A multi-follower bilevel stochastic programming approach for energy management of combined heat and power micro-grids

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Article history:
Received 4 September 2017
Received in revised form 2 February 2018
Accepted 4 February 2018
Available online 9 February 2018

Keywords:
Micro-grid owner
Multi-follower bilevel programming
Combined heat and power systems

Abstract
This paper presents a multi-follower bilevel programming approach to solve the 24-h decision-making problem faced by a combined heat and power (CHP) based micro-grid (MG). The framework contains the interests of two different agents: the MG operator/owner (MGO), who procures the maximization of total profit incurred in attending the forecasted demand of consumers via demand response program (DRP) as well as day-ahead (DA) and real-time (RT) markets participation, and the various CHP owners (CHPOs) who procure the maximization of the profits obtained from the thermal and electrical energy sales. The interaction between the entities is determined in a bilateral contract. Further, to deal with various uncertainties, each level is formulated as a stochastic two-stage problem, where the volatility nature of consumers’ loads, RT market price and wind speed uncertainties are modeled using autoregressive moving average (ARMA) technique. In this paper, in order to consider realistic model of the problem, on the contrary to the most CHP-based MG scheduling literature, the network operation constraints such as voltage magnitude of buses and line flow limits are taken into account.

1. Introduction
Demand response program (DRP) according to the U.S. Department of Energy (DOE) is described as residential, industrial and commercial customers’ proficiency to change energy-consumption schemes as a reaction to changes in the electricity price over time, or to incentive fees in order to fulfill reasonable prices and system reliability [1]. DRP also has great influence in reducing peak load, which aids to put off the requirement of generation capacity expansion as well as decreasing carbon footprint by operating a smaller number of generators to supply the peak load [2]. DRP research has recently attracted a great deal of interest from the research community. A review of the demand side management (DSM) policies in UK is presented in Ref. [3]. References [4] and [5] introduce different market clearing models regarding the DRP incorporation. A real-time (RT) DRP in response to time-varying market price has been proposed in Ref. [6] implementing robust optimization procedures. In Ref. [7], an optimization outline is addressed to maximize the DR aggregator profit in the day-ahead (DA) market.

Utilization of micro-grids (MGs) can assist effectual DSM and incorporation of renewable energy sources (RESs) at distribution level [8]. MG can be characterized as a cluster of distributed energy resources (DERs) and associated load that can maintain the operation while being connected or disconnected from the main grid in different circumstances, considering certain constraints [2]. Research on MG concerns has received lots of attention in recent years. In Refs. [9,10], various models have been proposed to optimize the operation of MGs. On the other hand, increasing penetration of volatile RESs in MGs as well as volatile demand and real-time market prices pose several challenges to MGs in order to retain the energy (electricity/heat) production and consumption balance [11]. Hence, a proper wind speed, price and load forecast has a decisive influence on decision making strategies of the MG master.

Recently, using combined heat and power (CHP) systems in MGs has attracted more attention [12]. CHP system is a well-established technology that affords advantages with regard to environmental impact and efficiency. Therefore, CHP units are considered as a foremost technology for responding to MGs’ power and heat demands for more efficient energy resources as well as environmental apprehensions. However, to assist this generation technology to meet the MGs’ demands successfully, a precise and efficient model must be developed to optimize the strategy for selling electrical...
and heat energy to customers. In a CHP unit, the power production boundary depends on the heat generation of unit and the heat production boundary depends upon the power generation of the unit. In Refs. [13,14], the feasible operation region of CHP units has been modeled successfully. In Ref. [13], different sources of uncertainty have been modeled in MG scheduling problem, successfully. The major goal of [13] is to meet both electricity and heat demands of MGs equipped with CHP units as well as DRP while taking participation in the DA market. However, in Ref. [13], the MGO and CHPOs are considered as a unit agent and power network constraints are neglected.

New independent owners have invested in CHPs to transform power and heat to MGs in real-world [15]. The on-site CHP investors should supply the heat demand of local heat loads using their facilities. In these circumstances, finding the optimal set points of energy resources for profit maximization of MGO along with different CHP investors is of important issues. Energy management of MGOs along with CHPOs has not been studied in previous works. A hierarchical formulation is required for studying the response of various agents when acting in a constantly changing environment. Supply demand driven by fluctuating prices, which is defined by a supplier confronting with the continuously changing power market price, entails inevitable challenges in the operation of an MC, necessitating new functionalities and tools. Bilevel programming, originally describing a Stackelberg game [16], is extended by several researchers, and appears preferably appropriate for establishing real market circumstances where different autonomous, possibly inconsistent, decision makers perform in a hierarchical framework. The Bilevel programming is suitable for modeling problems, in which one agent, namely, the leader, optimizes its fitness function (upper-level problem) taking into account a second agent, namely, the follower, will respond by optimizing its own fitness function (lower-level problem) [17]. A multi-objective bilevel optimal operation model for distribution network with grid-connected MGs is presented in Ref. [18]. In this paper, distribution network, as upper-level model, determines the optimal dispatch of distribution network to attain its voltage profile improvement and power loss reduction. On the other hand, MG, as a lower-level model, determines the optimal operation strategy of distributed generators in MGs regarding dispatch requirements of upper-level.

This technique is applicable in situations that the performance of the follower will influence the decision making of the leader. Hence, in the CHP-based MGs energy management problem, as the revenue of the MGO (leader) from market participation depends on the offering strategy of various CHPOs (followers), which is decided by the CHPOs based on the bidding strategy of MGO, the bilevel programming will be appropriate. The current paper employs multi-follower bilevel programing (MFBP) for optimal energy management of CHP-based MGs. The proposed framework constitutes a Stackelberg game, in which MGO is the leader and CHPOs are the followers. The interaction between the CHPOs and MGO are determined in a bilateral contract. In the most of CHP-based MG scheduling literature, the physical restrictions of electric power transmission known as power flow constraints, have not been addressed. This simplification may result in a solution that is not technically accurate. In this regard, the network operation constraints such as voltage magnitude of buses and line flow limits are taken into account in this paper. Moreover, DRP has been proposed to serve the electricity and heat demands of the consumers with minimum cost. It is presumed that the MGO will receive (pay) for increased (decreased) consumption of consumers, which is proportionate to the DA market price. In addition, in the proposed DRP, the total power demand of MG at different buses can be supplied, with or without curtailed load.

