Pl}\) is the covariance matrix of system noise, \(\hat{x}_i(k) \in \mathbb{R}^{Pl}\) is the posteriori estimate of \(x(k)\), \(P_i(k|k-1) \in \mathbb{R}^{Pl \times Pl}\) is the priori covariance matrix of estimation errors, \(P_i^{-1}(k) \in \mathbb{R}^{Pl \times Pl}\) is the inverse posteriori covariance of estimation errors known as information matrix. In the presented integrated navigation system, \(i = 1, 2, m\), where \(m\) stands for MF.

One of the interesting things to note about the navigation error residual covariance can be seen from Eq. (7). This implies that the updated navigation error residual covariance is less than either the updated navigation error residual covariance or the measurement error covariance. Master filter (fusion) equations are as follows:

\[
P_{f^{-1}}(k) = P_{m^{-1}}(k) + \sum_{i=1}^{l} P_{l^{-1}}(k)
\]

(9)

\[
P_{f^{-1}}(k) \hat{x}_f(k) = P_{m^{-1}}(k) \hat{x}_m(k) + \sum_{i=1}^{l} P_{l^{-1}}(k) \hat{x}_i(k)
\]

(10)
where \( \hat{x}_m(k) = \hat{x}_m(k|k-1) \) and \( \hat{x}_m(k|k-1) = \hat{x}_f(k|k-1) \), \( P_f^{-1}(k) \in \mathbb{R}^{nf \times nf} \) is the inverse of the fused covariance, \( \hat{x}_f(k) \in \mathbb{R}^{nf} \) is the fused state estimate.

In fact, the estimates of the LFs are correlated because they use a common dynamic system. In order to eliminate this correlation, the process noise and the state vector covariance are set to their upper bounds i.e.

\[
Q_i(k) = \beta_1^{-1}(k)Q, \quad i = 1, 2, \cdots, l, m
\]

\[
P_i(k) = \beta_1^{-1}(k)P_f(k), \quad i = 1, 2, \cdots, l, m
\]

where \( \beta_1(\geq 0) \) is an information sharing coefficient satisfying the following condition:

\[
\beta_1 + \beta_2 + \cdots + \beta_l + \beta_m = 1
\]

When the fusion reset mode is realized, the covariance matrices of LFs and MF are reset by using Eqs. (11) and (12) and the estimates of the MF and LFs are also reset by the global solution i.e.

\[
\hat{x}_i(k) = \hat{x}_f(k), \quad i = 1, 2, \cdots, l, m
\]

### C. Information Sharing in FKF

Applying the maxim of conservation to the estimated error covariance results in information sharing among the filters. The information residing in the estimated error covariance can be construed to be the memory of the filter system. To remain optimal, the filter must combine the local estimates into a single estimate every cycle. After the combination step, at the start of the next cycle, the estimate or memory can be fed back to the local filters with the master filter retaining a part or none of the information. At this point, all estimates in the system are equal and the information is distributed. If feedback is implemented, conservation of information dictates that the net sum of the information in the filter system before and after the feedback operation must remain the same; simply but, every feedback operation requires an adjustment of the covariances to reflect information sharing.

In general, when the information is redistributed about the local and master filters, the sum of local and master filter matrices after feedback must be equal the master filter information before feedback. The simplest form of parametric control is to choose parameters \( \beta_i \) and \( \beta_m \) with summation one and set the error covariance of the individual and master filters after feedback respectively equal to \( \beta_i \) and \( \beta_m \) times the master filter covariance before feedback.

In the FKF implementation, the system dynamic model is applied to each filter, but the statistical weight of the dynamics driving noise is distributed such that information is preserved. The individual LFs are considered sub-optimal for this reason. The LFs then combine to give an optimal global estimate through the MF. The information can be returned to the LFs, or kept in the MF, according to several modes described below. The FKF reset modes are summarized in table 1.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Reset Information</th>
<th>MF Reset</th>
<th>Information Resides</th>
</tr>
</thead>
<tbody>
<tr>
<td>No reset</td>
<td>None</td>
<td>Yes</td>
<td>LFs</td>
</tr>
<tr>
<td>Fusion reset</td>
<td>( P ) and ( x )</td>
<td>Yes</td>
<td>LFs</td>
</tr>
<tr>
<td>Zero reset</td>
<td>( P=0 )</td>
<td>No</td>
<td>MF</td>
</tr>
<tr>
<td>Rescale</td>
<td>Rescaled ( P )</td>
<td>No</td>
<td>Shared LFs/MF</td>
</tr>
</tbody>
</table>

Our aim is to use fusion rest mode and find the optimized information sharing coefficients for the FKF. Suppose that we start with a fused solution \( P_f^{-1}(k), \hat{x}_f(k) \). Now, divide that solution so that the \( i = 1, 2 \) LFs plus MF each receive fractions \( \beta_i \) of the total information in accordance with Eq. (12).

