Wax deposition modeling is complicated under oil/gas two-phase pipe flow and therefore remains poorly understood. One-dimensional empirical heat and mass transfer correlations are unreliable for deposition modeling in stratified flow, due to non-uniform deposit across the pipe circumference. A mathematical model has been developed to predict the deposit thickness and the wax fraction of deposit in oil/gas stratified pipe flow using a unidirectional flow analysis of non-isothermal hydrodynamics and heat/mass transfer. The predictions for wax deposition are found to compare satisfactorily with experimental data with three different oils for single phase and oil/gas stratified pipe flow. In particular, the reason that the deposit forming a crescent shape at the cross section of pipe observed in different experiments is revealed, based on the non-uniform circumferential distributions of two most important parameters for the wax deposition, diffusivity at oil–deposit interface, and the solubility gradient at the oil–deposit interface at different time.

Wax deposition mechanism

In the case of single phase oil flow, several mechanisms for predicting wax deposition have been proposed, such as molecular diffusion, shear dispersion, heat transfer, Brownian diffusion, gravity settling, and shear removable. Molecular diffusion and heat transfer have been widely believed to be the most prevalent mechanisms. These two mechanisms for modeling the wax deposition differ in the treatment of the oil–deposit interface temperature. The heat transfer modeling approach assumes that the oil-interface temperature remains constant and equal to the wax appearance temperature (WAT) of the oil phase throughout the deposit growth process and considers the deposit process to be analogous to liquid-to-solid phase transformation with crystallization process. However, this important assumption in the heat transfer approach, that the oil–deposit interface temperature is constant, has not been verified experimentally in pipe flow without significantly interrupting the flow and heat-transfer profile.

In contrast, the molecular diffusion approach assumes that the oil–deposit interface temperature is variable, less than the WAT for the deposition to occurs, and increases from close to the pipe wall temperature initially to the WAT at steady state. And the deposit growth stops once the oil–deposit interface temperature becomes equal to WAT. The increase of the oil–deposit interface temperature with time is due to the insulation effect of the buildup deposit layer. This effect has been validated from the increase of the outlet temperature, even though the decrease in the coolant temperature, which has been observed multiple times.

Wax deposition in single phase flow

Singh et al. developed a comprehensive mass balance model which could study the effects of aging in wax
Wax deposition in oil/gas stratified flow

In recent years, more and more attention has been paid to the study on wax deposition in multiphase flow, with the exploration of deep-water oilfields. The study on wax deposition under the oil/gas two-phase flow condition encountered in most subsea lines and the additional gas phase adds to the complexity of understanding wax deposition phenomena. Wax deposition for oil/gas two-phase flow has been proved to be flow pattern dependent. A typical flow map of an oil/gas two-phase flow in horizontal pipeline is shown in Figure 1.

Oil/gas stratified flow will be examined in this research because it is considered to be among the simple and fundamental flow configurations in horizontal and slightly downward inclined two-phase systems of a finite density difference, and it is frequently encountered in practical applications and theoretical points. Additionally, the oil phase has a relatively large contact area with the pipe wall, allowing deposition to occur. There have been very few published studies reported on the effect of oil/gas stratified two-phase flow on the phenomena of wax deposition. Only two experimental studies on oil/gas stratified flow wax deposition are currently available, such as Matzain et al. and Gong et al. The observation results of the experiment proves that there is no wax deposition on the wall contacting with gas, and deposits only occurs on the wall contacting with oil. And the deposition forms a crescent shape at the cross section, in the both two studies. Furthermore, Hoffmann et al. observed the irregular shape of the deposit clearly for stratified oil–water flow where wax only deposited in the upper half of the pipe, which is no longer circular. And it is determined by the laser light projection and the caliper measurements. However, they didn’t give the reason why the deposition forms a crescent shape. The shape of the deposit is important to correctly determine the pressure-drop, temperature-drop especially during high oil holdup. The crescent shape may significantly increase the interfacial and wetted wall perimeters by wax deposit relative to the planner oil–deposit surface, even though the total amount of wax deposit may remain the same. Gong et al. reported that the equivalent thickness (deposit volume) of the deposit for the stratified flow increased with the superficial gas velocity increasing under the same liquid superficial velocity conditions (i.e., oil holdup decreasing). The explanation of this trend speculated by Gong et al. is the combination of the deposition-hindering factor and deposition-
promoting factor. However, there is no available model developed for stratified flow in their study. So, a wax deposit model for stratified two-phase flow is developed to elucidate it.

There are very few models to predict stratified oil/gas two-phase wax deposition. The currently available models, such as Apte et al., simply use the heat transfer correlation made by Kim et al. Predictions of simulator are tuned to experimental data by adjusting the film heat transfer, diffusion coefficients, and the thermal conductivity of the wax deposit. Furthermore, these models can only give the average thickness of wax deposition for whole inner wall of test section without accounting for non-uniform deposit across the pipe circumference, except the study of Huang et al., proposed a deposition model strictly for stratified oil–water channel flow.

There is no model available that accounts for the effects of multiphase flow on wax deposition mechanisms for oil/gas two-phase pipe flow. In wax deposition modeling, knowledge of the hydrodynamics and heat/mass transfer are the major challenges in two-phase flow. Recent advances in computational fluid dynamics have made it possible to calculate multiphase transport phenomena and multiphase wax deposition without using correlations, such as the work of Huang et al. The model is only appropriate for oil–water flow between two parallel plates, not in the circular pipe used in subsea field-scale. The practicality of the model is limited.

In this research, a three-dimensional (3D) model is established using a unidirectional flow assumption, to reduce the computational intensity, to predict wax deposition in oil/gas stratified flow in the circular pipe, as shown in Figure 2. In this case, wax deposition occurs only at the lower part of the pipe wall in contact with oil. There was no deposit on the top portion of the pipe.

The model is applied to simulate an experimental flow loop for wax deposition, and the predicting results of deposit thickness along the pipe circumference forming a crescent shape at the cross section and compare favorably with experimental measurements. Furthermore, the explanation for the thickness along the circumference in a crescent shape can be quantified and reasonably described.

**Wax Deposition Model Development**

The two phases in stratified gas–liquid two-phase flow may result in laminar–laminar (L–L), laminar–turbulent (L–T), turbulent–laminar (T–L), or turbulent–turbulent (T–T) regimes. In general, the gas phase result in turbulent and the liquid (i.e., oil) phase result in lamina or turbulent. Here, a deposition model is developed for non-isothermal laminar flow of gas and turbulent flow of gas in oil/gas stratified flow in the circular pipe. Based on the recommendations of Huang et al., To model oil/gas two-phase wax deposition, non-isothermal hydrodynamics, heat and mass transport phenomena need to be calculated for three phases (oil, gas, and deposit). As the deposit grows on the pipe wall, the oil–gas interface and oil–deposit interface are two moving boundaries that need to be updated at each time step.

