The consequences of the minimum wage when other wages are bargained over

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Abstract

This paper examines the impact of a binding minimum wage in a situation where unions dominated by skilled workers set wages. It is shown that the relationship between the minimum wage, the bargained wage and employment of skilled and unskilled workers depends on the magnitude of the elasticity of substitution between skilled and unskilled labor. Cases where minimum wage hikes increase both overall and unskilled employment are exhibited. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The simple theory of supply and demand suggests that increases in the minimum wage should decrease employment. Although this simple idea is widely accepted among economists, it has not gone unchallenged, especially in recent years. Card and Kruger (1995) claim, in a book that contains a huge amount of information, that the data give no support to the view that minimum wage increases have adverse effects on employment. Dolado et al. (1996) find

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very mixed evidence about the effects of minimum wages on European employment. Specifically, the sign of the effect varies across categories of workers: While negative for some of them (e.g. the youth), it is positive for others, and turns out to be positive for total employment.\footnote{To be fair, this claim is not new. Kennan (1995), in his review of Card and Krueger’s book, recalls that Obenauer and Nienburg (1915) have already found that increases in the minimum wage for women in Oregon in 1913 and 1914 reduced adult female employment, but significantly increased teenage female employment. They concluded that a higher minimum wage had elusive effects on total employment.}

Besides, a number of theoretical contributions have questioned the idea of an inverse relationship between employment and minimum wages. Most of them are built on the monopsony model (Stigler, 1946). However, in its simplest form, this model is open to the criticism that its prediction of a positive employment effect of raising the minimum wage holds only in the absence of unemployment, for it rests on a labor supply-determined level of employment. Replies to that criticism came with the introduction of efficiency wages (Rebitzer and Taylor, 1995), dual labor markets (Jones, 1987), or search unemployment or/and on-the-job search (Burdett and Mortensen, 1989; Chalkey, 1991; Manning, 1995) in the simple monopsony model. All these extensions feature monopsony power together with unemployment (see Boal and Ransom, 1997, for a survey).

Surprisingly, as far as we know, the issue of the effect of minimum wage on employment has not been examined in a context where wages are negotiated by collective institutions, such as trade unions. Yet, collective wage bargaining is a very important feature of labor markets, especially in Europe, where nearly 80\% of the labor force is covered by collective agreements. Accordingly, this feature is obviously worth taking into account if we want to understand how the structure of wages and employment is influenced by minimum wage legislation.

Our paper takes a first step in this direction. We consider a model with one firm and two types of workers: Unskilled workers, who are paid the minimum wage, and skilled workers, who get a wage determined through bilateral bargaining between the firm and a trade union. It is assumed that the union represents the interest of the skilled workers only. This is a simple way to capture the fact that some categories of low-skilled workers do not have the possibility to raise their wages through bargaining, their ‘bargaining power’ being simply too low. It is also worth noting that this assumption illustrates some important features of labor markets. For instance, in France, there is evidence that highly skilled and experienced workers are over-represented in the ‘comités d’entreprises’ (Folques, 1996). Moreover, the minimum wage is binding in more than 50\% of the industries that are covered by collective agreements (Barrat and Folques, 1995). This suggests that industry-level wage bargaining by trade unions improves the situation of the sole skilled workers. Brosnan and Bignell (1994) also emphasize that the question of skills is quite crucial to the
development of firm-level bargaining in the Australian industry. While unemployment remains high, they give examples of occupations (such as metal and electrical trades) suffering from skill shortages that provided the skilled workers with additional rent. According to these authors (p. 140),

It seems, then, that wage restraint has been exercised under the Accord by allowing skilled workers to win pay increases, but prohibiting flow-on to the other workers less capable of winning pay rises.

The model is presented in Section 2. Theoretical results are derived in Section 3. It is shown that minimum wage increases have ambiguous effects on the employment of skilled and unskilled workers, the degree of substitutability between these two types of workers playing a crucial role. Section 4 is devoted to some simulation exercises assessing which one of the effects of minimum wage increases are likely to be dominant under standard specifications for the technology and for the trade union objectives. It is shown that minimum wage increases can raise both overall and unskilled employment when the minimum wage is low, if skilled and unskilled workers are highly substitutable.