### D. Process Sharing

To uphold the optimal filter, process noise information must be warily accounted for. The master filter must recombine the LF estimates whenever process noise is injected, because the nature of the KF is to retain information about the process noise only for the current cycle. The total process information, represented by the matrix \( Q \) shared among the LFs, must sum up to the true net process information. To preserve optimality, the parameters controlling the process noise must be related to the parameters controlling the information sharing; using the same parameters for information and process sharing in the respective filters satisfies the required relationship and is the preferential method because of its simplicity. The process sharing is realized using Eq. (11).
E. Optimized Information Sharing Coefficients

Design engineers are occasionally required to select, from among a set of feasible designs, the design that is supreme in some sense, such as the least expensive design or the design with the highest performance. This process is referred to as optimal design or simply optimization. Here, we apply the principle of optimality to derive a relationship for optimized information sharing coefficients for the covariances those are fed back to the LFs.

The covariance matrix $P$ and its propagation in time are vital in both describing and analyzing physical test results and comparing them to theoretical predictions. Error budgets for nearly all costly and complex systems imply recognition of the covariance matrix $P$. Note that the sum of diagonals of this matrix $P$ is the root-sum-squared of all of the variances in the stochastic processes. This number is a significant indication of the overall performance of the system and often is employed as “metric to minimize” when making decisions about how to consider navigation data.

The sum of all the elements on the diagonal is called the trace of the covariance matrix $P$. To end with, note the physical significance of $P$ and realize that the diagonals must always be positive if they denote variances. Mathematicians add a further constriction and state that $P$ must be positive definite (PD). To solve state space system error equations, covariance matrix is propagated continuously so that error estimates and covariances are available at the discrete time when the measurements transpire.  

With the use of measurement data given in Eq. (2), the least square estimator yields the following state estimation error (SEE) covariance matrix

$$P(k) = E[(x - \hat{x}(k))(x - \hat{x}(k))^T]$$  \hspace{1cm} (15)

If the measurement system is observable then all elements of the SEE covariance matrix are finite, and then state estimation error can be directly calculated by using the SEE covariance matrix $P$ as follows:

$$\mathcal{R}(k) = E[(x - \hat{x}(k))^T(x - \hat{x}(k))] = \text{trace}\left( E[(x - \hat{x}(k))(x - \hat{x}(k))^T]\right) = \text{trace}(P(k))$$  \hspace{1cm} (16)

The performance of state estimator is appraised by this state estimation error, and thus it is one potential method to decide on a measurement set which minimizes the state estimation error. Therefore, the estimation errors of the filter states will be taken as the performance index of the state estimator. Then the performance index of the state estimator can be evaluated as follows:

$$C(k) = E[(x - \hat{x}(k))^TW(x - \hat{x}(k))]$$  \hspace{1cm} (17)

where $W$ is the positive semi-definite weighting matrix. As the optimal estimate is independent of $W$, we may opt $W=I$, where $I$ is the identity matrix, yielding

$$C(k) = \text{trace}(P(k))$$  \hspace{1cm} (18)

This is equivalent to minimizing the length of the estimation error vector and we wish to minimize the trace of $P$ because it is the sum of mean square errors in the estimates of all the elements of the state vector. If there are no uncertainties in the process and measurement noise covariances, the performance index $C$ attains a global minimum using the standard Kalman filter. But if there were uncertainties in $Q$ and $R$, $C$ would not attain a minimum. Here, we presume that there are no uncertainties in $Q$ and $R$. Minimization of the cost function given in Eq. (18) is equivalent to maximization of $\text{trace}(P^{-1}(k))$ i.e.

$$C(k) = \text{trace}(P(k)) \rightarrow \min\bigg\{ C(k) = \text{trace}(P^{-1}(k)) \rightarrow \max \bigg\}$$  \hspace{1cm} (19)