**Hydrodynamics in non-isothermal flow**

As the temperature of the fluid decreases along the pipe, the oil phase flows with smaller velocity and larger cross sectional area and the gas phase flows with larger velocity and smaller cross sectional area. For the oil and gas flow in circular cross-section pipe is shown schematically in Figure 2.

The non-circular and irregular oil and gas domains in stratified pipe flow, as shown schematically in Figure 3, are conveniently modeled with the bipolar coordinate system, described analytically by

\[ x = B \frac{\sinh \eta}{\cosh \eta - \cos \zeta} \]  
\[ y = B \frac{\sin \zeta}{\cosh \eta - \cos \zeta} \]  
\[ z = z \]

where the length \( B \) is calculated by

\[ B = R \sin \theta_0 \]

where \( \theta_0 \) represents the upper section of the pipe wall, which bounds the gas phase.

The scale factors of bipolar coordinate system are described as follows:

\[ l_x = \frac{B}{\cosh \eta - \cos \zeta} \]  
\[ l_y = \frac{B}{\cosh \eta - \cos \zeta} \]  
\[ l_z = 1 \]

The change of hydrodynamics along pipe is gradual due to negligible decrease in the fluid temperatures for a calculation pipe section. The assumption that a unidirectional-flow at every axial position is available, so the velocity profile can be found from momentum balance equations shown in Eq. 8 in the bipolar coordinate system.

\[ \frac{1}{l_x l_z} \frac{\partial}{\partial \zeta} (\Gamma_w l_x \frac{\partial w}{\partial \zeta}) + \frac{1}{l_y l_z} \frac{\partial}{\partial \eta} \left( \Gamma_w \left( l_z \frac{\partial w}{\partial \eta} \right) \right) = \frac{dP}{dz} \]  

The quantity \( \Gamma_w \) is the effective viscosity defined as the sum of the molecular viscosity \( \mu_m \) and a turbulent viscosity \( \mu_t \), \( \Gamma_w = \mu_m + \mu_t \). For laminar flow \( \mu_t = 0 \). For turbulent flow, the
Prandtl mixing length theory is used here for the eddy viscosity, which views the Reynolds stresses as an effective turbulent viscosity,

\[ \mu_t = \mu_{m}^{l} \left| \frac{\partial w}{\partial x} \right| = \mu_{m}^{l} \left| \frac{\partial \cos \eta - \cos \xi}{B} \right| \]  \hspace{1cm} (9)

where \( l_m \) is the mixing length, which can be calculated by the method of Shoham and Taitel.\(^3\)

It is noted that a good prediction of interface shape is important when performing stratified gas–liquid two-phase flow calculations.\(^3\) The interface of gas–liquid two-phase flow in pipe can be assumed as arc shape that suggested by Li et al.,\(^37\), as shown in Figure 3. The upper section of the pipe wall, which bounds the gas phase, is represented by \( \theta = \theta_0 \). The bottom of the pipe wall, which bounds the liquid phase, is represented by \( \theta = \theta_0 + \pi \). The interface is represented by a circular segment centered at \( O_2 \) and is defined by \( \theta = \theta^* \). In particular, when the interface is planar, it will be \( \theta^* = \pi \).

To handle the problem of deriving the interface configuration, the “double circle” model, propose by Chen et al.,\(^35\), is used to estimate the perimeter of the concave interface. The relationship between the oil phase flow area, \( H_L \), the central angle, \( \theta_0 \), and \( \theta^* \) is expressed:

\[ \begin{align*}
\theta_0 &= \frac{1}{2} \sin \left( \theta^* - \frac{1}{2} \sin \left( 2 \theta^* \right) \right) \pi H_L \\
R_2 \sin \left( \pi - \theta^* \right) &= R_1 \sin \theta_0
\end{align*} \]  \hspace{1cm} (10)

Based on extensive experimental studies of gas–liquid flow, Grolman\(^38\) related the wetted wall fraction to the liquid holdup:

\[ \Theta = \frac{\theta_0}{\pi} = \frac{\Theta_0 (\sigma_{\text{water}}/\sigma)}{0.15 + \rho_{\text{gas}}/\rho_{\text{oil}} - \rho_{\text{gas}}/\rho_{\text{oil}} + (\rho_{\text{oil}} \rho_{\text{gas}}^2 D)^{0.25} \left( \frac{w_{\text{gas}}^2}{(1 - H_L)^2} \right)^{0.8}} \]  \hspace{1cm} (12)

where \( \Theta_0 \) is the minimum wetted wall fraction corresponding to a flat interface.

So, when the liquid holdup \( H_L \) is given, the interface represented by \( \theta^* \) can be calculated by using the above method.

As shown in Eqs. 1 and 2. The bipolar coordinate is orthogonal, yielding an infinitely long. The two-phase fluid domain is defined as:

**Gas calculation domain:**

\[ \theta_0 < \xi < \theta^*, \quad -\infty < \eta < +\infty \]  \hspace{1cm} (13)

**Liquid calculation domain:**

\[ \theta^* < \xi < \pi + \theta_0, \quad -\infty < \eta < +\infty \]  \hspace{1cm} (14)

The hydrodynamic boundary condition assumes no slip condition at pipe wall, wax deposit, and oil–gas interface.

\[ \begin{align*}
\frac{\partial w}{\partial \eta} &= 0, & \text{at } \eta &= 0 \\
w_{\text{int.Gas}} &= w_{\text{int.Oil}}, & \text{at } \xi &= \theta^* \\
w_{\text{Gas}} &= 0, & \text{at } \xi &= \theta_0 \\
w_{\text{Oil}} &= 0, & \text{at } \xi &= \xi_{\text{inner}}
\end{align*} \]  \hspace{1cm} (15)

where \( \xi_{\text{coll}} \) and \( \xi_{\text{inner}} \) represent the oil–deposit interface and the inner pipe wall for the \( \xi \) coordinate, respectively. For liquid phase, \( \xi_{\text{inner}} = \pi + \theta_0 \).

The flow equations are solved separately for each phase, coupled between two phases by these boundary conditions.

For the gas phase region, the velocity boundary condition at the interface is obtained from the calculation of the liquid flow. The interfacial shear stresses calculated from the calculated gas flow are then used to provide a boundary condition for the liquid region, by assuming

\[ \mu_{\text{Gas}} \frac{\partial w_{\text{Gas}}}{\partial \xi} \bigg|_{\text{int}} = \mu_{\text{Gas}} \frac{\partial w_{\text{Gas}}}{\partial \xi} \bigg|_{\text{int}} \]  \hspace{1cm} (16)

Evidently, it is necessary to have a finite limit on the maximum value of \( \eta \) to perform the numerical simulation. This maximum is set to \( \eta_{\text{max}} = 6 \), suggested by Newton and Behnia.\(^39\)

Note that the problem described above is not closed: the pressure gradient, \( dp/\partial z \) and interface location, represented by liquid holdup, \( H_L \), are unknown. The equation that close the model come from the requirement to meet the imposed flow rates of oil and gas phases, \( Q_{\text{oil}} \) and \( Q_{\text{Gas}} \):

\[ Q_{\text{Gas}} = 2 \int_{\eta_{\text{max}}} w(x, y) dA \]  \hspace{1cm} (17)

\[ Q_{\text{oil}} = 2 \int_{\eta_{\text{min}}} w(x, y) dA \]  \hspace{1cm} (18)

where \( A_{\text{oil}} \) and \( A_{\text{Gas}} \) are the computational area of oil and gas phase, respectively.

