Stability analysis of the tip vortices of a wind turbine

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ABSTRACT

The aim of the present paper is to obtain a better understanding of the stability properties of wakes generated by wind turbine rotors. To accomplish this, a numerical study on the stability of the tip vortices of the Tjaereborg wind turbine has been carried out. The numerical model is based on large eddy simulations of the Navier–Stokes equations using the actuator line method to generate the wake and the tip vortices. To determine critical frequencies, the flow is disturbed by inserting harmonic perturbations, giving rise to spatially developing instabilities. The results show that the instability is dispersive and that growth arises only for some specific frequencies and type of modes, in agreement with previous instability studies. The result indicates two types of modes; one where oscillations of neighboring vortex spirals are out of phase and one where oscillations in every vortex spiral in phase. The mode with spirals out of phase results in the largest growth with the main extension of the disturbance waves in radial and downstream directions. The out-of-phase disturbance leads to vortex pairing once the development leaves the linear stage. The study also provides evidence of a relationship between the turbulence intensity and the length of the near wake. The relationship, however, needs to be calibrated against measurements. Copyright © 2010 John Wiley & Sons, Ltd.

KEYWORDS

wakes; CFD; EllipSys3D, wake length, Horns Rev, Actuator line, Harmonic perturbations, Wind turbines

1. INTRODUCTION

Interaction between wakes of wind turbines in wind farms is an issue addressed by researchers using increasingly enhanced methods. The overall goal is to develop tools for optimizing the energy production of wind farms and by this reduce the energy cost. In order to do so, detailed knowledge of the unsteady flow development of interacting wakes is needed. Wakes originate from the trailing vortices shed from the spinning blades forming concentrated helical tip and root vortex structures. The development and lifetime of tip vortices are subjects of interest when considering the impact from upstream turbines on downstream turbines. The tip vortices break up due to self induced instabilities and ambient atmospheric turbulence. Thus, the life time of tip vortices and the process of how they break up is valuable knowledge when considering the aerodynamic loads and fatigue on affected turbines.

When considering three-bladed turbines shedding three tip and root vortices, studies have shown that the root vortex structure is destroyed rapidly since the three vortices are close together and the nacelle disturbs the flow. The emphasis has therefore been to quantify the basic mechanisms behind the breakdown of the tip vortex structure.

Joukowski proposed a model for a two-bladed propeller in 1912. His model basically consisted of two rotating horseshoe vortices.1 Recent inviscid studies have shown that the wake resulting by Joukowski is unconditionally unstable.2 Navier–Stokes simulations, however, have indicated that the wake might be stable under some conditions.3 Inviscid studies by Widnall indicate three different modes: a long and a short wave instability, and a mutual inductance mode.4 The latter occurs when the pitch of the helix decreases and the neighbouring turns of the filament begin to interact strongly. Recent studies by Walther et al.5
confirm the stability of the inviscid model, but predict a breakdown of the vortex system due to viscosity. Leishman et al.\textsuperscript{6} and Bhagwat et al.\textsuperscript{7} performed a temporal instability analysis and identified unstable modes in the wake structure originating from a helicopter by using free-vortex wake calculations. They found that a maximum in the growth rate was found to occur at wave numbers equal to half-integer multiples of the number of blades, for a three blade configuration occurring at \( \frac{1}{2}, \frac{4}{2}, \frac{7}{2}, \ldots \). In the middle of those values minima in the growth rate exist.

The present study focuses on the stability of shed tip vortices and the process of how they break up using a technique where controlled harmonic perturbations are applied in the neighbourhood of the shed tip vortices. The method is used together with the actuator line technique combined with the unsteady incompressible Navier–Stokes equations. In contrast to some of the earlier studies where the growth in time of the instability was calculated, this study represents a spatial stability analysis where the disturbances are followed all the way to the non-linear vortex pairing stage.

2. SIMULATIONS

As basic solution for the stability analysis, simulations were performed to reach a basic steady flow field.

Together with a well-defined superimposed disturbance, this forms the initial condition for the subsequent stability analysis. To simplify the analysis only flow cases with constant axial inflow are considered. However, about 10 million mesh points are needed to perform the simulations at realistic Reynolds numbers. The point of onset and the growth of the instabilities are evaluated in detail.