Using the principle of information fusion reset for the LFs and MF, we write

$$P_i(k|k-1) = \beta_i^{-1}(k)P_f(k|k-1), \quad i=1,2,\ldots,l,m$$  \hspace{1cm} (20)

Now, using the definition given by Eq. (19), Eq. (20) becomes

$$\beta_i^{-1}(k)\text{trace}\left(P_f(k|k-1)\right) = \text{trace}\left(P_i(k|k-1)\right)$$

or

$$\beta_i^{-1}(k) = \frac{\text{trace}\left(P_i(k|k-1)\right)}{\text{trace}\left(P_f(k|k-1)\right)}$$  \hspace{1cm} (21)

The optimized information sharing coefficients estimated through Eq. (21) ensure the conservation of information as given by

$$\beta_{\text{opt}}(k) + \sum_{i=1}^{l} \beta_i(k) = 1$$  \hspace{1cm} (22)
F. Summary of the FKF Algorithm

In summary, the FKF implementation for obtaining an optimal global estimate can be summarized as follows:\textsuperscript{23}:
1) Initialize FKF; input system and measurement noise covariance, and information sharing coefficients
2) Using Eqs. (11), (12) and (14), compute initial state vector and covariances for process and measurement noises of LFs and MF
3) For \( k=1 \), accomplish time update of LFs and MF using Eqs. (5) and (6)
4) Perform measurement update of LFs using Eqs. (7) and (8)
5) Realize the information fusion via Eqs. (9) and (10)
6) Using Eq. (21), calculate optimized information sharing coefficients for the LFs for the next cycle
7) Increment \( k \) an go to step 2

III. Simulation and Results

The primary sensor subsystems used in the integrated navigation system described in this paper are SINS, ANS and GPS. SINS used in the simulation generates position, velocity and attitude information, ANS provides attitude information, and GPS outputs position and velocity solutions. The aspiration of the integrated system is to endow with increased accuracy and improved estimates of the SINS error sources. Velocity and position information available from GPS in conjunction with SINS yields observation to the LF1. The ANS provides attitude of the vehicle which is coalesced with the SINS attitude information to output an observation to the LF2. ANS augmentation comes into effective 40 seconds after lift off of the missile when it attains altitude above 22 km from onwards stars are observable unobstructed\textsuperscript{24}.

The powered flight phase of the vehicle is the most decisive phase during which, with the help of navigation information, vehicle is placed on a trajectory with flight conditions which are appropriate for the desired target. Consequently, simulation is carried out for the powered flight trajectory of the ballistic missile which in rectangular coordinates as expressed in the local level (East-North-Up) frame is shown in Fig. 2. A random measurement noise is also included in the simulations. Here, the SINS computations are carried out at 80 Hz, the ANS aiding is provided at 1 Hz and the GPS updates are available at 1Hz.

The discrete FKF realization used in this paper is the direct feedback where the estimated attitude errors are fed back to the SINS, thus minimizing the evolution of the observed attitude, velocity and position errors those are to be delivered as an observation to the LFs. In this simulation, quaternion is obtained from the corrected attitude matrix and is fed back for attitude error compensation.

A. Coordinate Frames

Inertial navigation system theory necessitates precise description of the coordinate frames. Definitions of the coordinate frames employed in this paper are as follows:
1) The Earth-centered Earth fixed frame (\( e \)-frame) is used for position location definition. Its origin is at the Earth center, \( x \)-axis points to the point of intersection of Greenwich meridian and equatorial plane, \( z \)-axis is perpendicular to equatorial plane pointing to North pole and \( y \)-axis is defined by right hand rule.
2) The Earth-centered inertial frame (\( i \)-frame) is fixed in an inertial space (i.e. fixed relative to the “fixed stars”). It is a right handed system with its origin at Earth center with \( z \)-axis normal to the equatorial plane, \( x \)-axis in equatorial plan and \( y \)-axis complements the right handed system. In this paper, \( x \)-axis of \( i \)-frame is chosen to coincide with \( x \)-axis of the \( e \)-frame.
3) The launch-inertial frame (\( il \)-frame) has its origin at launch point. Its \( y \)-axis is vertical upward, \( x \)-axis is horizontal and lies in nominal launch plane and \( z \)-axis is horizontal and perpendicular to nominal launch plane, rightward.
4) The body frame (\( b \)-frame) has its \( x \)-axis along longitudinal axis of the vehicle, \( z \)-axis is perpendicular to longitudinal plan of symmetry and \( y \)-axis complements the right handed system.