It should be mentioned that enhancing the operational flexibility of CHP units can introduce noteworthy benefits in the uncertain environment. Potential solutions to inflexibility contain installing heat pumps [19], boilers [20], and equipping with heat storage tanks [21]. In addition, these solutions have been reported to potentially diminish wind curtailments and recover the operation efficiency [19–21]. Hence, in this study, the CHPOs are equipped with CHP units along with heat-only unit and heat buffer tank (HBT). They possess different types of CHP units, where the heat-power dual dependency characteristic is represented implementing a combination of extreme points in the polyhedral feasible operating region. The solution of proposed scheduling model will determine the optimal set point of CHPOs’ facilities to maximize their profit. In addition, the model maximizes the MGO profit by defining the performance of its wind turbine (WT) and conventional power generation unit as well as determining the amount of exchanging power with the main grid and CHPOs in order to supply whole electrical and thermal demand of the MG.

The uncertainties pertaining to RT market price, consumers demand and WT generation are regarded as scenarios. Furthermore, fast backward reduction technique is applied to enhance a trade-off between the accuracy of the solution and the computational burden as scenario reduction method. Three case studies have been scrutinized in the paper in order to highlight the economic advantages of implementing the proposed strategy. In the first case, the effect of centralized control in which only one entity manages both the MGs’ and CHP units’ constraints has been studied. The second case studies the benefits of applying a two level structure in the scheduling problem. Finally, deterministic formulation of the proposed model has been investigated.

The main contributions of the paper may be summarized as follows:

1) Power flow constrained energy management of CHPOs as well as MGOs has been presented in which electric power grid constraints, i.e. voltage magnitude of buses and line flow limits are well-considered.

2) Stochastic multi-follower bilevel formulation of the framework which contains the interests of multiple agents, i.e. MGO and different CHPOs has been presented. In the proposed model CHPO is equipped with CHP units as well as boiler unit and HBT.

3) The CHP-based MG will take participate in both RT and DA markets, simultaneously. The real-time environment is unraveled using two-stage mathematic formulation.

4) MFBP with modeling each level as a two-stage problem is proposed. The uncertainty and variability of WT outputs, consumers load and RT market prices are fully considered.

This paper is organized as follows. In Section 2, the CHP-based MG model as well as DRP, power flow (PF) constraints and scenario generation process of the model is introduced. The mathematical program with equilibrium constraints (MPEC) is formulated to handle the MFBP model in Section 3. In Section 4, case studies are conducted to demonstrate the benefit and effectiveness of the proposed method. Conclusion is finally given in Section 5.

2. Problem formulation and modeling approach

2.1. System model

The design objective of the CHP-based MG scheduling is maximizing the profit of MGO as well as CHP units owners considering uncertainties. The MGO profit could be acquired from DA and RT electricity markets revenues over 24-h time horizon by activating DRP. The MGO is assumed to possess power-only units and WTs.
Also, it is assumed that the CHPOs sell the electricity and heat energy to the MG at a predefined retail rate.

In the proposed framework, MGO acts as leader and each CHPO that offers to the MGO is a follower. Hence, to express the equilibrium problem, first, a stochastic MFBP model is formulated. This hierarchical formulation includes MGO profit maximization problem incorporating power flow, DRP and facilities constraints as the upper-level model and each CHPO profit maximization as the part of lower-level problem.

To handle such problem, two typical procedures are often employed [22]. In the first one, each participant’s individual profit-maximization problem is solved iteratively until a Nash stationary point is attained, which is so-called diagonalization methods. The second one assembles the Karush-Kuhn-Tucker (KKT) conditions of all the contributors’ individual profit-maximization problems and unravels them together. Regarding the lack of adaptability of the diagonalization methods [22–24], the KKT method is adopted in this paper. In this regard, the MPEC is derived for MGO by bearing in mind the KKT conditions of the lower-level model. The framework of proposed MFBP procedure is presented in Fig. 1.

2.2. Demand response model

In the paper, the MGO can adjust the loads’ consumption level regarding the DA and RT market prices in order to maximize the market participation revenue. The load consumption \( load_i \) at each bus \( i \) may be stated as:

\[
load_i^t = \left(1 - DR_i^t \right) \times load_{i,0}^t + SL_i^t
\]

(1)

in which, \( load_{i,0}^t \) represents the primary electricity demand of bus \( i \) at time \( t \) and scenario \( s \), \( DR_i^t \) indicates the percentage of load consumption shifting from time slot \( t \) and \( SL_i^t \) is the shifted load from other intervals to hour \( t \). The increased load, \( IL_i^t \), in each time interval is defined as:

\[
IL_i^t = SL_i^t - \left(DR_i^t \times load_{i,0}^t \right)
\]

(2)

In order to address the load curtailment or increment in 24-h time interval, the coefficient \( \rho_i \) has been employed with the limit \( \gamma_i \) \((\gamma_i \geq 1)\) as follows:

\[
\sum_{t=1}^{24} SL_i^t = \rho_i \sum_{t=1}^{24} DR_i^t \times load_{i,0}^t
\]

(3)

To indicate the behaviors of coefficient \( \rho_i \), if \( 0 \leq \rho_i < 1 \) the consumer of bus \( i \) will reduce the consumption as required at the real-time stage. If \( 1 < \rho_i \leq \gamma_i \) the consumer of bus \( i \) consumes more than its prediction.