2. The model

We consider a firm producing a single good with two types of labor: skilled and unskilled, denoted by $\ell_q$ and $\ell_n$, respectively. We assume that wages are bargained over by a trade union that represents the interest of the skilled workers only. The collective bargain is represented by the ‘right-to-manage’ model (Nickell and Andrews, 1983). The sequence of decisions is the following:

1. The firm and the trade union bargain over the skilled and unskilled wages, denoted by $w_q$ and $w_n$, respectively.
2. The employer maximizes his profit and hires skilled and unskilled workers. Production occurs and workers get paid.

The firm’s contribution to the bargain depends on the optimal labor demand given wages. Our first concern is therefore the employer’s employment decisions.

2.1. The firm

Following the ‘right-to-manage’ hypothesis, the employer has the unilateral power to set skilled and unskilled employment after the wage-bargain. The firm is endowed with a technology represented by a production function $F(\ell_q, \ell_n)$ which is twice continuously differentiable, increasing, concave and homogeneous of degree $\alpha \in ]0,1[$ with respect to $\ell_q$ and $\ell_n$. The other inputs, such as
capital, are kept implicit. One can thus define the cost of labor as

$$C(w_q, w_n, Y) = \operatorname{Min} w_q \ell_q + w_n \ell_n \quad \text{s.t.} \quad F(\ell_q, \ell_n) \geq Y,$$

given output $Y$. The homogeneity of the production function allows us to write

$$C(w_q, w_n, Y) = W(w_q, w_n) L \quad \text{with} \quad L \equiv Y^{1/\alpha}.$$

The cost function can be seen as the product of an indicator $\ell$ of aggregate labor times a wage index $W(w_q, w_n)$. General properties of cost functions imply that this wage index is increasing, concave and homogeneous of degree one with respect to $(w_q, w_n)$. Furthermore, Shephard’s lemma states that

$$\frac{\partial C}{\partial w_i} = L \frac{\partial W}{\partial w_i} = \ell_i \quad \text{for} \quad i = q, n. \quad (1)$$

Let $\eta$ be the partial skilled wage elasticity of the wage index. Using relation (1), one gets

$$\eta \equiv \frac{\partial W}{\partial w_q} \frac{w_q}{W} = \frac{w_q \ell_q}{WL} \quad \text{and thus} \quad 1 - \eta = \frac{\partial W}{\partial w_n} \frac{w_n}{W} = \frac{w_n \ell_n}{WL}. \quad (2)$$

As the function $W$ is homogeneous of degree one with respect to $(w_q, w_n)$, the elasticity $\eta$ is homogeneous of degree zero with respect to $(w_q, w_n)$ and therefore only depends on the ratio $s \equiv w_n/w_q$ so that it can be denoted by $\eta(s)$. Finally, restraining ourselves to the cases where both types of labor are employed (that is $\ell_q > 0$ and $\ell_n > 0$), the last two expressions imply

$$\eta(s) \in ]0, 1[ \quad \text{and} \quad \frac{\ell_q}{\ell_n} = s \cdot \frac{\eta(s)}{1 - \eta(s)}. \quad (3)$$

The firm is assumed to act as a monopolist and to face a demand with a constant price elasticity, the absolute value of which is denoted by $e > 1$. The profit function can thus be written as $\Pi = PY - WL$, with $P = AY^{-1/e}$ and $A > 0$ is a shift demand parameter. Profit maximization with respect to the labor aggregate implies

$$L = \left(\frac{vW}{A}\right)^{-v/(v - 1)}, \quad (4)$$

where $v = e/(e - 1) > 1$ is the mark-up of the price over marginal costs. It is possible to derive the partial skilled and unskilled wage elasticities of the demand for skilled, denoted by $\varepsilon_q \equiv (\partial \ell_q/\partial w_q)(w_q/\ell_q)$ and $\varepsilon_n \equiv (\partial \ell_q/\partial w_n)(w_n/\ell_q)$, respectively. Using relations (2) and (4), one gets the skilled employment level $\ell_q = \frac{\text{cst} \cdot \eta W^{-1/(v - 1)}/w_q}$. Because both elasticities $\varepsilon_i$ depend on the derivative of the function $\eta$, the next thing we need is its
expression.