B. Notion of Inertial Navigation

The elemental equation of inertial navigation comes directly from Newton's Second law as
\[
\ddot{r}^i = \dot{v}^i = c_b^i \dot{r}^b + g^i
\] (23)
where \( r \) is the position vector in \( il \)-frame, \( C^i_b \) is the transformation matrix between indicated frames, \( f \) is the total specific force vector, and \( g \) is the acceleration due to gravity. This equation is integrated once to obtain velocity and a second time to obtain position. Accelerometers measure specific force, which does not include forces due to gravity. Gyros measure the angular motion of the SINS body with respect to inertial space. SINS uses an internal gravity model to estimate the acceleration due to gravity based on its current estimate of position and attitude. The major sources of error in SINS include initialization errors, system alignment errors, sensor errors, gravity model errors and quantization and computational errors. In this contemporary error analysis of SINS, errors equations for attitude, velocity and position are presented in the subsequent sections.

C. System Dynamical Model

In this paper, space-stabilized mechanization is used for the SINS implementation. It is conceptually the simplest of all possible system implementations, since Newton’s laws are most simply stated in an inertial frame of reference. The space-stabilized inertial navigation system outputs navigation parameters in an inertially non-rotating frame. Thus, this mechanization is free of Earth’s rotation and transport rate\(^{25}\). This section will give error analysis of the space-stabilized SINS which will consider the effect of all of the known major error sources i.e. gyro drift, accelerometer bias, and gravity anomalies.

1) \textbf{Attitude Error Model}

The attitude error for the space-stabilized SINS mechanization is the orthogonal transformation error between the body and \( il \)-frame. In this perspective, attitude error equation can simply be articulated as

\[
\Phi^i_b = C^i_b (\Phi^b_i + \omega^b_i) \tag{24}
\]

where \( \Phi \) represents SINS axes misalignment angle or attitude error; \( \epsilon \) and \( w_\epsilon \) denote gyros constant and random drifts respectively; \( g \) in the subscript stands for gyro.

2) \textbf{Velocity Error Model}

Velocity error equation in the space-stabilized mechanization is given as

\[
\delta V^i_b = V^i_b \times \Phi^i_b + C^i_b (\delta \Phi^b_i + \delta w_a^b_i) \tag{25}
\]

where \( f \) is the specific force measured by accelerometers; \( \delta g \) is the acceleration error due to gravity; \( \nabla \) and \( w_a \) represent accelerometer constant and random bias respectively; \( a \) in the subscript stands for accelerometer.

In the simulation of SINS, more accurate expression for the gravitational acceleration is employed with an ellipsoidal (or more precisely, spheroidal) Earth model. Such a more precise approximation accounts for the Earth oblateness by including the second order gravitational harmonic term of the Earth’s gravitational field model. While deriving expression for the acceleration error due to gravity, inverse square gravity model is considered for the spherical Earth model. It is due to avoid complexity in the velocity error model. The gravity model used in the derivation of error equation is given as

\[
g^i_b = -\frac{\mu}{r^3} \left[ \begin{array}{c} r_x^i \\ r_y^i \\ r_z^i \end{array} \right] \tag{26}
\]

where: \( \mu = \text{product of the Earth’s mass and universal gravitational constant} = (3.9860305 \times 10^{14}) [m^3/s^2] \)

\( r = \text{magnitude of the position vector} = (r_x^i)^2 + (r_y^i + r_0)^2 + (r_z^i)^2 \)

\( r_0 = \text{the mean Earth’s radius} = 6371004 [m] \) and \( r_x^i, r_y^i, r_z^i \) = components of the position vector