In order to comply with the technical constraints associated with the maximum amount of increased load, the increased load in each time slot should be in a definite bound, which can be represented as:

\[
0 \leq IL_i^t \leq inc_{\text{max}} \times load_{i,0}^t
\]

(4)

where, \( inc_{\text{max}} \) is the maximum percentage of increased load. The portion of the consumption that is portable in each time slot might be different and assigns the busbar load participation factor. The following restraint limits the portion of load consumption that can be shifted to other time intervals:

\[
DR_i^t \leq DR_{\text{max}}
\]

(5)

where, \( DR_{\text{max}} \) stands for the maximum participation factor in DRP. The linear function of demand is one of the simplest and most extensively implemented models of the responsive load [25,26] which addresses the customer’s demand as a linear function of the pool market price as follows [25,26]:

\[
load_i^t = \alpha_{i,\text{lin}}^t + \beta_{i,\text{lin}}^t \cdot p_{\text{pool}}
\]

(6)
in which, \(a_{i,lin}^s\) and \(\beta_{i,lin}^s\) are the constants of linear demand versus the price term. By fitting a linear curve to the historic price and load data, the price can be specified as:

\[
\lambda_{i,t}^s = \frac{1}{\beta_{i,lin}^s} (load_{i,t}^s - a_{i,lin}^s) 
\]

(7)

It should be mentioned that the variable \(\lambda_{i,t}^s\) depends on the \(load_{i,t}^s\), which will be determined according to the real-time demand. By summing the multiplication of the difference between the responded and initial load, \(\lambda_{i,t}^s\) in (7), the profit (cost) function of total increased (curtailed) load that the MGO should receive from (to) consumers, \(C_{t}^{ds}\), can be obtained as:

\[
C_{t}^{ds} = \sum_{i=1}^{N_{bus}} \lambda_{i,t}^s \times \lambda_{i,t}^s 
\]

(8)

2.3. Power flow equations

The PF equations in the CHP-based MG scheduling problem is used to simulate more realistic framework of the problem. The following equations characterize the flow of power throughout the system which are determined by Kirchoff’s laws:

\[
P_{t}^{grid,s} + P_{t}^{s} - P_{t}^{D,s} = \sum_{j=1}^{N_{bus}} \left( |V_{t,j}^s| |V_{t,j}^s| Y_{ij} \cos(\theta_{ij,t} - \delta_{ij,t} + \phi_{ij,t}) \right) 
\]

(9)

\[
Q_{t}^{grid,s} + Q_{t}^{s} - Q_{t}^{D,s} = \sum_{j=1}^{N_{bus}} \left( |V_{t,j}^s| |V_{t,j}^s| Y_{ij} \sin(\theta_{ij,t} - \delta_{ij,t} + \phi_{ij,t}) \right) 
\]

(10)

which show the active and reactive power flow equations, respectively. \(N_{bus}\) is the number of buses of the MG. Also, \(P_{t}^{grid,s}\) and \(Q_{t}^{grid,s}\) are the active and reactive power flows of DERs located on bus \(i\), respectively. \(P_{t}^{s}\) and \(Q_{t}^{s}\) stand for active and reactive powers bought from the utility through the bus which is connected to the main grid at time \(t\), respectively. It should be noted that these variable \((P_{t}^{grid,s}\) and \(Q_{t}^{grid,s}\)) will get values in the grid connected mode, otherwise these variables will be zero. \(V_{t,j}^s\) is the voltage of bus \(i\) at time interval \(t\). \(Y_{ij}\) and \(\theta_{ij,t}\) are the magnitude and phase angle of feeder’s admittance. \(P_{t}^{D,s}\) and \(Q_{t}^{D,s}\) are active and reactive loads of bus \(i\) at time \(t\), respectively.

2.3.1. Voltage limits

Voltage limits refer to the requirement of system bus voltage magnitudes, \(V_{i,t}^s\), which to be kept at permissible range. Moreover, the voltage magnitude for substation buses, \(V_{Sub}\), should be maintained at nominal value \(V_{Sub}^n\), which can be represented as:

\[
V_{min} \leq |V_{i,t}^s| \leq V_{max} 
\]

(11)

\[
|V_{Sub}| = V_{Sub}^n 
\]

(12)

2.3.2. Exchangeable power limit

Exchangeable apparent power with the main grid has to be in a limited bound in order to have the stable operation [27].

\[
\sqrt{P_{grid}^{s2} + Q_{grid}^{s2}} \leq S_{max}^{grid} 
\]

(13)

where, \(S_{max}^{grid}\) is maximum apparent power which could be exchanged with the grid.

2.3.3. Apparent power flow limits for branches

It is essential to keep the apparent power flowing through each branch, \(S_{br,t}\), of MG in its admissible range:

\[
\sqrt{P_{br,t}^{s2} + Q_{br,t}^{s2}} \leq S_{max,br} 
\]

(14)

where, \(S_{max,br}\) is maximum apparent power flowing through branch \(br\).

2.4. Stochastic optimization programming

The MG optimization problem faces various uncertainties in RT electricity market price, system load and wind energy generation when it makes the decision. To deal with these uncertainty sources, a two-stage scenario-based stochastic programming approach is employed in this paper. The stochastic optimization approach creates an appropriate tool to make decisions under uncertainty environment and exposes the fact that new material about the indeterminate data becomes recognized as time develops along the design horizon. Since the bilateral contracts of MGO and CHPOs should be assigned before realization of uncertain stochastic proceedings, the CHP units and boiler units production, HBT variables as well as the offering of heat and electrical energy of CHPOs are the here-and-now or first stage decision variables. In addition, the amount of power traded in the DA market is the first stage variable and does not depend upon the scenario realization. However, other state variables, i.e., generations of MGO’s units and the buying/selling of power from/to the RT market are wait-and-see or second stage decisions variables.