Let \( \sigma \equiv (d\ell/ds)\cdot (s/\ell) \) be the elasticity of substitution between skilled and unskilled workers, where \( \ell = \ell_q/\ell_n \). Using Eq. (3) one gets

\[
\eta'(s) = (\sigma - 1) \cdot \frac{\eta(s)(1 - \eta(s))}{s}
\]  

and this last relation, together with the skilled labor demand, implies

\[
\varepsilon_q(s) = -\sigma - \beta \eta(s) \quad \text{and} \quad \varepsilon_n(s) = -\beta(1 - \eta(s))
\]

with \( \beta = \frac{v}{v - \alpha} - \sigma \). (6)

Finally, the optimal profit, given wages, is

\[
\Pi^* = \left(1 - \frac{\alpha}{\nu}\right) A^{\nu/(\nu - \alpha)} \left(\frac{\nu W}{\alpha}\right)^{-\alpha/(\nu - \alpha)}
\]

2.2. The bargain

Skilled workers are represented by a union. The bargain is modelled by the generalized Nash solution. We assume that it takes place over skilled and unskilled wages and that the union may care about skilled employment, so that we may define its contribution to the Nash program as: \( \epsilon^*_q(v(w_q) - v(\bar{w})) \), where \( \chi \in [0, 1] \) measures the weight of employment in the trade union’s objective and \( v(w_q) \) the utility reached by a skilled worker when there is no strike. The function \( v \) is supposed to be increasing, concave and homogeneous.

In case of a strike, we assume that the production process stops and that no one gets paid. The skilled workers’ threat point \( v(\bar{w}) \) consists of earnings from temporary jobs that strikers may pick up in another firm during the strike. The firm’s contribution to the Nash program is just equal to its optimal profit, given wages. Using the Nash maximand, the bargained skilled and unskilled wages are therefore maximizing

\[
\left[\epsilon^*_q(v(w_q) - v(\bar{w}))\right]^{1 - \gamma} \cdot [\Pi^*]^{1 - \gamma} \quad \text{s.t.} \ w_n \geq \bar{w}_n,
\]

where \( \gamma \) measures the union’s bargaining power (0 < \( \gamma < 1 \)). Using Eqs. (6), the first-order conditions of that program can be written as

(i) \( 0 = -\chi[\sigma + \beta \eta(s)] + (v'(w_q)/v(w_q) - v(\bar{w})) - \mu \eta(s) \),

(ii) \( 0 = -[\chi \beta + \mu] + \lambda w_n \),

where \( \mu \equiv (1/\gamma - 1)/(\nu/\alpha - 1) > 0 \) and \( \lambda \geq 0 \) is the Kuhn and Tucker multiplier associated with the constraint \( w_n \geq \bar{w}_n \). From the Kuhn and Tucker theorem, the last equation together with Eq. (3) implies the following result.
Proposition 1. The skilled workers and the firm negotiate a wage for the unskilled workers equal to the minimum wage if and only if
\[
\chi \beta + \mu > 0.
\]

Proposition 1 deserves some comments. It is possible to distinguish two effects of an increase in the unskilled workers’ wage on the objectives of the firm and the trade union. On the one hand, a rise in the wage of the unskilled workers reduces profits. This induces the firm and the skilled workers to agree upon as low a wage as possible for the unskilled workers. On the other hand, the wage of the unskilled workers influences skilled employment, through two channels. First, a substitution effect that implies a positive relation between the unskilled wage and skilled employment, because unskilled workers are substituted to skilled workers when the unskilled wage is increased. This effect can induce skilled workers to increase the unskilled wage in order to protect their own jobs. But there is also a supply effect, that corresponds to a negative influence of the unskilled wage on skilled employment, because a rise in the unskilled wage increases the overall labor cost, which in turn raises the price set by the firm, and possibly decreases the amount of output produced.