Hence, the gravity gradient \( \delta g \) for a two body gravity field can be written as follows:

\[
\delta g^i_b = \frac{\delta g^i_b}{\delta r^i_b} \delta r^i_b = \left[ \begin{array}{ccc} \partial g^i_x / \partial r_x^i & \partial g^i_y / \partial r_y^i & \partial g^i_z / \partial r_z^i \\ \partial g^i_y / \partial r_x^i & \partial g^i_y / \partial r_y^i & \partial g^i_y / \partial r_z^i \\ \partial g^i_z / \partial r_x^i & \partial g^i_z / \partial r_y^i & \partial g^i_z / \partial r_z^i \end{array} \right] \left[ \begin{array}{c} \delta r_x^i \\ \delta r_y^i \\ \delta r_z^i \end{array} \right] = \left[ \begin{array}{ccc} 1 - 3(r_y^i)^2 / r^2 & 3(r_y^i + r_0) / r^2 & -3(r_z^i)^2 / r^2 \\ -3(r_x^i)^2 / r^2 & 1 - 3(r_x^i)^2 / r^2 & -3(r_z^i + r_0) / r^2 \\ -3(r_x^i + r_0)^2 / r^2 & 3(r_x^i)^2 / r^2 & 1 - 3(r_z^i)^2 / r^2 \end{array} \right] \left[ \begin{array}{c} \delta r_x^i \\ \delta r_y^i \\ \delta r_z^i \end{array} \right] \tag{27}
\]
3) Position Error Model

Position error equation in rectangular coordinates is given as

$$\delta \hat{r}^i = \delta \hat{r}^i$$

(28)

D. Conversion of Geodetic Position to Rectangular Coordinates

The customary reference frame for terrestrial navigation system is the $e$-frame expressed as a function of latitude, longitude and height. Note that height is defined as the perpendicular distance above an assumed Earth shape known as the geoid which is mathematically an ellipsoid of revolution meant to approximate the true shape of the Earth. The $e$-frame is the coordinate system most commonly associated with the GPS. Since longitude is measured relative to a fixed point on the surface of the Earth, $e$-frame rotates at the Earth rate. As an alternative reference frame, it is sometimes convenient to define the $e$-frame in rectangular coordinate system. The transformation of $e$-frame from geodetic to rectangular system is fairly easy as given in subsequent lines.

To precisely apply the geodetic $e$-frame coordinate system, the exact shape of the Earth’s ellipsoid must be given. This shape is derived from measurement, and has been defined differently by different groups. The assumed shape used in the GPS is known as WGS–84 and contains the parameters given in Table 2.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>3.1415926535898</td>
<td>Pi</td>
</tr>
<tr>
<td>$a$</td>
<td>6378137.0 m</td>
<td>Earth semi-major axis</td>
</tr>
<tr>
<td>$b$</td>
<td>6356752.3142 m</td>
<td>Earth semi-minor axis</td>
</tr>
<tr>
<td>$\omega_e$</td>
<td>$7.2921151467 \times 10^{-5}$ rad/s</td>
<td>Value for Earth rotation rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$3.986005 \times 10^{14}$ m$^3$/s$^2$</td>
<td>Value for Earth’s gravitational constant</td>
</tr>
</tbody>
</table>

The distance from the surface to the $z$-axis along the ellipsoid normal (the plumb line) is then found as a function of geographic latitude as

$$r_p(\phi) = \frac{a}{\sqrt{1 - e^2 \sin^2(\phi)}}$$

(29)

where the eccentricity is defined as $e = \sqrt{\sin^2(\phi)}$ and the ellipsoid flatness $f = (a - b)/a$.

Figure 3 shows the relationship of the east-west radius of curvature $r_p$, the height $h$ and the geographic latitude $\phi$ for a point $P$. Position vector in $e$-frame expressed as a function of geodetic coordinates is given as

$$\vec{r}^e = [(r_p + h)\cos \phi \cos \lambda, (r_p + h)\cos \phi \sin \lambda, [(1 - e^2)r_p + h]\sin \phi]^T$$

(30)

It is desired to further transform the position in $e$-coordinates into what will be termed here the $i$-$l$-frame. This transformed position vector is given as

$$\vec{r}^i = C^i_e C^e_r \vec{r}^e$$

(31)

where $C^i_e$ and $C^e_r$ are the matrices for transformation between the indicated frames.

