Mechanical Resonance Suppression in Servo System Based on The Fractional Order Low-pass Filter

Ming Yang, Yongjian Fu, Xin Lv, Dianguo Xu
(Dept of Electrical Engineering Harbin Institute of Technology, Harbin 150001, China)

Abstract—The conventional filter method could effectively suppress resonance in servo system without backlash. But when the system contains backlash, the methods may fail. In this paper, a fractional order low-pass filter substitutes the integer order low-pass in a dual-inertia servo system with backlash to solve the problem. Fractional algorithm provides greater flexibility for robust control design, because it makes the order of the low-pass filter from integer domain to real-number domain. Fractional order low-pass filter can achieve a better tradeoff between robustness and resonance suppression by selecting the appropriate order. For implementation of the fractional order, Oustaloup recursive approximation method is introduced. Experimental results show that fractional-order low-pass filter can improve robustness while suppressing system resonance.

Index Terms—Fractional order, Low-pass filter, Servo system, Backlash, Mechanical resonance.

I. INTRODUCTION

Servo system transmits power from the driving side to the load side via the gear box, couplings, shafts and other elastic mechanism, which cause mechanical resonance in the system. With the development of servo performance, the requirements of anti-resonance capability of servo system are also increasing.

There are many ways to avoid mechanical resonance, which can be divided into two categories: active and passive control. The active control changes the structure or parameters of the controller to eliminate the effect of resonance, which contains PI control (two degrees of freedom PI control, RRC) [1-2], State feedback control based on PI [3-5] and other advanced algorithms [6-8]. The passive control means that a filter is inserted between the output of speed loop and the input of current loop, while the others are not changed [9-11].

For the problem of mechanical resonance in servo system with backlash, margin loss is the main reason why the conventional filter methods fail. Some algorithms of backlash compensation have effect in resonance suppression, including linear control based on the torque feedback, adaptive control, nonlinear control, and speed control based on the magnitude identification of the backlash. The linear control mainly based on the observer of gear torque. In [12], an observer based on the dual-inertia model and a robust speed controller based on state feedback are proposed. State feedback is used to estimate the second shaft torque, load torque disturbance and load angular velocity, which can suppress the first-order shaft resonance effectively. The papers [13-14] deal with model-based control method, the control model of dynamic compensator consists of a linear reduced-order model and delay elements associated with backlash, providing a feedback signal by estimating the load speed to suppress transient vibration.

As with the linear control, adaptive and nonlinear control suppress the negative effects of the backlash by changing the controller structure and algorithm, the control performance is improved. As for the adaptive control, in [15], the method of neural network control is introduced. The paper [16] achieves online assessment and compensate of backlash, which has a good effect for system of variable backlash and parameters. However, there is high demand for hardware because of computational complexity. And for the nonlinear control, in [17-18], nonlinear controller gets better performance than the linear controller at designing the maximum limit cycles, closed loop characteristics especially in low bandwidth and low limit cycles. Lastly, the compensation control method based on the magnitude identification of the backlash, can not only improve the dynamic characteristics of the system, but also for positioning accuracy problems. The papers [19-20] proposed several different methods to identify backlash. And papers [21-22] take appropriate measures to compensate the backlash as the magnitude of the backlash is already known.

But the methods above are almost active control ways, which are complex design and difficult to achieve. There are few papers having been proposed to suppress mechanical resonance in the passive control way when backlash exists. While with the application of fractional algorithm, the proposed method—the fractional order low-pass filter, which belongs to the passive control way, could solve the problem effectively.

The order of fractional order controller can be any real numbers including the integer numbers, which provides greater flexibility for robust control design. Like [23-24], for traditional PID control, fractional algorithm is applied to the integral and differential links, which could get fast response and better robustness. In [25], fractional order disturbance observer is proposed. While in this paper, the fractional order low-pass filter can achieve a better tradeoff between robustness and resonance suppression by selecting the suitable order. Analysis and experiments
in this paper show the superiority of fractional order low-pass filter.

II. DUAL-INERTIA SERVOSYSTEM WITH BACKLASH

A dual-inertia servo system with backlash is depicted in Fig.1, where \( J_M, J_L \) are driving motor and load's inertia, \( \phi_M, \phi_L \) driving motor and load displacement, \( K \) shaft elastic coefficient and \( D \) damping coefficient. In the modeling, the gear backlash is simplified as a dead zone factor with backlash angle band \([-b, +b]\) and \( b \) is taken as 0.01 rad. Assuming that the transmission ratio is 1.