The computations are carried out as large eddy simulations (LES) employing the mixed sub-grid-scale model developed by Ta Phuoc.\textsuperscript{8} This model exploits the advantage of a closure combining vorticity and turbulent kinetic energy. In this model, the vorticity is derived directly from the filtered variables whereas the turbulent kinetic energy is determined by use of a test-filter that is twice as coarse as the computational grid. For more details about the mixed scale model, we refer to the text book by Sagaut.\textsuperscript{9}

2.1. Grid

In the computations periodic boundary conditions are used, which reduces the mesh size to one third, i.e. a 120-degree slice. Using this approximation, each spiral segment is free to take any form with the condition that the three spirals from the three blades behave in the same way. For example, the radial and axial position of the spiral from the first blade after 60, 180, 300, \ldots degree of azimuthal travel must correspond to the position of the second and the third blade after 60, 180, 300, \ldots degree of azimuthal travel. Since this is a spatial instability with the vortices free to move in the downstream direction, the symmetry does not restrict the wave lengths possible to simulate to multiples of 120 degrees, i.e. each spiral does not have to return to its value after 120 degrees to preserve the symmetry, instead the periodicity is satisfied because the spiral from the next blade takes on the value of the first blade 120 degrees later in the azimuthal direction.

The mesh consists of 40 blocks that are distributed both in axial and radial directions, allowing the simulations to be run on up to 40 processors. Figure 1 shows how the mesh is designed. The well-resolved area stretches 7 rotor radii behind the turbine.

2.2. Numerical method

The computations are carried out using the so-called Actuator Line Method (ACL) introduced by Sørensen and Shen.\textsuperscript{10} In this method, the body forces are distributed along the blades as line sources. In contrast to this method, full CFD simulation would require a great number of mesh points along the blades to resolve the boundary layer on the surface. With the actuator line method, these mesh points are not needed and the method therefore opens new possibilities for turbine simulations with a well resolved wake. The drawbacks are that the method is based on tabulated airfoil data from which \( C_D \) and \( C_L \) are functions of \( \alpha \). The method is implemented into the EllipSys3D code, a general purpose 3D Navier–Stokes solver developed by N.N. Sørensen and Michelsen.\textsuperscript{11–14} The flow solver is a multiblock, finite volume discretization of the Navier–Stokes equations in general curvilinear coordinates. The code is formulated in primitive variables, i.e. in pressure and velocity variables, in a collocated storage arrangement.
ment. Rhie/Chow interpolation is used to avoid odd/even pressure decoupling. The presence of the rotor is modelled through body forces, determined from local flow and airfoil data. The Navier–Stokes equations are formulated as:

\[
\frac{\partial u}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + f_{body} + f_{c,i}
\]

where \( f_{body} \) represents the forces acting on the blades, \( f_c \) the coriolis force, \( \nu \) is the kinematic viscosity and \( \nu_t \) the eddy viscosity that is modelled through the sub-grid-scale model, using the LES method. The numerical method uses a blend of third order QUICK (10\%) and fourth order CDS (90\%) difference scheme for the convective terms and second order central difference scheme for the remaining terms.

The aerodynamic forces are distributed along the actuator line and smeared among neighbouring node points in a Gaussian manner. This is done by taking the convolution of the computed load \( f_{th} \) and the regularization kernel \( \eta_h \).

\[
f_i = f_{th} * \eta_h
\]

where \( h \) is a blade number index. The regularization kernel is defined as:

\[
\eta_h(p) = \frac{1}{\varepsilon \pi^{1/2}} \mathrm{e}^{-p^2/\varepsilon^2}
\]

where \( p \) is the distance between cell centred grid points and points on the actuator line. The 3D Gaussian distribution is controlled by the parameter \( \varepsilon \).

The choice of \( \varepsilon \) will also affect the numerical discontinuity at the tip, which has been investigated by Ivanell et al.\(^{15}\) In the present study, \( \varepsilon \) is set to 1, which corresponds to a smearing of about two cell sizes, which is a suitable choice according to Ivanell et al.\(^{15}\)

A comment on the restriction of the 120-degree symmetry on the solution of the non-linear equations is in order. Note that once an initial condition has been chosen that satisfies this symmetry, the complete non-linear development will continue to satisfy this symmetry, i.e. we are not making any approximation regarding the non-linear development of the instabilities. In a real flow case where initial disturbances may exist which do not have 120-degree symmetry, they will contaminate the flow and as the flow becomes turbulent, the initial symmetry will eventually break down.