It is noted that the assumption that the volumetric flow rates of the oil and gas are constants over the entire pipeline, is valid in the modeling of flow-loop experiments where the pipeline is relatively short or the experiment mediums are oil and air. This assumption can be invalid in the modeling of wax deposition in field scale pipelines. In a field scale pipe, pressure and temperature can change significantly along the axial direction. Depressurization at downstream locations can leads to vaporization of the light components and therefore increase the gas flow rate and decrease the liquid flow rate. Cooling of the fluid can leads to condensation of the vapor and therefore leads to an increase in the liquid flow rate and decrease in gas flow rate. Furthermore, the pressure and temperature have effect on the liquid phase composition, and thus affects the solubility of wax molecules in the liquid phase. In the end, the wax deposition profile is affected.

As the temperature of the fluid decreases along the pipe, to maintain the conservation of the mass flow rate, the pressure gradient, \( dp/\partial z \), the liquid holdup, \( H_L \), and the interface shape, \( \theta^* \), along the pipe must be calculated instead of being specified a priori. The solution is obtained by using an iterative process that combines two numerical techniques. The first is an external Newton-Raphson method aimed to adjust \( dp/\partial z \) and \( H_L \), to satisfy Eqs. 17 and 18. The second, which we call the flow solver, runs internally and involves the finite difference scheme of the nonlinear problem given by Eq. 8, for given values of \( dp/\partial z \) and \( H_L \).

**Heat and mass transfer**

The temperature profile is calculated using the energy balance, Eq. 19, assuming a quasi-steady state in which axial conduction is neglected.

\[ \frac{1}{l_p} \frac{\partial}{\partial \xi} \left( \Gamma_T \frac{l_p}{l_p} \frac{\partial T}{\partial \xi} \right) + \frac{1}{l_p} \frac{\partial}{\partial \eta} \left( \Gamma_T \frac{l_p}{l_p} \frac{\partial T}{\partial \eta} \right) = w \frac{\partial T}{\partial z} \]  \hspace{1cm} (19)

where \( \Gamma_T \) is the effective thermal diffusivity which is defined as the sum of the molecular thermal diffusion \( \mu_{m}/Pr \) and eddy diffusion of heat transfer \( \mu_{e}/Pr_T \). \( \Gamma_T = \mu_{m}/Pr + \mu_{e}/Pr_T \). \( Pr \) is the Prandtl number. \( Pr_T \) is the turbulent Prandtl number, which is calculated by Kay\(^40\) and Mansoori et al.\(^41\).
The thermal boundary condition assumes a constant inlet temperature and a continuous heat fluxes at the walls.

\[
T = T_b, \quad \text{at} \quad z = 0
\]

\[
\partial T / \partial \eta = 0, \quad \text{at} \quad \eta = 0
\]

\[
T_{\text{int.Gas}} = T_{\text{int.Oil}}, \quad \text{at} \quad \xi = 0^*
\]

\[
- \lambda_{\text{Gas}} (\partial T / \partial \xi) = l_1 h_z (T_{\text{envir}} - T_{\text{wall}}), \quad \text{at} \quad \xi = \theta_0
\]

\[
- \lambda_{\text{dep}} (\partial T / \partial \xi) = l_2 h_z (T_{\text{wall}} - T_{\text{envir}}), \quad \text{at} \quad \xi = \xi_{\text{inner}}
\]

The thermal boundary condition assumes a continuous heat flux at the wall with external heat-transfer coefficient, \(h_z\), accounting for the thermal resistance of the insulation material and the coolant. Due to the computational complexity of oil–gas stratified flow, the heat transfer coefficient is assumed to be known and constant in subsequent numerical simulation.

\[
\frac{1}{h_z \xi_{\text{inner}}} = \frac{1}{\lambda_{\text{pipe}}} \ln \frac{D_{\text{out.in}}}{D_{\text{inner}}} + \frac{1}{h_o \rho_{\text{out.in}}}
\]

The thermal conductivity of the gel deposit is assumed to be a function of its wax content, as given by Singh et al.,

\[
\lambda_{\text{dep}} = \frac{[2 \lambda_{\text{wax}} + \lambda_{\text{oil}} + (\lambda_{\text{wax}} - \lambda_{\text{oil}})F_w]}{2 \lambda_{\text{wax}} + \lambda_{\text{oil}} - 2(\lambda_{\text{wax}} - \lambda_{\text{oil}})F_w}
\]

The external convective heat transfer coefficient, \(h_o\), can be calculated from correlations specific to submerged, buried, or unburied pipelines.

Due to the low temperature gradient, a slow precipitation rate is neglected, which is valid for laminar flow. 1 So, assume that there is no wax precipitation in the bulk after the oil flows pass the inlet and there is a complete precipitation at the inlet and the oil–deposit interface. The concentration profile is calculated by using Eq. 24, in which axial diffusion is neglected. Assuming that the heat and mass transfer analogy is valid here, the mass transfer equation of wax molecules in bipolar coordinate system has been shown by Eq. 24.

\[
\frac{1}{l_q \xi} \frac{\partial}{\partial \xi} \left( \Gamma_C \frac{1}{l_q} \frac{\partial C}{\partial \xi} \right) + \frac{1}{l_q \xi} \frac{\partial}{\partial \eta} \left( \Gamma_C \frac{1}{l_q} \frac{\partial C}{\partial \eta} \right) = \frac{D_{\text{eff}}}{l_q} \frac{\partial C}{\partial \xi}
\]

where \(\Gamma_C\) is the effective mass transfer diffusivity, \(\Gamma_C = \mu_m / Sc + \mu_t / Sc, Sc\) is the mass transfer diffusivity resulted by turbulence; \(Sc\) represents the Schmidt number. \(Sc = \rho_m / D_{\text{wo}}\).