This description of the consequences of variations in the unskilled wage suggests that the minimum wage should always be binding if the trade union does not care about employment. It can easily be checked that this is the case if \( \chi = 0 \). If the trade union cares about employment, it can theoretically be the case that the substitution effect is so large that skilled workers are encouraged to negotiate a high unskilled wage in order to protect skilled employment (this happens when \( \beta < 0 \)). However, for reasonable values of the elasticity of substitution \( \sigma \) between skilled and unskilled labor, the supply effect should overcome the substitution effect (the value of which is given by \( \psi(v - z) \)) if the firm’s market power on its product market is not too strong. For instance, assuming perfect competition (i.e. \( v = 1 \)), the standard value of \( z \) being about \( \frac{2}{3} \), one has \( \beta \geq 0 \) for \( \sigma \leq 3 \). Conversely, assuming a maximum monopoly power (\( v \to \infty \)) yields a floor value for the supply effect which implies a positive impact of the unskilled wage on skilled employment if the elasticity of substitution between the two types of workers is larger than one. Moreover, it should be noted that if the firm has a sufficient monopoly power (i.e. if \( v \) is large) to imply that skilled employment increases with the unskilled wage, the minimum wage will not be binding only if the trade union pays enough attention to employment and has enough bargaining power. This may arise in some very extreme situations that we assume away.

Accordingly, we shall henceforth focus on a situation where the minimum wage is binding. As a conclusion, the bargained skilled wage must satisfy
\[
\Psi(w_q, \bar{w}_n, \mu, \chi, \bar{w}) = V(w_q) - \mu \eta(s) + \chi \nu_q(s) = 0, \tag{7}
\]
with

\[ V(w_q) = \frac{w_q v'(w_q)}{v(w_q) - v(\bar{w})} \quad \text{and} \quad s = \frac{\bar{w}_n}{w_q} \]

and the second-order condition, \( \partial \Psi / \partial w_q < 0 \). These conditions show that when the bargain succeeds, \( V(w_q) > 0 \) and union members receive more than they could get elsewhere. Moreover, assuming that the second-order condition is fulfilled, Eq. (7) implies that

\[ \hat{\partial} w_q / \hat{\partial} x = - (\hat{\partial} \Psi / \hat{\partial} x)/(\hat{\partial} \Psi / \hat{\partial} w_q), \]

\( x = \mu, \chi, \bar{v}, \bar{w}_n \), has the same sign as \( \hat{\partial} \Psi / \hat{\partial} x \). Therefore, the surplus obtained by the employees increases with respect to the union power \( \gamma \) but decreases with respect to the weight of employment in the trade-union objective \( \chi \). The same reasoning shows that any increase in the absolute value of the skilled wage elasticities of profits or skilled employment (which equal \( \eta(s)/(v - z) \) and \( - \varepsilon_q(s) \), respectively) should lead to a fall in the skilled wage.

3. Employment effects of the minimum wage

In this section, we analyze the effect of a minimum wage increase on skilled and unskilled employment. It will be shown that those effects depend crucially on the degree of substitutability between skilled and unskilled labor. Moreover, the workings of the model are best described in two special cases that correspond, respectively, to the following two assumptions:

**H1**: The total cost function is CES, i.e. the elasticity of substitution between skilled and unskilled labor is a constant denoted by \( \sigma \geq 0 \).

**H2**: The union’s members are insiders and employment is not part of the union’s objective during the bargain, i.e. \( \chi = 0 \).

In the present section all properties and comments are conditional either on H1 or H2, but we shall show in the last section in a numerical example that none is essential to the results.

3.1. Skilled wage and employment

Minimum wage hikes impinge on the bargain outcome because the skilled wage elasticities of profits and of skilled employment depend on the minimum wage. The outcome of the wage bargain shows that any increase in the absolute value of these elasticities entails a decrease in the wage negotiated by the trade-union. We thus have to understand how these elasticities vary with respect to the minimum wage to characterize the skilled wage and employment changes induced by minimum wage hikes. It appears that the degree of substitutability between skilled and unskilled workers plays a crucial role.
Indeed, it can be seen that the firm’s profit gets more sensitive to the skilled wage when the minimum wage increases if the elasticity of substitution between skilled and unskilled workers is larger than one, a situation that will be referred to as the case of substitutability. That property stems from the behavior of the partial skilled wage elasticity of the wage index, equal to the share of skilled labor in the total labor cost (see Eq. (2)), which is increasing in the minimum wage when substitutability prevails (see Eq. (5)). Therefore, in that case, the mark-up obtained by the union drops when the minimum wage is raised. In the case of complementarity, i.e. that of an elasticity of substitution smaller than one, the partial skilled wage elasticity of the wage index is decreasing in the minimum wage and thus the mark-up obtained by the union rises. Under H2, that is if the union does not care about employment, the sole effect of the partial skilled wage elasticity of the wage index explains the bargained wage variations.