The paper assumes that the MGO can acquire predicted data from a local prediction center or the MGO has appropriate tools to predict the MG uncertainty aspects with a high degree of accuracy. State-of-the-art forecasting techniques, i.e., time series prediction, support vector machines and artificial neural networks can be exploited for forecasting the load, electricity prices, and wind speed [8, 28]. In other words, the paper presumes that the forecasts for uncertainty sources in the considered optimization model could be attained, whose detailed scheme is out of the scope of this paper.

2.4.1. Wind speed states

In order to study the effect of wind power generation uncertainty on the MGO decision making process, the wind speed is forecasted employing the autoregressive moving average (ARMA) based models [13]. An ARMA (p, q) process \(Y\) can be expressed as [28]:

\[
y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \epsilon_t - \sum_{j=1}^{q} \theta_j \epsilon_{t-j} 
\]

(15)

where, \(p\) stands for the number of autoregressive parameters \(\phi_1, \phi_2, \ldots, \phi_p\), and \(q\) is the number of moving average parameters \(\theta_1, \theta_2, \ldots, \theta_q\). The term \(\epsilon_t\) is the error term and states a normal stochastic process with zero mean and variance equal to \(\sigma^2\). After exploiting ARMA model, the error of the states are compared and
updated pursuant to some realized quantities of wind speed. Next, the error would be added to the forecasted quantity. Afterwards, the estimated wind speed frequency distribution is made for each time slot of optimization horizon that tails a Weibull distribution. Fig. 2 depicts the estimated wind speed frequency distribution and pertinent probability density function (pdf) of the fitted Weibull. Finally, wind power states for each time slot of the day are produced by means of scale and shape parameters of Weibull distribution.

### 2.4.2. RT market price and demand scenarios

The paper employs seasonal autoregressive integrated moving average models (SARIMA) technique [13] to model the stochastic processes of RT price and the MG’s electricity demand. SARIMA processes are stochastic practices which are implemented to analyze time series prediction, taking into account the daily and weekly seasonality of both series. SARIMA process of future RT prices and demand scenarios can be formulated as following [28,29]:

\[
\left(1 - \phi_1 B - \phi_2 B^2\right)\left(1 - \phi_{16} B^{16}\right)\log(\lambda_t) \\
= \left(1 - \phi_1 B - \phi_2 B^2\right)\left(1 - \phi_{16} B^{16}\right)\epsilon_t^p
\]

\[
\left(1 - \phi_1 B - \phi_{16} B^{16}\right)\log\left(l_{0t}\right) \\
= \left(1 - \phi_1 B - \phi_{16} B^{16}\right)\epsilon_t^l
\]

where, superscripts \(pr\) and \(l\) indicate the price and load. \(B\) is a backshift operator. The historical data have been engaged to approximate the model parameters using MATLAB software.

### 2.4.3. Scenario reduction

Since the computational requirements for solving the stochastic programming problems depend upon the number of scenarios, an effective scenario reduction method is essential for solving large scale problems. The reduction procedure would be a scenario-based approximation with a smaller number of scenarios and close to the original system.

It is favorable to create a large number of scenarios to reliably imprison the fundamental system uncertainty factors. However, since the computational complexity of handling the stochastic optimization problems directly depend on the number of scenarios, the tradeoff between computational intricacy and modeling accuracy requires to be considered. Guided by some in-sample and out-of-sample stability tests, number of scenarios in the reduced scenario tree have been selected [8]. In this regard, fast backward reduction technique is applied as scenario reduction method [30]. In the paper, the SCENRED tool under GAMS environment [31] is used to generate the reduced scenarios with pertinent probabilities.

### 3. Multi-follower bilevel programming

In this section, the objective functions of MGO and CHPOs are formulated in more details. Then the optimization MFBB model in view point of MGO as leader is presented and the relaxation of followers using KKT technique is implemented to solve the problem.

#### 3.1. Upper-level model

The upper-level model is the profit-maximization problem of MGO. The MGO aims to maximize the total profit, while supplying the total power and heat demand of the MG. It should be mentioned that the MG is assumed in grid-connected mode. Therefore, the revenue of the MGO is originated from selling the excess electrical energy to the DA and RT market, participating in DRPs pursuant to (8) and selling energy to the consumers. The cost of the MGO comprises the MGO’s units operational cost as well as the cost of buying energy from markets and also the thermal and electrical energy from CHPOs through bilateral contracts. In the proposed model, the MGO will participate in the DRP in order to maximize total profit. Furthermore, the PF constraints should be applied to the MGO problem to have more practical situation. Hence, the objective function can be written as:

\[
\text{Maximize } F^u = \sum_{t=1}^{24} \left( \sum_{s=0}^{S_t} x^s \left( p^{RTsale}_t \times p_{RTsale} \right) \\
- \left( p^{RTbuy}_t \times p_{RTbuy} \right) + d^r_s \right) \\
\left( p^{DrDA}_s \times g_s \right) \\
\left( p^{DrDA}_s \times g_s \right) \\
\left( p^{DrDA}_s \times g_s \right)
\]

where, \(x^s\) is the probability of scenario \(s\), \(l, m, k\) represent the indices for power-only units, boiler units and cogeneration units, respectively. \(p_{RTbuy}\) is the price of selling energy to the consumers and \(d^r_s\) and \(H^r_s\) are the total electricity and heat demands of consumers. \(p^{DrDA}_s\) and \(p^{DrDA}_s\) refer to the amount of electricity sold/bought in the RT and DA markets at time \(t\), respectively. \(n^t_{lth}\) and \(j^t_{lth}\) indicate the forecasted RT market, DA market prices, and \(p^{RTsale}_{lth} = H^r_{lth} + p^{CS}_{lth} + p^{PC}_{lth}\) are the offers of the \(k\)th CHPO for power and heat at hour \(t\), respectively. \(p^{CS}_{lth} = H^r_{lth}\) are extremal-value of CHP units power production and sold heat which optimize the lower-level objective. The MGO obtains the optimal value of its variables using the optimal value of \(p^{CS}_{lth}\) and \(H^r_{lth}\) computed in the lower-level problems. \(C^{p,s}_{lth}\) is the cost function of \(p^s\) power-only unit at time \(t\) and is defined as follows.