The boundary conditions of mass transfer equation are described in the Eq. 25. At the pipeline inlet, it is assumed that the distribution of wax concentration remains constant at the cross section of pipeline inlet, \(C=C_{\text{inlet}}\). For the deposit phase, it is assumed that the wax concentration governed by solid–liquid equilibrium, i.e., the equilibrium concentration \(C_{\text{wo}(T)}\).

\[
C = C_{\text{inlet}}, \quad \text{at} \quad z = 0
\]

\[
\partial C / \partial \eta = 0, \quad \text{at} \quad \eta = 0
\]

\[
\partial C / \partial \xi = 0, \quad \text{at} \quad \xi = 0^*
\]

\[
C = 0, \quad \text{at} \quad \theta_0 \leq \xi < \theta^*
\]

\[
C = C_{\text{wo}(T)}, \quad \text{at} \quad \xi_{\text{odi}} \leq \xi \leq \pi + \theta_0
\]

**Deposition mechanism**

The molecular diffusion generally considered to be the dominant mechanism 1,12 in single phase, is used in the modeling for oil/gas stratified smooth pipe flow. The wax deposit model of this case is 3D because the deposit is non-uniform across the pipe circumference and along the pipe axial direction. It is more complex than the single phase because the wax mass flux caused by the concentration gradient includes two items, the mass flux from the bulk oil to the oil–deposit interface in the direction of \(\xi\) and \(\eta\), \(J_{A,\xi}\) and \(J_{A,\eta}\), as given by Eqs. 26 and 27. Especially, because the oil–deposit interface is not always on a line of a constant \(\xi\) and the wax concentration \(C\) is a function of both \(\xi\) and \(\eta\), the concentration gradient in the direction of \(\eta\) also contributes to the wax mass flux.

\[
J_{A,\xi} = - \left( -D_{\text{wo}} \frac{1}{l_q} \frac{\partial C}{\partial \xi} \right) \text{from oil to interface} \quad (26)
\]

\[
J_{A,\eta} = - \left( -D_{\text{wo}} \frac{1}{l_q} \frac{\partial C}{\partial \eta} \right) \text{from oil to interface} \quad (27)
\]

Similarly, the diffusive mass flux from oil–deposit interface into the deposit in the direction of \(\xi\) and \(\eta\), \(J_{B,\xi}\) and \(J_{B,\eta}\), given by Eqs. 28 and 29.

\[
J_{B,\xi} = -D_{\text{eff}} \frac{1}{l_q} \frac{\partial C}{\partial \xi} \text{from interface into deposit} \quad (28)
\]

\[
J_{B,\eta} = -D_{\text{eff}} \frac{1}{l_q} \frac{\partial C}{\partial \eta} \text{from interface into deposit} \quad (29)
\]

As shown in Figure 4, the difference between \(J_{A,\xi}\) and \(J_{B,\xi}\) represents the growth of the deposit thickness in the direction of \(\xi\), \(d\xi / dt\) given as

\[
\frac{d\xi}{dt} = \frac{1}{\rho_{\text{dep}} F_w(t)} (J_{A,\xi} - J_{B,\xi})
\]

\[
= \frac{1}{\rho_{\text{dep}} F_w(t)} \left( -D_{\text{wo}} \frac{1}{l_q} \frac{\partial C}{\partial \xi} \right) \text{from oil to interface} \quad (30)
\]

\[
+ D_{\text{eff}} \frac{1}{l_q} \frac{\partial C}{\partial \xi} \text{from interface into deposit}
\]

Meanwhile the difference between \(J_{A,\eta}\) and \(J_{B,\eta}\) represents the growth of the deposit thickness in the direction of \(\eta\), \(d\eta / dt\) given as

\[
\frac{d\eta}{dt} = \frac{1}{\rho_{\text{dep}} F_w(t)} (J_{A,\eta} - J_{B,\eta})
\]

\[
= \frac{1}{\rho_{\text{dep}} F_w(t)} \left( -D_{\text{wo}} \frac{1}{l_q} \frac{\partial C}{\partial \eta} \right) \text{from oil to interface}
\]

\[
+ D_{\text{eff}} \frac{1}{l_q} \frac{\partial C}{\partial \eta} \text{from interface into deposit}
\]
For \( \frac{d \delta_j}{dt} \), the deposit surface is generated by the difference of the adjacent computational grid of growth of the deposit thickness and the concentration gradient in the direction of \( \eta \) provide the driving force.

The increase of the wax fraction in the deposit is depended on the diffusive mass flux, \( J_B \), and \( J_{B,j} \) into the deposit.

\[
\frac{d F_w(t)}{dt} = \frac{1}{\rho_{dep}} \left( \frac{J_B}{\delta_i} + \frac{J_{B,j}}{\delta_j} \right)
\]

\[
= \frac{1}{\rho_{dep}} \left( -D_{wo} \frac{1}{l_0} \frac{d C}{d \eta} \right) \text{ from oil to interface}
+ D_{eff} \frac{1}{l_0} \frac{d C}{d \eta} \text{ from interface into deposit} \tag{31}
\]

For \( \frac{d \delta_j}{dt} \), the deposit surface is generated by the difference of the adjacent computational grid of growth of the deposit thickness and the concentration gradient in the direction of \( \eta \) provide the driving force.

The increase of the wax fraction in the deposit is depended on the diffusive mass flux, \( J_B \), and \( J_{B,j} \) into the deposit.

\[
\frac{d F_w(t)}{dt} = \frac{1}{\rho_{dep}} \left( \frac{J_B}{\delta_i} + \frac{J_{B,j}}{\delta_j} \right)
\]

\[
= \frac{1}{\rho_{dep}} \left( -D_{eff} \frac{1}{l_0} \frac{d C}{d \eta} \right) \text{ from interface into deposit}
+ D_{eff} \frac{1}{l_0} \frac{d C}{d \eta} \text{ from interface into deposit} \tag{32}
\]

\[
D_{wo} = 13.3 \times 10^{-12} \times \frac{T^{1.47}}{V_A^{0.71}} \times \gamma = 10.2 \times \frac{V_A}{V_A} - 0.791 \tag{33}
\]

\[
D_{eff} = \frac{D_{wo}}{1 + \alpha \frac{F_w}{1 - F_w}} \tag{34}
\]

where \( \alpha \) is the equivalent crystal aspect ratio. Singh et al.\(^7\) assumed that the \( \alpha \) varies linearly with the wax fraction given as

\[
\alpha = 1 + k_w \frac{F_w}{1 - F_w} \tag{35}
\]

where \( k_w \) is obtained by fitting the prediction with the available experimental results. It changes with the operating conditions.

It is noted that the wax fraction across the deposit layer is assumed to be constant. This is inconsistent with the layer structure of Eskin et al.\(^12\), which is wax fraction increasing nearly linearly from the deposit layer surface toward the wall during the entire deposit layer formation process. However, this deficiency does not strongly affect calculating results in our work because we do not consider thick deposits and large time scales. In practical applications, the deposit thickness is usually thin, and so assumption of the invariable wax fraction over the layer is acceptable.