2.3. Numerical setup

The simulation was performed on a Linux PC cluster using MPI.

All simulations have been performed with a wind speed of 10 m/s corresponding to a tip speed ratio of 7.07 and a \( C_T \) of 0.79. The actuator line is fixed in the mesh. The rotation is performed by a rotating reference frame.

Since the actuator line method uses tabulated airfoil data, good quality data must be used. Data from the Tjaereborg turbine have been used for all simulations in this project. The Tjaereborg turbine was operational between 1988 and 1998. The Tjaereborg wind turbine was equipped with a three-bladed upwind horizontal axis rotor. The blade profiles consisted of NACA 44xx airfoil sections with a blade length of 29 m giving a rotor diameter of 61 m. The chord length was 0.9 m at the tip, increasing linearly to 3.3 m at a hub radius of 6 m. The blades were twisted 1° per 3 m. The tip speed was 70.7 m/s. The rated power was 2 MW and the output was controlled by continuously varying the pitch angle between 0 and 35 degrees in production mode. The hub height was 60 m.

Two different meshes have been used in the study: a coarse grid with 48 nodes on each block side (i.e. \( 48^3 \cdot 40 \approx 4.5 \cdot 10^6 \) nodes in total) and a fine mesh. The coarse mesh was used in a pre-study where the objective was to identify a dispersive instability. The second resolution (from now on referred to as the finer grid) with 64 nodes on each block side, i.e. \( 64^3 \cdot 40 \approx 10.5 \cdot 10^6 \) nodes, has been used for further investigations of identified modes. The number of node points along the actuator line is 50 for the coarse mesh and 70 for the finer mesh. In both cases the points are equidistantly distributed.

3. FOURIER ANALYSIS

A Fourier analysis has been performed to identify growth of introduced perturbations.

3.1. Perturbation

The aim is to study which modes are present and to what extent they grow in order to quantify frequencies leading to a vortex spiral breakdown. Figure 2 shows the structure of the vortex spiral. By disturbing these vortex spirals one may trigger modes leading to instability growth and thereby a vortex spiral breakdown. Figure 3 illustrates the concept of introducing a small sinusoidal perturbation close to the tip of the blade by adding a time dependent body force. The perturbation is performed in axial direction only. The perturbation is performed according to:

\[
F_z = a \sin(\omega t)
\]

where \( a \) is the applied amplitude and \( \omega \) is the perturbation frequency. \( a = 1 \) corresponds to an amplitude resulting in a velocity change of \( \pm U_{\infty} \) close to the disturbance. The disturbance is positioned close behind the tip and results in a spatially developing disturbance wave on the spiral vortices. This is in contrast to most of the previous
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3.2. Evaluation by Fourier series

By evaluation of $N$ velocity fields, $\Phi_n$, taken equidistantly over one period, the response from a specific perturbation frequency can be evaluated. In the present study we are using $N = 8$. By using Fourier series, the disturbance response can be divided into its frequency components and be expressed according to:

$$\hat{\Phi}_j = \frac{1}{N} \sum_{n=0}^{N-1} \Phi_n e^{-j2\pi \frac{n}{N}}$$  \hspace{1cm} (5)

where $\Phi_n$ is the complex eigenfunction with angular frequency:

$$\omega_n = \{0, \omega, 2\omega, \ldots, (N/2-1)\omega\}$$  \hspace{1cm} (6)

Here $i = \sqrt{-1}$ is the complex unit and index $j$ corresponds to the $j$:th velocity field. $\Phi_0$ corresponds to the basic flow, $\Phi_1$ the response from the perturbation frequency, $\omega$, and $\Phi_k$ the response from the first superharmonic, $2\omega$, and so on. First, a steady state solution is found after which a time resolved computation is performed to reach a periodic solution. $N$ fields, equispaced in time, are then extracted during one period and used in the calculations of the Fourier coefficients $\Phi_n$. In order to evaluate the growth along the spiral, here noted as $s$, the amplitude of the perturbation is needed along the vortex spiral. That is done by identifying the maximum response of the perturbation frequency, $\Phi_1$, at each $z$-position, i.e. at each position in the flow direction, according to:

$$\hat{\Phi}_m^s = \max_{x,y,z} |\Phi_1(x, y, z)|$$  \hspace{1cm} (7)