\[
C^{p,s}_{lth} = \psi_t \times p^{p,s}_{lth}
\]
where, $P_{t}^{s}$, $P_{t}^{max}$ and $P_{t}^{min}$ represent the produced power, minimum and maximum capacity of $i^{th}$ power-only unit at time $t$ and scenario $s$, respectively. Also, $\psi_{i}$ is the cost function coefficient of $i^{th}$ power-only unit. The available wind power of a WT can be modeled as follows [32]:

$$P_{t}^{WT,s} = \begin{cases} 0 & V_{t}^{WT,s} < V_{Cl} \lor V_{t}^{WT,s} > V_{CD} \\ P_{max}^{WT} \times \left( \frac{V_{t}^{WT,s} - V_{Cl}}{V_{r} - V_{Cl}} \right) & V_{Cl} \leq V_{t}^{WT,s} \leq V_{r} \\ P_{min}^{WT} & V_{r} \leq V_{t}^{WT,s} \leq V_{CD} \end{cases}$$  

where, $V_{r}, V_{CD}, V_{Cl}$ and $V_{t}^{WT,s}$ are rated, cut-out, cut-in, and instantaneous wind speed respectively. Also, $P_{max}^{WT}$ and $P_{min}^{WT}$ are the available and maximum wind power, respectively. In the WT model, wind power spillage is permitted. The algorithm will decide about employing renewable generation regarding the total profit and the constraints. The following restriction imposes this fact:

$$P_{t}^{WT,s} \leq P_{t}^{s}$$  

### 3.2. Lower level model

The lower-level model is the individual profit-maximization problems of various CHPOs. The CHPOs cost is incurred from operation cost of CHPs and the revenue comes from selling energy to MGO. Hence, the objective function of CHPOs can be formulated as following:

$$\text{Maximize } F^{l} = \sum_{t=1}^{N_{t}} \left\{ \sum_{k=1}^{N_{CHP}} \left( H_{k,t}^{Se} \times \gamma_{k,t}^{H} + P_{k,t}^{c} \times \gamma_{k,t}^{P} - C_{k,t}^{d} \right) \\ - \sum_{m=1}^{N_{B}} C_{m,t}^{d} \right\}$$  

$$= \sum_{d=1}^{D_{k}} \alpha_{d,k,t}^{d}$$  

where, $C_{k,t}^{d}$ and $C_{m,t}^{d}$ are the CHP units and boiler units cost functions, respectively. The CHPOs should satisfy the capacity limits of CHP units. The back-pressure type of CHP turbine usually has a linear relationship between its steam flow used for district heating and the electric power output, while for the extraction turbine, the extracted steam for heating and the electric power output form an operation region, as shown in Fig. 3. [33]. The feasible operating area of a CHP unit is shown in Fig. 3. The boundaries of AD, AB, BC, and CD represent the minimum limit of steam injection, maximum fuel consumption, maximum heat extraction, and minimum fuel consumption, respectively [34]. The power and heat productions of CHPOs are represented using a combination of extreme points in the polyhedral feasible operating region [35]. The relationship between heat production, power production and the coordinates of the corner point $d$ are stated by:

$$P_{k,t}^{d} = \sum_{d=1}^{D_{k}} \alpha_{d,k,t}^{d}$$  

$$H_{k,t}^{d} = \sum_{d=1}^{D_{k}} \alpha_{d,k,t}^{d}$$  

where, $D_{k}$ is the total number of corner points for the $k^{th}$ CHP unit and the coefficient $\alpha_{d,k,t}^{d}$ should satisfy the following constraints:

$$\sum_{d=1}^{D_{k}} \alpha_{d,k,t}^{d} = 1 : \lambda_{k,t}^{1}$$  

$$0 \leq \alpha_{d,k,t}^{d} \leq 1 : \left( \frac{\pi_{1}^{d} - \mu_{1}^{d}}{\pi_{2}^{d} - \mu_{2}^{d}} \right)$$  

where, $(x_{i}, y_{i})$ is the electricity and heat productions of the corner point $d$ (intersection of the boundaries). The linear cost function of CHP units is defined as:

$$C_{k,t}^{d} = \sum_{d=1}^{D_{k}} \alpha_{d,k,t}^{d}$$  

The capacity limit and cost function of boiler unit are expressed as:

$$H_{m,t}^{b} = \sum_{d=1}^{D_{m}} \alpha_{d,m,t}^{d}$$  

$$\sum_{m=1}^{N_{B}} \alpha_{d,m,t}^{d} = \sum_{d=1}^{D_{m}} \alpha_{d,m,t}^{d} = 1 : \lambda_{m,t}^{d}$$  

$$0 \leq \alpha_{d,m,t}^{d} \leq 1 : \left( \frac{\pi_{1}^{d} - \mu_{1}^{d}}{\pi_{2}^{d} - \mu_{2}^{d}} \right)$$  

where, $H_{m,t}^{b}$ represents the produced heat of $m^{th}$ boiler unit at time $t$. Also, $\lambda_{m,t}^{d}$ is the cost function coefficient of boiler unit.