**Computation procedure**

The computation includes coupling of four main modules: fluid flow hydrodynamic calculation, heat transfer calculation, mass transfer calculation, and wax deposit growth calculation. Hydrodynamic model is used to calculate the pressure gradient, liquid holdup, and velocity distribution in a section of the pipeline. The heat and mass transfer model then predicts fluid temperature and wax molecules concentration distribution. Some parameters at the deposition surface, such as the temperature, the concentration gradient of wax molecules, are obtained to get the circumference distribution of the growth rate of wax deposition and the growth rate of wax content in the deposition. According to the wax deposition model proposed, the thickness of wax deposition and the wax content in the deposition are acquired by integration simultaneously in the given time step with the Runge-Kutta-Fehlberg (RKF45) method,\(^45\) by solving the ordinary differential equations at the end of that time step. Finally, the calculation sequence is repeated, taking into consideration the newly deposited wax layer at the end of each time step, until the setting simulation time. The algorithm scheme for the calculation of the thickness and the wax content of wax deposition is shown in Figure 5.
The wax concentration in the bulk oil, $C_{\text{inlet}}$, continuously changes with time due to the depletion of wax content of the bulk oil as the wax-oil gel deposits on the wall. Then, the overall mass balance is performed to calculate the wax mass left for the deposition of the next time step. This overall mass balance is calculated from

$$C_{\text{inlet}} = C_{\text{ini, inlet}} - \frac{M_{\text{tot}}}{V_{\text{tot}}}$$  \hspace{1cm} (36)

where $C_{\text{ini, inlet}}$ is the initial inlet concentration that is obtained from wax solubility curve.

**Results and Discussion**

In this section, the accuracy of the developed wax deposition model will be verified first in a lab-scale single phase flow experiment and then by oil/gas stratified flow loop experiment, the wax deposit section shown as Figure 6. The properties of the oils and the operating conditions of these experiments are summarized in Table 1, whereas the viscosity and the solubility curves of these three oils are shown in Figure 7, respectively. These operating conditions and material properties are used as the inputs to the model simulation.

There are only one wax deposit experiment available in Matzain et al. and three experiments in Gong et al. for stratified smooth flow, as shown in Table 2. So, five simulations were carried out in this study including a single phase flow. The oil and gas flow rates for these simulations are shown in Table 2. It is noted that the transitional superficial gas velocity from stratified smooth to the stratified wavy flow is obtained by the analysis of Tzotzi and Andritsos. So flow patterns are all stratified smooth flow and the developed model is available for prediction.

**Deposition results for single phase oil flow**

The data from lab-scale single phase experiment is the average thickness and the average wax fraction based on the whole wax deposit section. To compare the result between experiment and calculation expeditiously, the average thickness and the average wax fraction calculated by the model are given as

$$\overline{\delta} = \frac{\sum_{k=0}^{P} (\delta_k h_k)}{\sum_{k=0}^{P} h_k}$$  \hspace{1cm} (37)

$$\overline{F}_w = \frac{\sum_{k=0}^{P} (\overline{F}_w, k \delta_k h_k)}{\sum_{k=0}^{P} (\delta_k h_k)}$$  \hspace{1cm} (38)

A lab-scale single phase flow experimental result from Singh et al., is first compared to the wax deposition model. This rather simple test consists of assigning the same fluid properties and flow rates for both phases. The oil–gas interface is fictitious. As expected, the oil–gas interface position is at the middle of the pipe and the interface shape is planar. A model oil is used in the experiment, which is a mixture of a 3:1 volume ratio of mineral oil (Blandol) and kerosene is blended with 0.67 wt % of food-grade wax.

Figure 8 compares the deposit thickness and the wax fraction of the deposit measured in the experiment with that predicted by the model. It is observed that the present model accurately calculates both the deposit thickness and the wax content in the deposit. It is noted that while the growth of the deposit virtually stops after 24 h, the wax fraction of the deposit continues to increase and the present model successfully gives this result. As the deposit thickness grows, the insulating effect of deposited gel increases, thus the heat transfer rate decreases, and the mass driving force for further deposition significantly diminishes. Once the deposit interface temperature reach the WAT, the thickness will stop growing. Although the thickness stops growing, there is still a concentration gradient across the deposit. This gradient causes an

---

**Table 1. Summary of the Oil Properties and Operating Conditions in the Experiments**

<table>
<thead>
<tr>
<th></th>
<th>Singh et al.</th>
<th>Matzain et al.</th>
<th>Gong et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAT (°C)</td>
<td>13.9</td>
<td>51.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Wax content (wt %)</td>
<td>0.67%</td>
<td>6.6%</td>
<td>23.98%</td>
</tr>
<tr>
<td>Length of the pipe (m)</td>
<td>2.44</td>
<td>7.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Inside pipe dia. (mm)</td>
<td>14.4</td>
<td>51.8</td>
<td>25.4</td>
</tr>
<tr>
<td>Outside pipe dia. (mm)</td>
<td>15.87</td>
<td>57.9</td>
<td>33.7</td>
</tr>
<tr>
<td>Oil/gas mixture inlet temperature (°C)</td>
<td>22.2</td>
<td>40.56</td>
<td>50.0</td>
</tr>
<tr>
<td>Coolant inlet temperature (°C)</td>
<td>8.3</td>
<td>15.56</td>
<td>40.0</td>
</tr>
<tr>
<td>Test duration (h)</td>
<td>120.0</td>
<td>24.0</td>
<td>24.0</td>
</tr>
</tbody>
</table>

---

**Figure 7. Solubility of wax in the oil and viscosity of the oil.**

(a) Solubility of wax in oil. (b) Viscosity of oil.
internal mass flux in deposit, which in turn is result in the wax fraction of the deposit to continue to increase with time.

**Deposition results for oil/gas two-phase flow**

A more general check for oil/gas two phase flow has been carried out by comparison the model to the experiments, one carried out by Matzain et al. (i.e., Simulation 2) and the other by Gong et al. (i.e., Simulation 3, 4, and 5). Two crude oils (the South Pelto oil and Daqing oil) are used in this case.

Matzain et al. and Gong et al. both find in experiments that the deposit was crescent shaped with two identifiable zones, where there is no wax deposition on the wall contacting with gas phase, while the wax deposition only occurs on the lower part of the wall contacting with oil, as shown in Figure 9. It shows the comparison between deposit thickness at cross section from experiment and that calculated by the present model, for Simulation 2. It is should be pointed out that the liquid holdup in the oil/gas stratified flow calculated by the model is 0.54, being consistent with the liquid holdup, 0.55, measured by Gamma densimeter. The accurate prediction on hydraulics is essential to estimate the pipe wall where wax deposit occurs.

Figure 10 gives the thickness and wax fraction in deposit angular distribution around the circumference. As seen in Figure 10, both the experiments and model predictions reveals that the thickness and wax fraction in deposit circumferential distribution on pipe wall is non-uniform, and at the bottom of the pipe (i.e., γ=0), the thickness reached its largest value, and the nearer the position gets close to A and B, two point where the oil/gas interface contacts the inner wall, the thinner the wax deposition. However the wax fraction of deposit presents an opposite tendency, a increasing wax fraction from bottom to the two points A, B. The result is in accordance with what was observed in the Matzain et al.’s experiment. It may be attributed to the fact that a higher thickness results in a lower temperature gradient across the deposit, resulting in a lower diffusive flux of wax inside the deposit, leading to a lower wax fraction in the deposit. It is important to note that since the thickness measured using a caliper has a degree of uncertainties, the difference between the experiment and calculation could be a result of the impact of the residual oil.