Under H1, the effect of the partial skilled wage elasticity of the skilled labor demand adds itself to the previous effect. In fact, when the minimum wage increases, the sensitivity of the skilled labor demand with respect to the skilled wage can vary in the opposite way to the sensitivity of the wage index when the elasticity of substitution between skilled and unskilled workers is not a constant, which makes the final result ambiguous. That is why we have chosen to illustrate our results under either H1 or H2.

Proposition 2. Under H1 or H2, the bargained wage decreases with respect to the minimum wage if and only if skilled and unskilled workers are substitutes. The relative wage of unskilled workers always increases with respect to the minimum wage.

Proof.

- Keeping implicit the last three arguments μ, χ and \( \bar{w} \) of the function \( \Psi(\cdot) \) defined in (7), the first-order condition of the bargaining program is written as
  \[
  \Psi(w_q, \bar{w}_n) \equiv V(w_q) - \mu \eta(s) + \chi \varepsilon_q(s) \equiv \Phi(w_q, s) = 0.
  \]

  The implicit functions theorem implies \( \frac{dw_q}{d\bar{w}_n} = -\frac{\partial \Psi/\partial \bar{w}_n}{\partial \Psi/\partial w_q} \) and \( \frac{dw_q}{ds} = -\frac{\partial \Phi/\partial s}{\partial \Phi/\partial w_q} \). The second-order condition implies that \( \frac{\partial^2 \Psi}{\partial w_q^2} < 0 \) and one also has \( \frac{\partial \Phi/\partial w_q}{\partial \Phi/\partial \bar{w}_n} = -\frac{\nu(\bar{w})}{\nu(w_q)} < 0 \), owing to the homogeneity of the utility function \( v(\cdot) \). Therefore, since \( \frac{\partial \Psi}{\partial \bar{w}_n} = w_q^{-1}\partial \Phi/\partial s \), \( \partial \Psi/\partial \bar{w}_n \) has the same sign as \( \partial \Phi/\partial s \). One gets
  \[
  \text{sign}(\frac{dw_q}{d\bar{w}_n}) = \text{sign}(\frac{dw_q}{ds}) = \text{sign}(\frac{\partial \Phi}{\partial s}).
  \]

  Since \( \frac{dw_q}{d\bar{w}_n} \) has the same sign as \( \frac{dw_q}{ds} \), one necessarily gets \( \frac{ds}{d\bar{w}_n} > 0 \).

- Using Eqs. (5) and (6), we have
  \[
  \eta'(s) > 0 \iff \sigma > 1 \quad \text{and} \quad \varepsilon_q'(s) = -\beta \eta'(s) - (1 - \eta(s))\varepsilon \sigma/\partial s.
  \]
Under H1 or H2, the last two relations imply

\[ \hat{\sigma} \frac{\partial \sigma}{\partial s} = - [\mu + \chi \beta] \eta'(s) - \chi (1 - \eta(s)) \hat{\sigma} \frac{\partial \sigma}{\partial s} < 0 \iff \sigma > 1. \]

One thus has, since the minimum wage is assumed to be binding,

\[ \frac{d w_q}{d \bar{\omega}_n} < 0 \iff \sigma > 1. \]

Proposition 2 shows that the impact of the minimum wage on the wage structure can be fully characterized under fairly general assumptions.

First, a rise in the minimum wage should always narrow wage dispersion, even if the wage bargained by the trade union is increased.