where $\hat{\Phi}_m^s$ is a vector defining the amplitude along the spiral for mode $k = 1$. From $\Phi_1^s$ we can extract the positions of the spiral. We define $\Phi_m^s$ as the amplitude along the spiral for mode $k$, where the spiral position has been defined by the maximum response for mode $k = 1$. Figure 4 shows an example of the development of the disturbance for $f_c = 2$, where $f_c \equiv f_{sp}R \frac{c}{U_{\infty}}$. $f_c$ is therefore non-dimensional frequency in computational space and $f_{sp}$ the frequency in physical space. Note the exponential growth until about $z = 17$, i.e. 3.5 rotor radii behind the turbine, where the second harmonic is about half the magnitude of the primary wave and the system becomes non-linear.

The absolute value of $\hat{\Phi}_m^s$ represents the amplitude of the disturbance, and the slope of the curve represents the growth rate. $k = 1$ corresponds to the response of the frequency used to perturb the flow, $k = 2$ corresponds to the response of the first superharmonic, and so on. From the real and imaginary parts of the vector $\hat{\Phi}_m^s$ the spatial wave-length can be found.

Figures 5 and 6 show the development of the amplitude of the first harmonic due to disturbances for $f_c = 2$ and $f_c = 5$ using different perturbation amplitudes. Figure 5 illustrates the results in linear scale, while Figure 6 shows the results in logarithmic scale. From the figures it is clearly seen that the amplitudes have an exponential growth. In Figure 6, the data are normalized by the amplitude at $z = 14$ to compare growth from computations with different amplitudes. The result shows that the growth rate is independent of the amplitude of the perturbation since the slope of the data from all perturbation amplitudes are equal up to the point where non-linear effects start to become important. The data from all perturbation amplitudes are showing exponential development until they reach a specific value of $Abs(\hat{\Phi}_m^s)$ of about 0.1 m/s (see

Figure 2. The vortex spiral structure of the wake behind a three-bladed turbine.

Figure 3. Conceptual development of the sinusoidal perturbation introduced close behind the turbine blade. Only one spiral is shown for simplicity.
Figure 5). This can be considered as the starting point of non-linear development of the perturbation. When choosing a smaller amplitude, the linear part of the extracted signal stretches further downstream, since it takes longer time for the instabilities to grow to the extent that they reach a non-linear state. There is, however, a limit to how small the amplitude can be. When the amplitude becomes too small, truncation errors will become of the same order as the disturbance amplitude and will trigger instabilities without applying any perturbation. The dotted curve in Figure 5 illustrates the growth when no perturbation is applied. As a result, that curve gives the lower limit of the disturbance input. As seen in Figure 5, the curves representing development of instabilities originating from amplitudes 0.05–0.0005 are all of higher orders than the numerical errors when using the finer grid. When using the
coarser grid, not shown in the figure, the numerical errors are in the same order as the response to the perturbation amplitude of 0.0005. Therefore, in the analysis of the results from the simulations with the coarse grid an amplitude of 0.005 has been used.

4. RESULTS AND DISCUSSION

A numerical stability investigation has been performed introducing frequencies $f_c = 0.5$ to $f_c = 8$ in computational time scale. Figure 7(a),(b) shows the growth of the instability for the perturbation frequencies along the spiral $s$ in a logarithmic scale for the fine mesh. Figure 7(c) shows the growth rate along the spiral $s$ as a function of the frequency for results from both grid sizes. The growth has been computed as the derivative of a curve fit to $\Phi_k^m$ in the linear regime (see Figure 7(b)). In Figure 7(c), the results from the coarse mesh is identified by "$\ast\ast\ast\$" and results from the fine mesh is identified by "$\ast\ast\$". The results from the simulation with finer mesh generally resulted in higher growth rates compared to the results with the coarse mesh.

The result shows that the growth rate reaches a maxima at two different frequencies of about $f_c = 2$ and $f_c = 5$, respectively.