As mentioned, the CHPO is equipped with HBT. The HBT is disposed to the CHP units and the heat-only units of CHPOs. In the employed facility, the heat storage is possible as well. The available heat in the HBT, $B_{t}$, by taking into account the heat loss rate $\eta$, could be computed as:

$$B_{t} = (1 - \eta)B_{t-1} + \sum_{k=1}^{N_{CHP}} \sum_{d=1}^{D_{k}} \alpha_{d,k,t}^{d} + \sum_{m=1}^{N_{B}} H_{m,t}^{b} - H_{m,t}^{b} : \left( \lambda_{m,t}^{d} \right)$$  

In addition, the capacity of HBT is limited as:

$$B_{\text{min}} \leq B_{t} \leq B_{\text{max}} : \left( \frac{\pi_{1}^{1} - \mu_{1}^{1}}{\pi_{2}^{1} - \mu_{2}^{1}} \right)$$  

where, $B_{\text{max}}$ and $B_{\text{min}}$ are maximum and minimum HBT capacities,
respectively. In the paper, the ramping up/down rates have been considered to simulate more practical condition of HBT, as follow:

\[ B_t - B_{t-1} \leq B^\text{charge}_{\max} : (\mu^4_t) \]  \hspace{1cm} (33)

\[ B_t - B_{t-1} \leq B^\text{discharge}_{\max} : (\mu^5_t) \]  \hspace{1cm} (34)

where, \( B^\text{charge}_{\max} \) and \( B^\text{discharge}_{\max} \) are the maximum charge and discharge rates of HBT, respectively.

\[ \lambda^1_{k,t}, \lambda^2_{k,t}, \lambda^3_{k,t}, \lambda^4_{k,t}, \lambda^5_{k,t}, \mu^4_t, \mu^5_t \] represent the dual variables of the problem defined in (26)–(27), (29) and (31)–(34). Specifically, \( \lambda^1_{k,t} \) and \( \lambda^2_{k,t} \) correspond to constraints (26) and (31), respectively. \( \mu^4_t, \mu^5_t \) correspond to constraint (27), \( \pi^2_{m,t}, \pi^3_{m,t} \) correspond to constraint (29), \( \pi^3_t, \mu^3_t \) correspond to (32), \( \mu^2_t \) and \( \mu^2_t \) correspond to (33) and (34), respectively. By substituting (24), (28) and (30) in (23), one can write:

\[
\text{Maximize} \quad F^t = \sum_{i=1}^{N_c} \left( \sum_{k=1}^{N_c} \left( \sum_{i=1}^{N_c} \left( \frac{\partial P^\text{Sale}_{k,i}}{\partial P^\text{Sale}_{k,i}} \times \gamma^R_{k,t} + \sum_{i=1}^{N_c} \frac{\partial P^\text{Sale}_{k,i}}{\partial P^\text{Sale}_{k,i}} \times \gamma^R_{k,t} \right) \right) \right) + \sum_{i=1}^{N_c} \left( \sum_{i=1}^{N_c} \left( \frac{\partial P^\text{Sale}_{k,i}}{\partial P^\text{Sale}_{k,i}} \times \gamma^R_{k,t} \right) \right) \right) 
- \sum_{i=1}^{N_c} \left( \sum_{i=1}^{N_c} \left( \frac{\partial P^\text{Sale}_{k,i}}{\partial P^\text{Sale}_{k,i}} \times \gamma^R_{k,t} \right) \right) \right) 
- \sum_{i=1}^{N_c} \left( \sum_{i=1}^{N_c} \left( \frac{\partial P^\text{Sale}_{k,i}}{\partial P^\text{Sale}_{k,i}} \times \gamma^R_{k,t} \right) \right) \right) 

(35)

3.3. Reformulation of MPEC

3.3.1. Individual single-level optimization model-MPEC

Using mathematical programming with equilibrium constraints (MPEC), the bilevel model of each CHPO can be transformed into a single-level optimization model. Each lower-level problem is substituted by its KKT conditions. Considering the limited space, the generic mathematical formulation of MFBP and equilibrium MPEC is presented in Appendix A.

Maximize \( F^t = \sum_{i=1}^{N_c} \left( \sum_{k=1}^{N_c} \left( \frac{\partial P^\text{Sale}_{k,i}}{\partial P^\text{Sale}_{k,i}} \times \gamma^R_{k,t} \right) \right) + \sum_{i=1}^{N_c} \left( \sum_{i=1}^{N_c} \left( \frac{\partial P^\text{Sale}_{k,i}}{\partial P^\text{Sale}_{k,i}} \times \gamma^R_{k,t} \right) \right) \right) 
- \sum_{i=1}^{N_c} \left( \sum_{i=1}^{N_c} \left( \frac{\partial P^\text{Sale}_{k,i}}{\partial P^\text{Sale}_{k,i}} \times \gamma^R_{k,t} \right) \right) \right) 
- \sum_{i=1}^{N_c} \left( \sum_{i=1}^{N_c} \left( \frac{\partial P^\text{Sale}_{k,i}}{\partial P^\text{Sale}_{k,i}} \times \gamma^R_{k,t} \right) \right) \right) 

\text{s.t.}

\text{Constraints(1)–(8), (9)–(14) and (19)–(22)}

\[ \nabla_{\lambda^1_{k,t}} F^t + \mu^1_{k,t} = 0 \]  \hspace{1cm} (36)

(37)

4. Simulation studies

In this section, firstly, the structure of studied MG is introduced and afterwards the simulation outcomes of optimal energy management are presented.

4.1. Data

As shown in Fig. 4, a six bus meshed MG has been implemented as the test bed here. In the studied case, bus 1 is connected to the main grid and the MGO is able to procure the power from the grid according to the RT and DA market prices. The hourly data of price is used to model the framework [38]. In addition, facilities of CHPOs are located at buses 2 and 5. The MGO has bilateral contracts to buy...
power and heat from CHPOs according to Table 1 prices, which are adopted from Ref. [39]. The MGO owns a conventional power-only unit and a WT along with the electrical and thermal loads. The location of all units is illustrated in Fig. 4. The fundamental network data including impedance of branches are presented in Table 2 [40]. The heat demand of MG is developed from Refs. [13,40] and provided in Fig. 5. The cogeneration units data are adopted from Ref. [34], where data are scaled down by 100 (see Fig. 6). Input data is provided in Table 3. According to Table 3 the value of \( r_i \) is set to 1, which means that neither increment nor decrement in the total load will occur. The cost functions of power-only and heat-only units are supposed to be linear and stated in Eqs. (54) and (55), respectively. The maximum exchangeable power with the RT and DA markets is assumed to be 250 kW. The WT has a rated power of 100 kW, and the cut-in, cut-out, and rated wind speeds parameters are 3.5, 25, and 11.9 m/s, respectively [41]. In addition, it is assumed that the MGO sells electricity and heat to the consumers at a fixed retail price of 0.055 $/kWh [8]. Mathematical modeling of the PF constrained CHP-based MG scheduling problem is solved by using Simple Branch-and-Bound (SBB) solver [42] under General Algebraic Modeling System (GAMS) environment [31].