Second, if the trade union does not care about employment, any minimum wage hike that increases the absolute value of the skilled wage elasticity of profits reduces the negotiated wage. According to Eq. (5), one can then deduce that in the case of substitutability (resp. complementarity) between skilled and unskilled labor, a minimum wage increase will always raise (resp. lower) the share of skilled labor, therefore increasing (resp. decreasing) the elasticity of profit and reducing (resp. raising) the wage of skilled workers.

Third, if the trade union is concerned by employment, the bargained wage is influenced by the wage elasticities of both profit and skilled employment. The relation between the minimum wage and the skilled wage elasticity of skilled employment has an ambiguous slope (see Eq. (6)). But assuming a constant elasticity of substitution between skilled and unskilled labor, a minimum wage increase either raises or lowers the skilled wage elasticities (in absolute values) of both skilled employment and profits. Accordingly, in that case, a minimum wage rise reduces the skilled wage if and only if skilled and unskilled labor are substitutes.

Proposition 3. Under H1 or H2, the skilled employment level decreases with respect to the minimum wage if skilled and unskilled workers are complements. When they are substitutes, the effect of a minimum wage increase is ambiguous and can be positive if the skilled wage falls sufficiently.

Proof. Using the expression of \( \varepsilon_q(s) \) given by Eqs. (6), one has

\[ \frac{d \varepsilon_q}{d \bar{\omega}_n} = - \ell_q \left[ \frac{\sigma + \beta \eta(s)}{w_q} \frac{d w_q}{d \bar{\omega}_n} - \frac{\beta (1 - \eta(s)) \bar{w}_n}{w_n} \right]. \]

According to Proposition 2, \( \sigma < 1 \Rightarrow \frac{d \varepsilon_q}{d \bar{\omega}_n} < 0 \), since \( \frac{d w_q}{d \bar{\omega}_n} > 0 \) for a large enough decrease in the skilled wage (\( \frac{d w_q}{d \bar{\omega}_n} \) is negative in this case).

The response of the skilled labor demand to a minimum wage increase results from the combination of two effects. The first one is the response to the skilled
wage variation. In case of complementarity, the bargained wage rises, which depresses the demand for skilled labor. Since the bargained wage decreases when skilled and unskilled workers are substitutable, the skilled employment level tends to increase. The second effect is the response to the unskilled wage variation, which itself results from two counteracting effects. First, a positive substitution effect: given the skilled wage, when the unskilled wage increases, employers find it worthwhile to substitute skilled workers for unskilled workers. On the other hand, a minimum wage increase tends to raise the wage index and therefore has an adverse effect on the skilled employment level because it reduces aggregate employment.

When skilled and unskilled labor are complements, the skilled labor demand always responds negatively to changes in the minimum wage. When the two types of labor are substitutes, the skilled employment level can increase if the skilled wage decreases by a sufficient amount.

3.2. Unskilled employment

Proposition 4. Under H1 or H2, the unskilled employment level decreases with the minimum wage if skilled and unskilled workers are complements. In case of substitutability, the effect of a minimum wage increase is ambiguous and can be positive if the skilled wage falls sufficiently.

Proof. Eqs. (2), (4) and (5) define $\ell_n$ as a function of $\tilde{w}_n$ and $w_q$, which we denote by $\ell_n(\tilde{w}_n, w_q)$. From these equations, one gets

$$\frac{\partial \ell_n}{\partial \tilde{w}_n} = - \frac{\ell_n}{\tilde{w}_n} \left[ \sigma + \beta(1 - \eta(s)) \right] \quad \text{and} \quad \frac{\partial \ell_n}{\partial w_q} = - \frac{\ell_n}{w_q} \beta \eta(s)$$

and therefore,

$$\frac{d\ell_n}{d\tilde{w}_n} = - \ell_n \left[ \frac{\beta \eta(s)}{w_q} \frac{d w_q}{d \tilde{w}_n} + \frac{\sigma + \beta(1 - \eta(s))}{\tilde{w}_n} \right].$$

According to Proposition 2, $\sigma < 1 \Rightarrow d\ell_n/d\tilde{w}_n < 0$, since $dw_q/d\tilde{w}_n$ and $\beta$ are positive in case of complementarity. For $\sigma > 1$, when the supply effect dominates the substitution effect (that is if $\beta > 0$), one can have $d\ell_n/d\tilde{w}_n > 0$ for a large enough decrease in the skilled wage ($dw_q/d\tilde{w}_n$ is negative in this case).□

Let us describe the mechanism that can lead to a rise in unskilled employment when the minimum wage is increased. In the case of substitutability, a minimum wage increase diminishes the skilled wage, giving rise to a drop in the total labor cost if the change in the skilled wage is sufficient. This drop in total labor cost may in turn cause unskilled employment to rise in spite of the minimum wage
increase. Hence, it can be the case that a minimum wage increase raises both unskilled and total employment.