To better understand the dynamics of the instabilities three of the unstable modes are illustrated conceptually in Figure 8. The figure shows the full domain divided into three 120-degree subdomains. The computation uses 120-degree symmetry and is, therefore, only performed in a 1/3 subdomain with periodic boundary conditions. The nine figures illustrate the evaluation of three different modes during one complete revolution, resulting from three different frequencies. The first row of figures illustrates a mode with 1.5 wavelengths along a 360-degree turn. The second row of figures illustrates a mode with 3 wavelengths in one revolution and the third row a wave with 4.5 wavelengths in one revolution. The grey area signifies the phase of the disturbance of every spiral arc at one azimuthal position. Note that for waves with 1.5 and 4.5 wavelengths during one complete revolution the oscillations of every other vortex spiral is out of phase, whereas for a wavelength with three periods in one revolution they are in phase. It will be seen that it is the out of phase perturbations that are the most unstable, possibly due to the fact that they will result in a closer proximity between the vortices than the in phase perturbations.

In order to relate spatial structure of the disturbance waves to specific perturbation frequencies, the real part of the field $\Phi_k^m$ has been studied. By analyzing an iso-surface of the real part of the response from the perturbation frequency the spatial structure of the wave can be analysed. Figure 9 shows the spatial extension of the response from the perturbation frequency at a perturbation frequency $f_c = 5$ in computational space. Note that there exists 9 nodes per revolution, indicating that the result from a perturbation frequency $f_c = 5$ corresponds to a spatial wavelength of 4.5 wavelengths in one revolution. A similar analysis for the case with a perturbation frequency of $f_c = 2$ results in 3 nodes in each revolution indicating 1.5 wavelengths in one revolution (compare with the filled circles at the first row in Figure 8).
The results show that row 1 in Figure 8 corresponds to a perturbation frequency of about $f_c = 2$ in computational space and that the third row in Figure 8 corresponds to a perturbation frequency of about $f_c = 5$ in computational space. It is important to note that growth mainly occurs when every other vortex spiral is out of phase. That is when the arrows in Figure 8, as discussed earlier, are pointing in opposite directions at every other axial position.

Figure 10 illustrates that the main disturbance velocity components for both types of modes are in the radial and downstream direction. The figure also shows a phase shift of 180 degrees between the axial and radial components. This implies that when the wave extends in positive radial direction, it also extends in negative flow direction and vice versa. The figure further demonstrates that the amplitude of the velocity fluctuations in these two directions is of the same order. Figure 11(a),(b) shows the vortex spiral. The spiral is identified by an iso-surface of the vorticity. The figure shows the breakdown of the spiral structure for two different frequencies, $f_c = 2$ and $f_c = 5$. The result clearly indicates that there is an out of phase interaction between the different spirals for the case with $f_c = 2$ and $f_c = 5$. For the other frequencies it is difficult to identify any clear interaction between the spirals.

Earlier work performed by Leishman et al. on a helicopter rotor, using free-vortex wake calculations, shows the same type of modes as identified in this study. Leishman et al. shows that the wave numbers corresponding to maximum growth of the instability occurs at $(j + 1/2)N$, $j = 0, 1, 2 \ldots$ for a N-bladed rotor. This exactly corresponds to the first and third mode explained in Figure 8. That is, the modes with every other spiral out of phase. Leishman et al. further shows that minimum growth occurs at wave numbers of $jN$, $j = 0, 1, 2 \ldots$, which corresponds to the second case in Figure 8; that is, the less unstable case with spirals in phase.

In Figure 12, the unfolded spiral structure for $f_c = 5$ case is depicted as a curved plane through one 120-degree slice of the vortex spiral. The colour contours illustrate the vorticity. The modes where every other vortex spiral is out of phase is clearly seen. In Figure 13 a plane cutting through the spiral structure in a radial-axial direction is seen, where we initially observe the regularly spaced vortices, after which vortex pairing occur. Note that at the point where pairing occurs, the phase shift between the radial and the down stream components has disappeared. Instead there is an extension in both positive radial and positive flow directions and vice versa. The same behaviour has been shown in early experiments by Alfredsson and Dahlberg (see Figure 14). Wie et al. have also observed vortex pairing phenomenon in their recent paper.

The frequency in computational space has been recalculated to physical space (see Table I), showing that the frequencies resulting in the mode previously described are in the order of 0.66 Hz and 1.64 Hz. This is also the order of common wind field fluctuations. One may therefore expect these modes to occur in everyday situations for wind turbines.

Assuming that disturbance amplitudes and frequencies are related to the turbulence intensity and frequencies of a wind field, makes it possible to model the length of the stable wake.