\[
C_{p,t}\left(p^p\right) = 0.5 \times p_{p,t}^{p,s}
\]  

(54)

\[
C_{h,t}\left(H^h\right) = 0.234 \times H_{m,t}^h
\]  

(55)

Table 1
Offers of the CHPOs.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>CHPO 1 ($/kWh)</th>
<th>CHPO 2 ($/kWh)</th>
<th>Heat price of CHPOs ($/kWth)</th>
<th>Time (h)</th>
<th>CHPO 1 ($/kWh)</th>
<th>CHPO 2 ($/kWh)</th>
<th>Heat price of CHPOs ($/kWth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0415</td>
<td>0.0427</td>
<td>0.0152</td>
<td>13</td>
<td>0.0425</td>
<td>0.0467</td>
<td>0.0192</td>
</tr>
<tr>
<td>2</td>
<td>0.0415</td>
<td>0.044</td>
<td>0.0182</td>
<td>14</td>
<td>0.0425</td>
<td>0.0465</td>
<td>0.0198</td>
</tr>
<tr>
<td>3</td>
<td>0.0417</td>
<td>0.044</td>
<td>0.0188</td>
<td>15</td>
<td>0.0425</td>
<td>0.0467</td>
<td>0.0190</td>
</tr>
<tr>
<td>4</td>
<td>0.0419</td>
<td>0.0442</td>
<td>0.0192</td>
<td>16</td>
<td>0.0427</td>
<td>0.0467</td>
<td>0.0186</td>
</tr>
<tr>
<td>5</td>
<td>0.0417</td>
<td>0.0445</td>
<td>0.0194</td>
<td>17</td>
<td>0.0432</td>
<td>0.0472</td>
<td>0.0182</td>
</tr>
<tr>
<td>6</td>
<td>0.042</td>
<td>0.0447</td>
<td>0.0202</td>
<td>18</td>
<td>0.0432</td>
<td>0.0475</td>
<td>0.0192</td>
</tr>
<tr>
<td>7</td>
<td>0.042</td>
<td>0.045</td>
<td>0.0178</td>
<td>19</td>
<td>0.0435</td>
<td>0.0477</td>
<td>0.0182</td>
</tr>
<tr>
<td>8</td>
<td>0.0422</td>
<td>0.0452</td>
<td>0.0182</td>
<td>20</td>
<td>0.0432</td>
<td>0.0472</td>
<td>0.0176</td>
</tr>
<tr>
<td>9</td>
<td>0.0423</td>
<td>0.0457</td>
<td>0.0192</td>
<td>21</td>
<td>0.0427</td>
<td>0.0465</td>
<td>0.0156</td>
</tr>
<tr>
<td>10</td>
<td>0.0427</td>
<td>0.0465</td>
<td>0.0202</td>
<td>22</td>
<td>0.0425</td>
<td>0.0462</td>
<td>0.0152</td>
</tr>
<tr>
<td>11</td>
<td>0.043</td>
<td>0.0467</td>
<td>0.0243</td>
<td>23</td>
<td>0.0422</td>
<td>0.0457</td>
<td>0.0142</td>
</tr>
<tr>
<td>12</td>
<td>0.0427</td>
<td>0.047</td>
<td>0.0202</td>
<td>24</td>
<td>0.0417</td>
<td>0.0455</td>
<td>0.0138</td>
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</table>

Table 2
Line data (in per unit).

<table>
<thead>
<tr>
<th>Line no.</th>
<th>Start bus</th>
<th>End bus</th>
<th>R</th>
<th>X</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0.0342</td>
<td>0.18</td>
<td>0.0106</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>0.114</td>
<td>0.6</td>
<td>0.0352</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0.0912</td>
<td>0.48</td>
<td>0.0252</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>0.0228</td>
<td>0.12</td>
<td>0.0071</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>5</td>
<td>0.0228</td>
<td>0.12</td>
<td>0.0071</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>3</td>
<td>0.0342</td>
<td>0.18</td>
<td>0.0106</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>4</td>
<td>0.114</td>
<td>0.60</td>
<td>0.0352</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>5</td>
<td>0.0228</td>
<td>0.12</td>
<td>0.0071</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>6</td>
<td>0.0228</td>
<td>0.12</td>
<td>0.0071</td>
</tr>
</tbody>
</table>

Fig. 4. Single line diagram of six-bus meshed MG.
To further scrutinize the well performance of the proposed model, the paper discusses three case studies: 1) stochastic MFBP; 2) deterministic MFBP; and 3) stochastic centralized formulation. In the centralized scheme, the fitness functions of MGO and each CHPO are the same as (18) and (23), respectively. To formulate a new fitness function, the objectives of all entities are equally augmented. The constraints are (1)–(17), (19)–(22) and (24)–(34).

4.2. Results and discussion

One thousand scenarios are generated implementing the presented methods. As discussed in the previous section, fast backward scenario reduction method is applied to reduce the computation exertions while maintaining the solution precision. The 1000 produced scenarios are reduced to 50 scenarios.