The purpose of the next section is to show that minimum wage hikes may actually rise unskilled employment for sensible parameter values and standard specifications for technology and preferences.

4. Numerical example

As we have previously claimed, the assumption that employment is not a part of the trade-union objectives (that is H2) was used for expositional convenience but is in no way essential. To make that point more clear, the assumption is relaxed in the numerical example. We also present a case in which the elasticity of substitution between the two types of labor depends on the wage ratio, using a translog cost function to represent the technology. We thus place ourselves in a case where the results of the theoretical model are ambiguous (that is without H1 nor H2).

For simplicity, we specify the minimum wage rule in terms of rigidity of the unskilled workers’ relative wage. The government sets the coefficient of proportionality $s$. The skilled workers’ wage is determined through decentralized bargaining, therefore the parties do not take the indexation of the unskilled workers’ wage to the skilled workers’ into account during the bargain. We first discuss the parameter values before we turn to the simulation exercise.

4.1. Calibration

- We first consider the value of the elasticity of substitution between skilled and unskilled labor, $\sigma$. The estimates of this parameter are quite approximative. In his large overview of the literature, Hamermesh (1993, Chapter 3) notes that the estimated values of the elasticity of substitution between blue and white collar in manufacturing industries are positive in most cases but may vary between 0.14 and 6! We shall focus on a situation where this parameter is larger than unity in order to shed light on the ambiguous results delivered by the theoretical model. More precisely, we choose an intermediate value, $\sigma \approx 1.2$, which is still reasonable regarding the empirical results even though relatively large.
- The extent of the trade-union bargaining power, $\gamma$, is not well known. Abowd and Allain (1996) estimate a value for this parameter around 0.6, Cahuc et al. (1997) find 0.25: we choose an intermediate value, taking $\gamma = 0.4$.
- The magnitude of the mark-up of prices over marginal costs set by the firms, $\nu$, is estimated by Oliviera Martins et al. (1996) for manufacturing industries in several OECD countries between 1.1 and 1.8, taking a mean value approximately equal to 1.2 in most of those countries. We take $\nu = 1.3$. 
• We assume technology to be homogeneous of degree $\alpha = 0.65$ which corresponds to the output share of labor. The demand shift parameter, $A$, and the skilled workers’ threat point, $\bar{w}$, can be chosen arbitrarily since the values of those parameters determine the level of wages and employment but have no influence on their variations with respect to $s$ (both parameters are set to 1).

• Finally, we also need to clarify the distinction between skilled and unskilled workers in order to define the benchmark value of the ratio of unskilled to skilled wages, $s$. Considering that the skilled workers’ group corresponds to professional and managerial workers, and that the unskilled workers’ group corresponds to employees and blue collar workers, Sneessens and Shadman-Metha (1995) find a ratio of the unskilled to skilled wages approximately equal to 50% in France. In this configuration, some unskilled workers earn more than the minimum wage. Nevertheless, the minimum to unskilled wage ratio is almost constant over time and we can thus consider that the unskilled wage variations are closely related to those of the minimum wage. The corresponding proportion of skilled workers in total employment is about $\frac{2}{3}$ (Maillard and Sneessens, 1994). To summarize, we consider that $s = 0.5$ whereas $\ell_n/(\ell_q + \ell_n) = 0.67$.

Since we have no idea about the weight of employment in the trade-union objective, $\chi$, we postpone the discussion on its value to the simulations.