As shown in Figure 10a, the thickness ranges from 0 to 1.82 mm. Similarly to the thickness distribution, the smallest wax fraction in deposit occurs at the bottom of the deposition, and the wax content ranges from 18.97 to 22.89%, as shown in Figure 10b. The hardness of deposition heavily affected by the wax fraction of the deposition, is significant for wax cleaning in pipe, in terms of cleaning method designing and pigging operation. In addition, the thickness and wax fraction at the bottom of pipe measured at the end of the experiment, agree well with that predicted by the present model, with absolute average error of 9.1 and 5.97%. The model will be applied to analyze and explain the reason of the deposit forming a crescent shape at the cross section of pipe in next section.

In addition to the South Pelto crude oil, the present model is also applied to the prediction of the experiment with the Daqing crude oil, for Simulation 3, 4, and 5. Deposit thickness circumferential distribution on pipe wall is non-uniform for stratified flow, as shown in Figure 11. To compare the result between experiment and calculation expediently, the equivalent thickness is introduced, which is a thickness averaged over the entire surface area of the pipe wall in the whole wax deposit section, this is done by assuming that the deposit covers the circumference of the pipe completely and uniformly and is back-calculated from the wax deposit volumetric displacement method.

The equivalent thickness calculated by the model are given as

![Figure 8. Comparison between the experimental and the predicted deposit thickness and wax fraction of the deposit for single phase flow.](image)

![Figure 9. Comparison of wax deposit circumferential distribution on pipe wall for experiment and calculation.](image)

<table>
<thead>
<tr>
<th>Simulation number</th>
<th>Oil flow rate (m³/s)</th>
<th>Gas flow rate (m³/s)</th>
<th>Superficial gas velocity (m/s)</th>
<th>Superficial oil velocity (m/s)</th>
<th>Transitional superficial gas velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$6.30 \times 10^{-5}$</td>
<td>NA*</td>
<td>NA*</td>
<td>NA*</td>
<td>NA*</td>
</tr>
<tr>
<td>2</td>
<td>$1.24 \times 10^{-4}$</td>
<td>$6.18 \times 10^{-4}$</td>
<td>0.305</td>
<td>0.0610</td>
<td>1.91</td>
</tr>
<tr>
<td>3</td>
<td>$3.17 \times 10^{-5}$</td>
<td>$4.31 \times 10^{-5}$</td>
<td>8.500</td>
<td>0.0626</td>
<td>11.45</td>
</tr>
<tr>
<td>4</td>
<td>$3.17 \times 10^{-5}$</td>
<td>$5.22 \times 10^{-5}$</td>
<td>10.300</td>
<td>0.0626</td>
<td>11.35</td>
</tr>
<tr>
<td>5</td>
<td>$4.76 \times 10^{-5}$</td>
<td>$4.32 \times 10^{-5}$</td>
<td>8.500</td>
<td>0.0939</td>
<td>9.68</td>
</tr>
</tbody>
</table>

*In single phase oil flow, there is no gas phase.*
Figure 10. Predicted angular distribution of deposit thickness and wax fraction in the deposit, $\gamma = 0^\circ$ at the bottom of the pipe.
(a) Wax deposit thickness. (b) Wax fraction in the deposit.

Figure 11 gives the equivalent thickness as a function of time and comparison of that at the end of the experiment. Similar tendency of wax deposit variation with time to the single phase flow are found. Obviously, the equivalent thickness calculated by the present model are very close to the experimental value with great accuracy when time reaches 24 h. The equivalent thickness of experiment and prediction are 0.33 and 0.35 mm, respectively, with 6.06% relative error, for Simulation 3; and for Simulation 4, the values are 0.39 and 0.42 mm, with 7.69% relative error; and for Simulation 5, the values are 0.56 and 0.62 mm, with 10.71% relative error.

**Equivalent crystal aspect ratio**

It is noted that the thickness values in Figure 8 were measured from pressure-drop method. The thickness values in Figures 9 and 10 were measured using a caliper (also noted as a "depth gauge" in the original study$^{47}$). The thicknesses were measured using volumetric displacement method in Figure 11. Each of these methods has quite a degree of uncertainties, for example, non-Newtonian behavior on the viscosity for pressure-drop method, the impact of the volume of the residual thin film for the caliper and the displacement method. The thickness measured with deferent methods could lead to significant different results, especially for laminar flow. There are six existing wax thickness measurement methods, namely spool piece, pigging, pressure drop, heat transfer methods, liquid displacement-level detection method, ultrasonic transit time, and direct measurement critically reviewed by Creek et al.$^9$ and Chen et al.$^{48}$. To best fit between the simulation and the experimental data, the value of the fitting parameter $k_a$ has to be varied with the operating condition.

The effective diffusivity of wax in the deposit is a function of the equivalent crystal aspect ratio $\alpha$, as shown in Eq. 34. The relationship between the equivalent crystal aspect ratio and the wax fraction has not been fully investigated$^{38}$. Therefore, the linearly form of the relationship (i.e., Eq. 35) used by Singh et al.$^7$ is introduced here. The fitting parameter $k_a$ is adjusted until minimizing the error in the deposit thickness prediction and wax fraction. Tables 3 shows the fitting parameter $k_a$ used in the five simulations. It is observed that the equivalent aspect ratio of the wax particles decreases with the Reynolds number for the same oil sample. It can be seen that for the case of laminar flow, the fitting parameter decreases as the flow rate increases from the Simulation 3–5.

To clarify the impact of $k_a$ on wax deposit, Figure 12 shows the predictions for the growth of the deposit thickness and wax fraction in the deposit with time, using Simulation 3, for selected different values of the $k_a$. It can be seen that the model gives similar predictions for the deposit thickness for each value of $k_a$. The effect of $k_a$ on the growth of the deposit thickness is not as significant as wax fraction. The less sensitive of the thickness is attributed to the external convective flux of wax molecules from bulk oil to the oil–deposit interface far more than internal diffusion of molecules through the trapped oil. And the effect of $k_a$ on the internal diffusion of molecules is much more significant than the external convective flux. So, there is barely any change in the deposit thickness with various value of $k_a$ and an increase in $k_a$ causes a significant decrease in the wax fraction.

Figure 11. Comparison of equivalent thickness as a function of time between the experimental results and the model prediction.
Analysis for deposit with a crescent shape

The deposit forming a crescent shape at the cross section of pipe is observed in different experiments. In this section, the non-uniform wax thickness along the pipe circumferential direction is explained and here Simulation 2 is used as an example, as shown in Figure 9.

It is noted that the mass flux \( J_{w, \eta} \) was found to be significantly greater than the mass flux \( J_{A, \eta} \) at a computational grid \((\zeta, \eta)\) of the oil–wax deposit interface, because the concentration gradient \( dC/d\eta \) from oil to interface is significantly lower than \( dC/d\zeta \) from oil to interface in the current study. So the mass flux \( J_{A, \eta} \) can be negligible for contribution to wax deposit, as can be seen in Table 3. Due to the oil–deposit interface changing with simulation time and not at a line of a constant \( \zeta, \xi \) is different with the \( \eta \) changing for different grid point \((\zeta, \eta)\). The grid point \((\zeta, \eta)\) represents the computational grid at oil–deposit interface in bipolar coordinate when \( \eta \)-ordinate has certain value.