E. System Measurement Model

Measurements for the LFs:

$$z_1 = [V_{x,INS} - V_{x,GPS}, V_{y,INS} - V_{y,GPS}, V_{z,INS} - V_{z,GPS}, \delta V_{x,INS} - \delta V_{x,GPS}, \delta V_{y,INS} - \delta V_{y,GPS}, \delta V_{z,INS} - \delta V_{z,GPS}]^T$$

where $(\delta \phi_x, \delta \phi_y, \delta \phi_z)$ represent attitude errors of the ANS.

$$z_2 = [V_{x,INS}^i - V_{x,GPS}^i, V_{y,INS}^i - V_{y,GPS}^i, V_{z,INS}^i - V_{z,GPS}^i, \delta r_{x,INS}^i - \delta r_{x,GPS}^i, \delta r_{y,INS}^i - \delta r_{y,GPS}^i, \delta r_{z,INS}^i - \delta r_{z,GPS}^i]^T$$

$$z_2 = [\delta \nu_x + \sigma_x, \delta \nu_y + \sigma_y, \delta \nu_z + \sigma_z, \delta \sigma_x, \delta \sigma_y, \delta \sigma_z]^T$$

where $(\sigma_x, \sigma_y, \sigma_z)$ represent velocity and position errors of the GPS.
State vectors of LFs and MF:

\[ x_1 = x_2 = x_m = [\phi_x, \phi_y, \phi_z, \delta V_x, \delta V_y, \delta V_z, \delta r_x, \delta r_y, \delta r_z, \delta e_x, \delta e_y, \delta e_z, \nabla_x, \nabla_y, \nabla_z]^T \]  \hspace{1cm} (34)

Measurement matrices for LFs:

\[ H_1 = \begin{bmatrix} I_{3\times3} & 0_{3\times12} \end{bmatrix} \]
\[ H_2 = \begin{bmatrix} 0_{6\times3} & I_{6\times6} & 0_{6\times6} \end{bmatrix} \]  \hspace{1cm} (35)

**F. Initial Conditions**

In the simulation, the initial value \( \hat{x}_f(0) \) of state vector \( x_f \) is chosen as zero and the initial alignment errors are also assumed to be zero. \( P_f(0) \) and \( Q \) are chosen as follows:

\[
\begin{align*}
P_f(0) &= \text{diag}\left[ P_{\phi_i}, P_{r_i}, P_{e_i}, P_{\nabla_i}, P_{\delta_i} \right], \quad i = x, y, z \\
Q &= \text{diag}\left[ (10^{-3})^2, (0.1)^2, (10)^2, (0.01^\circ/h)^2 \quad \text{and} \quad P_{\delta_i} = (100\mu g)^2 \right] \\
R_1 &= \text{diag}\left[ (5^\circ)^2, (5^\circ)^2, (5^\circ)^2 \right], \quad R_2 = \text{diag}\left[ (0.2m/s)^2, (0.2m/s)^2, (0.2m/s)^2, (10m)^2, (10m)^2 \right] 
\end{align*}
\]

where \( P_{\phi_i} = (10^{-3})^2 \), \( P_{r_i} = (0.1)^2 \), \( P_{e_i} = (10)^2 \), \( P_{\delta_i} = (0.01^\circ/h)^2 \) and \( P_{\delta_i} = (100\mu g)^2 \).

Initial values of the information sharing coefficients are assumed to be \( \beta_1(0) = \beta_2(0) = 0.4 \) and \( \beta_m(0) = 0.2 \).

The LFs and the MF are initialized as

\[
\begin{align*}
Q_i(0) &= \beta_i^{-1}(0)Q \\
P_f(k) &= \beta_i^{-1}(0)P_f(0) \\
\dot{x}_i(0) &= \dot{x}_f(0) 
\end{align*}
\]

**G. Results**

Simulation results for all the system states are depicted in the following Figs. 4 - 9.

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**Figure 4. Axes misalignment angles**
Figure 5. Velocity errors

Figure 6. Position errors
Figure 7. Gyros drift

Figure 8. Accelerometers bias
IV. Conclusions

This paper investigates into SINS/ANS/GPS integration using FKF for enhanced navigational performance and greater observability of the desired navigation states. An algorithm for selection of optimized information sharing coefficients of FKF is presented that brings about no upshot on the global optimality of the filter. Simulation results show that the presented scheme can estimate and compensate the axes misalignment angles, the gyro drift and accelerometers bias. Since gyro induced drift errors are the only error variables that contribute to misalignments, it is these gyro errors that the ANS updates are effective in estimating and compensating for, as well as the position and velocity errors that occur due to the misalignments. Velocity and position updates from the GPS are capable to estimate and compensate velocity, position and accelerometers bias. The optimal implementation of integrated system is known as the direct feedback method where the estimated inertial sensors errors are fed back to the SINS, thus minimizing the growth of the observed error that is delivered as an observation to the FKF.

References

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