4.2. Simulations

We first need to define the skilled workers’ preferences in order to derive the bargained wage equation. They are represented by the function $r(w_q) = w_q^\rho$, which means the agents are risk averse if $0 < \rho < 1$. One thus has for the bargained wage

$$w_q = \left(1 - \frac{\rho}{\mu(s) - \eta(s)}\right)^{-1/\rho} \bar{w}.$$ 

We suppose that skilled workers are mildly risk averse, taking $\rho$ equal to 0.9 for the simulations. Taking a translog cost function for the technology leads to the following expressions for the wage index and the partial skilled wage elasticity of the wage index:

$$\ln W = \ln w_q + (1 - a) \ln s - \frac{b}{2}(\ln s)^2 \quad \text{and} \quad \eta(s) = a + b \ln s. \quad (8)$$

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2 Its observed value over the period 1980–1990 being about 0.65.
With this specification, the elasticity of substitution between skilled and unskilled labor depends on the relative wage \( s \) and is given by

\[
\sigma = 1 + \frac{b}{\eta(s)(1 - \eta(s))}.
\]  

Clearly, inputs are substitutes (resp. complements) when \( b > 0 \) (resp. \( b < 0 \)). Then, as expected following the discussion related to Proposition 2, the partial skilled wage elasticity of the wage index increases (resp. decreases) with respect to \( s \). Given the values of \( s \), \( \ell_a/\ell_n \) and \( \sigma \) stated above, Eqs. (3), (8) and (9) lead to values for the parameters \( a \) and \( b \) of 0.53 and 0.05, respectively. The weight of employment in the trade-union’s objective, \( \chi \), is supposed to be 0.15. We then use Mathematica to simulate the model.\(^3\) The simulations (see Fig. 1) show that the unskilled employment can increase with respect to the unskilled wage when workers are substitutable and the minimum wage is not excessively high.\(^4\)

The results that have been obtained rely on the assumption of a relatively low value of \( \chi \), which amounts to 0.15 in the case we have just presented. However it should be noted that if the elasticity of substitution between the two types of labor is strictly larger than unity, it is always possible to find values of the employment weight in the trade-union objective, \( \chi \), and of the mark-up, \( \nu \), under which the positive relationship between unskilled wage and employment appears. A higher value of \( \chi \) must be compensated by an increase in \( \nu \) to get our results. For instance, with the translog cost function that we have chosen, there is a decreasing relationship between the minimum wage and unskilled employment if \( \chi = 0.4 \) and \( \nu \) is around 1.8.

Finally, we would like to emphasize that our results do not rely on any specific feature of the translog cost function. The same type of results hold, taking a CES cost function as a representation of technology (see Cahuc et al., 1998).

5. Conclusion

In this paper, we have presented a simple theoretical model in which a positive relationship between minimum wage and unskilled employment can occur. The structure of the labor market places unskilled workers who are paid a minimum

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\(^3\) We have checked that the conditions of concavity are locally satisfied with those values of the parameters \( a \) and \( b \). The second-order condition of the wage bargaining program is also satisfied for this set of parameters and the minimum wage is binding. Finally, the elasticity of substitution between skilled and unskilled labor is quite constant when \( s \) varies from 0.5 to 1, taking values between 1.2000 (the starting value) and 1.2008. For more details, see Cahuc et al. (1998).

\(^4\) The last figure gives the variation (not the variation rate) of the proportion of skilled workers in overall employment.
wage, in the face of skilled workers who bargain for their wages with the employer. In such a context, an increase in the minimum wage affects the skilled wage, changing the firm’s conditions during the wage negotiation. The employer’s behavior is subject to technological constraints that are represented in our model by the degree of substitutability between skilled and unskilled labor. We find that if inputs are substitutable, minimum wage increases will allow the employer to restrict the wage pressure exerted by the union and hire new skilled workers. As lower wages for the skilled workers decrease total labor cost, it is therefore possible that the demand for unskilled labor also increases with the minimum wage.
This result is obtained with a really standard representation of the labor market: the bargain is represented by a 'right-to-manage' model and the technology by a standard cost function. Nevertheless, it is a partial equilibrium analysis that does not take account of capital adjustments. A deeper exploration of the impact of the minimum wage should involve both capital and the general equilibrium effects. Such an analysis is beyond the scope of our paper and is left for future research.

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References