Let the length of the wake correspond to the point where the development of the disturbance reaches its maximum (see Figure 5). This corresponds approximately to the point where the development of the first superharmonic frequency is affecting the development of the disturbance.
and the structure of the tip spiral is affected (see Figure 4). That is, non-linear terms are becoming important and the growth can no longer be considered a result of solely linear terms.

Since the instability is exponential there exists an exponential relationship between turbulence intensity and the length of a stable wake. For a real wind field let the amplitude of the perturbation be proportional to the turbulence intensity (\(T_u\)) with a proportionality constant \(I\).

Assuming that the most critical mode is present the tip spiral length can be expressed as:

\[
\tau = -0.45 \ln (I \times T_u) + 2, 0
\]

where \(\tau\) indicates the tip spiral length and the turbulence intensity (\(T_u\)) is defined as \(\frac{U}{u'}\) where \(U\) is the mean wind

**Figure 8.** The rows of the figure displays three different instability mode shapes projected on a plane perpendicular to the flow direction. In each column, the thick curve indicates the instability mode in a 120° sector (A' to B, B' to C and C' to D), corresponding to a disturbance that has been generated by blade b1. The dashed lines show how different points in space are connected and the dotted line illustrate the history of the tip spiral in the previous domain. The filled circles illustrate where the wave has its largest extension where the blue colour represent the disturbance originating from blade b1, the red colour the disturbance originating from blade b2 and the green colour the disturbance originating from blade b3 respectively. The arrows illustrate the direction of the radial extension of the wave. In the calculation there is a 120° symmetry imposed so that the solution in each 120° sector is identical, implying that the mode shapes represented by the thick curves coexist in all sectors, signifying vortices originating from the three different blades and located at different downstream positions. From this figure it can be deduced (see the gray oval regions) that the first mode, which complete a wavelength in a 240° turn, has an out of phase oscillation of neighbouring vortices; the second mode, which completes a wavelength in a 120° turn, has an in phase oscillation of neighbouring vortices; and the third mode, which completes a wavelength in a 80° turn, has an out of phase oscillation of neighbouring vortices.

**Figure 9.** Iso-surfaces of \(\text{Re}(\Phi_m^k)\) of \(f_c = 5, k = 1\).
speed and $u'$ is the velocity fluctuation. The equation has been derived from simulations of the disturbance growth, where the turbulence intensity has been related to the initial perturbation amplitude. The turbulence intensity is set to $\sqrt{2}/2$ times the perturbation amplitude and normalized to physical space, giving the factor of $-0.45$ in the relation above.

The end of the wake length is set to the point where non-linear terms are becoming important as discussed.
5. CONCLUSIONS

To determine the stability properties of wind turbine wakes, a numerical study on the stability of the tip vortices behind the Tjaereborg wind turbine has been carried out. The numerical model is based on large eddy simulations of the Navier–Stokes equations using the actuator line method to generate the wake and the tip vortices. To determine critical frequencies, the flow is perturbed by inserting a harmonic perturbation, with the aim of studying existing modes and to which extent they grow in order to quantify frequencies leading to a vortex spiral breakdown.

The results show that the instability is dispersive and that growth arises only for some specific frequencies and type of modes. The result indicates two types of modes: one where oscillations of every other vortex spiral is out of phase and one with oscillations in every vortex spiral in phase. The mode with spirals out of phase results in the largest growth and the main extension of the waves is in radial and downstream directions. There is a phase shift of 180° between the radial and downstream velocity components. Downstream of the non-linear development of the instability results in vortex pairing.

Table 1. Relation between the frequency in computational and physical space.

<table>
<thead>
<tr>
<th>$f_c$ [c](^{-1})</th>
<th>0.50</th>
<th>1.00</th>
<th>2.00</th>
<th>3.33</th>
<th>4.00</th>
<th>5.00</th>
<th>6.00</th>
<th>7.00</th>
<th>8.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ph}$ [Hz]</td>
<td>0.16</td>
<td>0.33</td>
<td>0.66</td>
<td>1.09</td>
<td>1.31</td>
<td>1.64</td>
<td>1.97</td>
<td>2.30</td>
<td>2.62</td>
</tr>
</tbody>
</table>

above. This relation indicates that the wake length, as defined here, corresponds to two turbine radii when the turbulence intensity is 1%, with the calibration parameter $I$ set to one.
The study provides evidence of a relationship between the turbulence intensity and the length at which the wake is stable. The relationship, however, needs to be calibrated with measurements.

REFERENCES

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