Table 4 provides the profits of all entities based on the simulation results. Referring to Table 4, the profits of the studied cases are different from each other. In addition, the centralized scheduling has a higher total profit than the MFBPs. This is due to the objective of centralized scheduling which is maximizing the profits of all agents, while the profits of some agents may be forwent to accomplish the equilibrium in game-theoretic formulations. In the MFBPs, the CHPOs and MGO attempt to optimize their own profits. Thus, comparing the stochastic centralized formulation and stochastic MFBP, it can be perceived that the profit of MGO is reduced in the MFBP from $238.381 to $229.205, while the profit of CHPO 2 is improved about 8.74%. This observation is due to maximizing the profit of each entity which will result in an equilibrium point where no entity can further enhance its benefit by altering its own operation point. In addition, it can be seen from Table 4 that the profits of deterministic MFBP programing are lower than those of the stochastic model as in the deterministic method the forecasted

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Input data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1%</td>
</tr>
<tr>
<td>$b_{\text{charge}}$</td>
<td>70 kWh</td>
</tr>
<tr>
<td>$b_{\text{discharge}}$</td>
<td>70 kWh</td>
</tr>
<tr>
<td>$b_{\text{min}}$</td>
<td>10 kWh</td>
</tr>
<tr>
<td>$p_{\text{max}}$</td>
<td>200 kWh</td>
</tr>
<tr>
<td>$V_{\text{max}}$</td>
<td>1.1 p.u</td>
</tr>
<tr>
<td>$V_{\text{min}}$</td>
<td>0.9 p.u</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Total profit of each entity.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scheduling Entity</td>
<td>MGO</td>
</tr>
<tr>
<td>Centralized formulation</td>
<td>$238.381$</td>
</tr>
<tr>
<td>Stochastic formulation</td>
<td>$229.205$</td>
</tr>
<tr>
<td>Deterministic formulation</td>
<td>$221.564$</td>
</tr>
</tbody>
</table>

Fig. 5. Thermal load of network.

Fig. 6. Power-heat feasible region for CHP units a) unit 1 b) unit 2.
mean values are implemented. Obviously, applying deterministic decisions in practical systems may result in worse performance due to the forecast errors. The profits of CHPO 2 are the same in both deterministic and stochastic MFBP models. This fact is due to existence of only CHP unit as heat supplier at bus 5 which will produce power regarding its heat production enhancing its objective function.

Fig. 7 presents the voltage magnitude of all buses for deterministic MFBP. According to this figure, the voltage magnitude of all buses is limited between 0.9 and 1.1 p.u. Fig. 8 illustrates the
expected value of daily load curve of bus 2 after and before applying DRP for stochastic MFBP. According to Fig. 8, bus 2 load will be shifted from hours that both RT and DA market prices are high (from 8:00 to 13:00 and 15:00 to 17:00) to low market price intervals (to 1:00 to 6:00 and 22:00 to 24:00). The heat productions of CHPO 1’s facilities are depicted in Fig. 9. CHP unit will supply heat productions at heat demand hours that both RT and DA market prices are high (from 1:00 to 6:00 and 22:00 to 24:00). The heat productions are shifted from hours that both RT and DA market prices are high to hours that both RT and DA market prices are low. The MGO will take advantage of market participation and selling power to another entity.

5. Conclusions

A multi follower bilevel programming approach for optimal energy management of a CHP-based MG is presented. The framework considers different decision-making entities, namely, the MGO and different CHPOs. The MGO profit could be achieved from DA and RT electricity markets revenues over 24-h time horizon by incorporation of DRP. In the proposed DRP, the MGO could receive (pay) for increased (decreased) consumption of consumers, which is proportional to the market prices. The benefits of all agents are expressed by different objective functions and related constraints. Therefore, the solution of the MFBP benefits all entities. Moreover, the paper envisages the stochastic nature of an MG along with the PF constraints of the network to model more realistic state of the system. The simulation results confirm the superiority of proposed game-theoretic model over centralized scheduling model.

Appendix A

In general, the multi follower bilevel programming problem can be considered as:

\[
\begin{align*}
\text{minimize} & \quad F^U(x, y_1^*, y_2^*, \ldots, y_n^*) \\
\text{s.t.} & \quad H^U(x, y_1^*, y_2^*, \ldots, y_n^*) = 0 \\
& \quad G^U(x, y_1^*, y_2^*, \ldots, y_n^*) \leq 0
\end{align*}
\]

\[
\{y_1^*, y_2^*, \ldots, y_n^*\} \in \text{arg}
\]

where superscripts ‘U’ and ‘L’ denote upper-level and lower-level, respectively. Multi follower problem is considered in MFBP with respect to BP. The above problem consists of an upper-level optimization problem, associated with multi lower-level optimization problems. The lower-level problems consider x as a parameter and obtain the optimal value of \(y_i = y_i^*\) that depends on parameter x.

The upper-level problem obtains the optimal value of x using the optimal value of \(y_i\) computed in the lower-level problems. It is not possible to solve the bilevel problem in this implicit form. The most common algorithmic approach to attack bilevel problems is based on solving the nonlinear problem obtained by replacing each lower-level problem with its Karush–Kuhn–Tucker conditions, (see Refs. [43,44]). The equilibrium MPEC using KKT condition can be achieved by:

\[
\begin{align*}
\text{minimize} & \quad F^U(x, y_1^*, y_2^*, \ldots, y_n^*) \\
\text{s.t.} & \quad H^U(x, y_1^*, y_2^*, \ldots, y_n^*) = 0 \\
& \quad G^U(x, y_1^*, y_2^*, \ldots, y_n^*) \leq 0
\end{align*}
\]

where, \(F^U\) is the objective function of total profit maximization of CHPO k, \(h_i\) and \(g_i\) are the sets of operational constraints of different CHPOs. \(F^U\) is the MGO objective function, \(H\) and \(G\) are the sets of operational constraints of MGOs.

References

[14] Nazari M, Ardelleh M. Profit-based unit commitment of integrated cHP-thermal heat only units in energy and spinning reserve markets with considerations for environmental CO2 emission cost and valve-point effects. Energy 2017;133(1):21–33.