At different simulation time, four calculation grids at the oil–deposit interface (the right of the selected grid point) are selected for the comparison. Therefore, we can approximate the growth rate of the wax deposit using \( J_{A, \zeta} \). And according to some studies\(^1\,\,^2\,\,^8\) the mass flux from the bulk oil to the deposit interface, \( J_{A, \zeta} \) is significantly greater than the diffusive mass flux into the deposit, \( J_{R, \zeta} \), because of the enhanced mass transfer in the oil phase by the movement of the fluid. So, approximately, the growth rate of wax deposit is essentially equal to \( J_{A, \zeta} \), which is the most dominant parameter to affect the behavior of the characteristic mass flux for wax deposit, given as

\[
\rho \frac{dF_w(t)}{dt} \approx \rho \frac{dF_w(t)}{dt} \frac{dC}{d\eta} \text{ from oil to interface} \approx J_{A, \zeta} = D_{w, \text{interface}} \left( \frac{-1}{T_{\text{C}} \frac{dC}{d\zeta} \text{ from oil to interface}} \right) \text{ Term 1} \frac{d\delta_i}{d\zeta} \text{ Term 2} \quad (40)
\]

The right hand side of Eq. 40 contains the most important parameters for wax deposit and is divided into terms 1 and 2. These two terms are used to analyze the reasons for the deposit forming a crescent shape. To understand the deposit thickness circumferential distribution, one can analyze the angular distribution of two terms separately, as shown in Eq. 40, i.e., Term 1, the diffusivity at oil–deposit interface, \( D_{w, \text{interface}} \), Term 2, a concentration gradient at the interface. According to the work of Huang et al.\(^5\,\,^42\) and Lu et al.\(^2\) the concentration gradient can be found to be directly proportional to \( C_{\text{bulk}} - C_{\text{interface}} \), which represents the concentration driving force for wax deposition. \( C_{\text{bulk}} \) and \( C_{\text{interface}} \) are the concentrations of wax at the bulk and, at the oil–deposit interface, respectively. \( C_{\text{interface}} \) is obtained by the solubility curve of the wax in the oil. \( C_{\text{bulk}} \) is calculated by taking a velocity weight average of the axial (\( z \)-direction) concentration profile throughout the oil phase.

\[
C_{\text{bulk}}(z) = \int_{A_{\text{oil}}} C(x, y)w_{\text{oil}}(x, y)\,dA \int_{A_{\text{oil}}} w_{\text{oil}}(x, y)\,dA \quad (41)
\]

The \( D_{w, \text{interface}} \) and \( C_{\text{interface}} \) are depend on the interface temperature, \( T_{\text{interface}} \), which can vary as time progresses because of the increase in the insulation effect as the deposit builds up on the pipe wall. Consequently, to get the circumferential distributions of these two terms at different time, are important to thoroughly understand the deposit forming a crescent shape at the cross section of pipe.

Table 5 summarizes the mass flux, \( J_{\text{wax}} \) and its corresponding parameters \( D_{w, \text{interface}} \), \( C_{\text{bulk}} \), and \( C_{\text{interface}} \) for the circumferential angle \( \gamma = 0^\circ, 30^\circ, 60^\circ, 90^\circ \) at \( t = 0 \) and \( \delta = 0 \), where deposition has not yet occurred. In this case, the parameters at oil-interface simply reduces to at the pipe wall, because the wall temperature, \( T_{\text{wall}} \) can be calculated by solving the heat

![Figure 12. Effect of fitting parameter \( k_a \) on the predicted of wax deposit.](Image)

(a) Wax deposit Equivalent thickness (b) Averaged Wax fraction in the deposit.

**Table 3. Fitting Parameter \( k_a \) for the Simulations**

<table>
<thead>
<tr>
<th>Simulation number</th>
<th>Oil flow rate (m³/s)</th>
<th>Gas flow rate (m³/s)</th>
<th>Oil Re</th>
<th>Final z</th>
<th>Fitting parameter ( k_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.30 × 10⁻⁵</td>
<td>NA*</td>
<td>540.78</td>
<td>24.20</td>
<td>1.325</td>
</tr>
<tr>
<td>2</td>
<td>1.29 × 10⁻⁴</td>
<td>6.43 × 10⁻⁴</td>
<td>526.23</td>
<td>25.50</td>
<td>1.191</td>
</tr>
<tr>
<td>3</td>
<td>3.17 × 10⁻⁵</td>
<td>4.48 × 10⁻³</td>
<td>240.27</td>
<td>78.26</td>
<td>1.353</td>
</tr>
<tr>
<td>4</td>
<td>3.17 × 10⁻³</td>
<td>5.43 × 10⁻³</td>
<td>249.45</td>
<td>76.83</td>
<td>1.259</td>
</tr>
<tr>
<td>5</td>
<td>4.76 × 10⁻³</td>
<td>4.48 × 10⁻³</td>
<td>342.64</td>
<td>66.54</td>
<td>1.051</td>
</tr>
</tbody>
</table>

*Single phase oil flow, there is no gas phase.
transfer and mass equation, the diffusivity $D$ can be determined through $T_{wall}$ using the Hayduk-Minhas correlation. The wax concentration at wall, $C_{wall}$, can be obtained from $T_{wall}$ through the solubility curve of the oil. The dimensionless parameters is defined as the ratio of the value at the variable $\gamma$, to that at the bottom of the pipe (i.e., $\gamma = 0^\circ$). Such as the de-dimensionalized $D_{wo, wall}$ is given as

$$d e - D_{wo, wall} = \frac{D_{wo, wall}}{D_{wo, wall}|_{\gamma=0^\circ}}$$

(42)

similarly deal with the other parameters.

It can be seen from Table 5 that, initially (at $t=0$ and $\delta=0$), where deposition has not yet occurred, $J_{wax}$ reaches maximum at the bottom of the pipe and decreases toward the gas–oil interface. From bottom to oil–gas interface along the pipe circumference, the decreases in heat transfer rate leads to an increase in $T_{wall}$, which affects multiple terms in the mass flux, $J_{wax}$. The increase in $T_{wall}$ increase the values of both $D_{wo, wall}$ and $C_{wall}$. The increase in $D_{wo, wall}$ tends to increase the characteristic mass flux, $J_{wax}$. As angular $\gamma$ increase from 0 to 80°, $D_{wo, wall}$ has increased 25.6%, but $(C_{bulk} - C_{wall})$ has decreased 74.0%, which outweighs the change in $D_{wo, wall}$. Therefore, the characteristic mass flux, $J_{wax}$, in the last row of Table 4, decreases. This decrease of $J_{wax}$ explains the deposition thickness decreases from bottom toward the oil–gas interface along the pipe circumference.