Frictions between components are neglected, the dual-inertia model with backlash in the control of speed loop depicted in Fig.2, where \( T_e, T_w, T_l \) are electromagnetic torque, shaft torque and load torque.

Backlash in servo system will rise the damage of the resonance severely, which can be considered to reduce the elastic stiffness of the system equivalently. As shown in Fig. 3, FFT analysis of the resonance with different backlash is presented. It can be seen that, with the increasing of the backlash, the resonant frequency decreases while the resonance amplitude increases.

III. FRACTIONAL ORDER LOW-PASS FILTER

As depicted in Fig.4, the fractional order low-pass filter is applied to the speed control of the elastic dual-inertia system. And the dual-inertia system is shown in Fig.2, just because the bandwidth of current loop is bigger than the speed loop, the current loop is taken as 1. In generally, \( T \) takes a small value just to ensure the sufficient bandwidth of the system. When the bandwidth is known, selecting an appropriate fractional order can achieve a better tradeoff between robustness and resonance suppression. Fractional order provides greater flexibility for system design, which can’t be achieved by the conventional filters.
There are two pending parameters for tuning in the fractional order low-pass filter, the time constant and the order. How to set the optimal time constant and order of $\alpha$ is shown as follows. The theoretical value of the time constant is given by

$$T = \frac{D}{K}$$  \hspace{1cm} (2)

Where $K$ is shaft elastic coefficient and $D$ is damping coefficient.

The order $\alpha$ is not just integer, but also real numbers, which makes greater freedom for order tuning. It can take margin constraint to tune $\alpha$ in theory, set the cutoff frequency $\omega_c$ and phase margin $\phi_m$.

A. Phase margin condition of the control system

As shown in (3):

$$\text{Arg}(G(j\omega))|_{\omega=\omega_1} = \text{Arg}(G_0(j\omega)G_{ASR}(j\omega)Q_\alpha(j\omega))|_{\omega=\omega_1}$$

and let

$$\text{Arg}(G(j\omega))|_{\omega=\omega_1} = -\pi + \phi_m$$  \hspace{1cm} (3)

B. Amplitude-frequency characteristics conditions of the control system

$$|G(j\omega)| = |G_0(j\omega)G_{ASR}(j\omega)Q_\alpha(j\omega)|_{\omega=\omega_1} = 1$$  \hspace{1cm} (4)

Where $G_0(j\omega)$ is transfer function of controlled plant, $G_{ASR}(j\omega)$ is transfer function of the speed controller, $Q_\alpha(j\omega)$ is transfer function for the fractional-order low-pass filter.

From (3) and (4), three unknowns that are phase margin $\phi_m$, the cut-off frequency $\omega_c$ and order of $\alpha$, when the phase margin is known, the order will get.

In practical application of engineering, time constant is determined according to the system bandwidth, while trial and error method is used for order tuning. With these methods, optimal parameters of fractional order low-pass filter can be achieved.

IV. IMPLEMENTATION METHOD

A. Continuous system

Of course the fractional controller design is relatively straightforward, but it is more complicated to implement. A variety of methods for this, such as the continued fraction approximation, broken-line approximation, Oustaloup recursive approximation method. Rational function is used for fractional approximation in these methods. In this paper, fractional order low-pass filter is expressed by Oustaloup recursive approximate method.

As shown in (1), it’s better to select a large range of frequencies for approximation. Set $[\omega_b, \omega_a]$ in which,

$$T = \frac{1}{\omega_b}$$  \hspace{1cm} (5)

$$G(s) = \left(\frac{1+s/\omega_a}{1+s/\omega_b}\right)^{\alpha}$$  \hspace{1cm} (6)

A rational function approximation is obtained as follows:

$$G_N(s) = \prod_{k=1}^{N} \frac{s + \omega_a^*}{s + \omega_k}$$  \hspace{1cm} (7)

Where

$$\begin{aligned}
\omega_a &= \omega_b \omega_a^{(2k-1-\alpha)/N} \\
\omega_k &= \omega_b \omega_a^{(2k+1+\alpha)/N} \\
\omega_a^* &= \sqrt{\frac{\omega_k}{\omega_b}}
\end{aligned}$$  \hspace{1cm} (8)

$N$ represents the order of the Oustaloup filter, the values of $N$ have a certain impact on approximation results. In general, it can get a more accurate approximation when $N = 5$, The results shown in Fig.6 ($\alpha = 0.4$ $\omega_b = 200$ Hz $\omega_a = 10000$ Hz).