As mentioned previously, the $D_{wo, wall}$ and $C_{wall}$ change as time progresses because they involve the behavior of the insulation effect of the deposit. It is found that the trend of angular distribution for these three terms at $t>0$ ($\delta>0$) also hold for $t>0$ ($\delta>0$). It was found that the $J_{wax}$ follows the same trend on the circumferential distribution as the deposit builds up, Table 4 also gives the parameters of $J_{wax}$ at $t>0$ ($\delta>0$). It can be seen that, from bottom to oil–gas interface, the $J_{wax}$ decreases with increasing $D_{wo, wall}$, decreasing $C_{wall}$ and $(C_{bulk} - C_{wall})$.

Conclusions

In practice, oil/gas two-phase flow is a common occurrence during oil and gas production. Wax deposition prediction under oil/gas flow conditions is flow pattern dependent and largely ignored. The current practice that use correlations for heat/mass transfer in single-phase wax deposition model fail to give reliable predictions for multiphase flow system, because most hydrodynamic and heat/mass transfer models did not consider the flow pattern effects rigorously.

In this work, wax deposition under non-isothermal oil/gas stratified pipe flow is studied, which is a dominant flow pattern, for a slightly downward flow ($-1^\circ$), and a prediction model was developed. Instead of using empirical correlations for the non-isothermal hydrodynamics and heat/mass transfer, the model solve the transport equations numerically. Due to the non-circular and irregular oil and gas domains in stratified pipe flow, computations are performed in the bipolar coordinate system for convenient mapping of the physical domain.

The predictions for wax deposition are found to compare satisfactorily with experimental data with three different oils for single phase and oil/gas stratified pipe flow. In particular, for stratified flow, the deposit thickness circumferential distribution on pipe wall is crescent shaped with two identifiable zones and non-uniform for stratified flow. At the bottom of the pipe, the thickness reached its largest value, and from bottom to interface along the pipe circumference, the thinner the wax deposition.

More importantly, this study has proposed three terms to analyze the reasons for the deposit forming a crescent shape at the cross section of pipe observed in different experiments. These two terms include the diffusivity at oil–deposit interface (Term 1), the concentration gradient at the interface (Term 2). From bottom to interface along the pipe circumference at different time, Term 1 tends to increase the growth rate of the wax deposit, while Term 2 tends to have the opposite effect. Terms 2 dominate the competition over the other one, which lead to a reduction in the deposit thickness.

Acknowledgments

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Notation

**Variables**

$A$ = cross-sectional area inside the pipe, $m^2$
$A_F, A_S, A_g, A_w, A_B$ = coefficients
$a, b, c, d, \delta$ = the coordinate value of grid point
$B$ = the length of oil–gas interface, $m$
$B_0$ = Bond number
$C$ = wax concentration in crude oil, kg/m$^3$
$D_{in,n}$ = inner diameter of inner pipe, $m$
$D_{out}$ = outer diameter of inner pipe, $m$
$D_{eff} = effective diffusivity in the deposit, m$^2$/s

---

**Table 4. The Comparison of Contribution for Deposit Between $J_{wax}$ and $J_{c}$ at Different Time**

<table>
<thead>
<tr>
<th>Simulation time hour</th>
<th>Grid point $(\xi, \eta)^*$</th>
<th>$d e - d e_{\text{sim}}$</th>
<th>$J_{wax}$</th>
<th>$J_{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$(\xi_0, 0)$</td>
<td>0.11</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>$(\xi_1, 1)$</td>
<td>0.094</td>
<td>0.092</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>$(\xi_2, 2)$</td>
<td>0.078</td>
<td>0.075</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>$(\xi_3, 3)$</td>
<td>0.047</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>18.0</td>
<td>$(\xi_4, 4)$</td>
<td>0.033</td>
<td>0.031</td>
<td></td>
</tr>
</tbody>
</table>

*The grid point $(\xi, \eta)$ represents the computational grid at oil–deposit interface in bipolar coordinate when $\eta$-ordinate has certain value.

**Table 5. Calculated Angular Distribution of $J_{wax}$, $\gamma = 0^\circ$ at the Bottom of the Pipe**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$, deg</td>
<td>0</td>
</tr>
<tr>
<td>$d e - T_{wall}$</td>
<td>1.000</td>
</tr>
<tr>
<td>$d e - D_{wo, wall}$</td>
<td>1.000</td>
</tr>
<tr>
<td>$d e - C_{wall}$</td>
<td>1.000</td>
</tr>
<tr>
<td>$d e - (C_{bulk} - C_{wall})$</td>
<td>1.000</td>
</tr>
<tr>
<td>$d e - J_{wax}$</td>
<td>1.000</td>
</tr>
<tr>
<td>$t = 3.5$ h</td>
<td>1.000</td>
</tr>
</tbody>
</table>

It can be seen that, from bottom to oil–gas interface, $J_{wax}$ decreases with increasing $D_{wo, wall}$, decreasing $C_{wall}$ and $(C_{bulk} - C_{wall})$.
Greek Letters

\( \pi \) = the equivalent crystal aspect ratio
\( \Gamma_\gamma \) = effective thermal diffusivity
\( \Gamma_v \) = effective viscosity, \( m^2/s \)
\( \Gamma_c \) = effective mass transfer
\( \Gamma_\phi \) = generalized diffusivity
\( \gamma \) = circumferential angle, \( ^\circ \)

\( \Delta x, \Delta y, \Delta z \) = the grid length in \( \xi, \eta, \zeta \) coordinate direction
\( \Delta \rho \) = density difference between oil and gas, \( kg/m^3 \)
\( \Delta E \) = total energy, \( J \)
\( \delta \) = deposit thickness, \( m \)
\( \eta \) = coordinate in bipolar system
\( \theta_0 \) = upper section of the pipe wall, rad
\( \theta \) = the angle for interface, rad
\( \lambda \) = heat conductivity coefficient, \( W/(m \cdot K) \)
\( \mu_k \) = kinematic viscosity coefficient, \( m^2/s \)
\( \mu_e \) = eddy diffusivity for momentum, \( m^2/s \)
\( \zeta \) = coordinate in bipolar system, rad
\( \rho \) = density, \( kg/m^3 \)
\( \sigma \) = interfacial tension between oil and gas, \( N/m \)
\( \sigma_t, \sigma_r, \sigma_c \) = coefficient
\( \phi \) = general variable

Subscripts

bulk = properties at the bulk
dep = properties of the deposit
envir = properties of the coolant liquid.
gas = properties of the gas
\( i, j, k \) = coordinate of mesh point at \( \xi, \eta, \zeta \) coordinate direction
ini = properties at initial time
inlet = properties at the inlet
inner = from bulk to the inner wall (for \( \xi \) coordinate)
int = properties at the oil–gas interface
interface = properties at the oil–deposit interface
oil = properties of the oil
odi = from bulk to the deposit (for \( \xi \) coordinate)
pipe = properties of the pipe
wall = properties at the pipe inner wall
wax = properties of the wax
ws = solubility of wax

Literature Cited


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