Fig.6: Bode plots of Oustaloup approximation

B. Discretization

There are lots of methods for continuous systems discretization, such as Tustin method, differential transform method, response invariants and so on. Response invariants deals with the following method: based on the external characteristics of the system, the sample value of discrete-time systems response under excitation of typical signal, equals that of continuous-time systems under corresponding input signal, that is

$$y(n) = y(t)|_{t=nT}$$  \hspace{1cm} (9)

In this paper, discrete-time implementation with impulse response invariants as follows:
\[ G(z) = Z[G(s)] \] (10)

Its essence is to take the Z-transform for \( G(s) \) directly.

This method ensures that the impulse response of discrete system is equivalent to the samples of continuous-time.

When \( \alpha = 0.4 \), \( T = 0.005 \text{s} \), discretization formula with impulse response invariants can be obtained.

Fig. 7 shows the comparison of impulse response and bode diagram. From (a), the impulse response of true and approximated is almost the same, also having a comparison with first order low-pass filter. From (b), the approximated mag is same with the true mag in low frequency, but little changes in high frequency.

It can be seen that the approximation of fractional order controller is almost identical with the actual controller to meet the accuracy requirements.

V. EXPERIMENTAL RESULTS

The servo experiment platform is depicted in Fig. 8, it is based on TMS320F28335, which comprises of two permanent magnet synchronous motors with the rated data: speed of 3000 r/min, current of 4.4 A and motor torque of 2.39 N·m and power of 750W. A flywheel is used on the load side to change the inertia ratio between load and drive-side inertia. The moment of the drive motor is \( J_m = 8.53 \times 10^{-5} \text{ kg·m}^2 \), the sum moment of the load including flywheel is \( J_L = 6.22 \times 10^{-3} \text{ kg·m}^2 \) and the value of backlash is 0.01 rad.

When \( \omega_c = 60 \text{ rad/s} \), \( \phi_0 = 60^\circ \), from (3) and (4), the tuning order of low-pass filter is about 0.42. Just to verify if it's correct, the experimental results as follows.

As shown in Fig. 9, the time responses with different \( \alpha \) order low-pass filters in the elastic dual-inertia servo system with backlash. It can be seen from case of (a), the high frequency speed ripple is obvious. The source for speed ripple is the high stiffness of speed controller, the elastic shaft and other nonlinear factors in system which cause the mechanical resonance. From case of (b) and (d), the high frequency speed ripples is weakened but can not be eliminated completely. The two figure show that there is little effect in resonance suppression but can’t be completely eliminated. Margin loss is the main reason for that. While using the tuning order 0.4, the system gets the best response as shown in case of (c). Which verify the tuning method is correct.

Changing the time constant of filter as 0.05s for contrast, subjoining and decreasing 0.5 times rated load when \( t=4\text{s} \) and \( t=7\text{s} \) separately. Fig. 10 shows the responses of load mutation with different \( \alpha \) order low-pass filter to study the robustness of the system. In the case of (a), torque ripples and high frequency speed ripple is the response of mechanical resonance. Taking \( \alpha \) as 0.4 in case (b) gives the best robust, in which the speed change and recovery time are very small, and the torque ripple is low. This shows that the tuning order is basically correct. Robustness of the other order like case (c) and (d) are bad especially case (d) when the order \( \alpha = 1.0 \). However, resonance is well suppressed when \( \alpha = 1.0 \), it says that the integer order low-pass filter may fail in resonance suppression with different time constant.

VI. CONCLUSION

The fractional order low-pass filter is applied to the speed control of the elastic dual-inertia system. Experimental results show that the conventional filter methods may fail in resonance suppression because of backlash and other nonlinear factors in system. The fractional order filters get well resonance suppression, and also provide greater flexibility for robust control design. Contradiction between resonance suppression and
robustness has been a problem, fractional order low-pass filter can achieve a better tradeoff between robustness and resonance suppression by selecting the suitable $\alpha$. Fractional order contains the integer, while the integer order represents the fractional order. On the other hand, tuning of time constant $T$ and order $\alpha$ of filter requires more theoretical research for optimized configuration.

ACKNOWLEDGMENT

The authors gratefully acknowledge National Natural Science Foundation of China (NSFC) for the financial support, Project No. 61273147.
REFERENCES


[23] Farzad Taham, Behzad Esmaeinejad Moghadam. Speed Control of Servo Drives with a Flexible Couplings Using Fractional Order State Feedback The 5th Power Electronics, Drive

