A Novel Magnetic Harmonic Gear

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Abstract—Magnetic gears offer several advantages compared to mechanical gears in terms of reduced maintenance, improved reliability, and inherent overload protection while having a high efficiency. This paper describes the principle of operation of a novel form of magnetic gear, which is particularly suited to applications for which a high gear ratio is required. The performance capability of such a magnetic gear is investigated, and it is shown that it transmits a ripple-free torque and that an active torque density of up to 150 kN·m/m³ per stage can be achieved when high-energy permanent magnets are employed. Simulation results on this novel gear are verified by experimental measurements on a prototype.

Index Terms—Magnetic gears, permanent magnets.

I. INTRODUCTION

MAGNETIC gears offer several advantages compared to mechanical gears in terms of reduced maintenance, improved reliability, tolerance to mechanical inaccuracies, and inherent overload protection, and several topologies have been proposed [1]–[5]. It has been shown that state-of-the-art magnetic gears can exhibit torque densities of up to 100 kN/m³ [6], [7], and methods of integrating such gears with permanent-magnet brushless machines in order to achieve a high-torque density "pseudo" direct drive are already being proposed [8], [9]. Unfortunately, the torque density of these gears tends to fall significantly for gear ratios higher than about 20:1. Therefore, this paper describes the principle of operation of a magnetic gear topology, which is similar to a mechanical harmonic gear and which is particularly suitable for gear ratios higher than about 20:1. It is shown that the transmitted torque is ripple free and that an active torque density of up to 150 kN·m/m³ per stage can be achieved when rare-earth permanent magnets are used. Applications of the harmonic magnetic gear topology can vary from robotics, aerospace to space applications where lubrication could be an issue. Furthermore, although, due to the high cost of permanent magnets, a magnetic harmonic gear may be more expensive than a mechanical gear, for applications where a direct drive must be employed, combining such a gear with a high-speed electrical machine would be a more size/weight and cost-effective solution.

II. PRINCIPLE OF OPERATION

Fundamental to the operation of the magnetic harmonic gear is the mechanism for producing a time-varying sinusoidal variation of the air-gap length between a set of permanent magnets which may be mounted on a flexible low-speed rotor and another set of permanent magnets which are mounted on a rigid outer cylindrical stator. To achieve this, the gear employs an appropriately profiled high-speed rotor, which is equivalent to the wave generator in a mechanical harmonic gear and which deforms the flexible low-speed rotor using a sliding contact such that the low-speed rotor assumes the same profile while rotating independently. As a result of the variable air-gap length, the magnetic fields, which are produced by both sets of permanent magnets, are modulated such that asynchronous space harmonics are generated by one set of magnets which have the same number of poles as the other set of permanent magnets, and vice versa.

Due to the sinusoidal profile of both the high-speed and low-speed rotors, the air-gap length between the stator and the low-speed rotor can be expressed as

\[ g = \frac{g_{\text{max}} + g_{\text{min}}}{2} + \frac{g_{\text{max}} - g_{\text{min}}}{2} \cos (p_w (\theta - \omega_h t)) \] (1)

where \( g_{\text{max}} \) and \( g_{\text{min}} \) are the maximum and minimum air-gap lengths, respectively, \( \omega_h \) is the angular velocity of the high-speed rotor, and \( p_w \) is the number of sinusoidal cycles which result in the air gap between the low-speed rotor and the stator. Fig. 1 shows magnetic harmonic gears with \( p_w = 1 \), \( p_w = 2 \), and \( p_w = 3 \).

The radial component of the flux density distribution due to the low-speed rotor magnets, at a radial distance \( r \), can be written in the following form:

\[ B_r(r, \theta) = \sum_{m=1,3,5,...} b_{rm}(r) \cos (mp(\theta - \omega_r t) + m\theta_0) \times \left( \lambda_0 + \lambda_1 \cos (p_w(\theta - \omega_h t)) \right) \]

\[ = \sum_{m=1,3,5,...} (b_{rm}(r) \cos (mp(\theta - \omega_r t) + m\theta_0) \lambda_0) + (b_{rm}(r) \cos (mp(\theta - \omega_r t) + m\theta_0) \lambda_1) \times \lambda_1 \cos (p_w(\theta - \omega_h t)) \]


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where $\omega_r$ and $p$ are the angular velocity and number of pole pairs of the low-speed rotor, respectively, and $\lambda_0$ and $\lambda_1$ are the first two Fourier coefficients for the modulating function which is associated with the radial component of flux density as a result of the sinusoidal variation of the air-gap length. Therefore, from (2), the number of pole pairs in the space harmonic flux density distribution produced by the low-speed permanent-magnet rotor is given by

$$q_{m,k} = mp + (-1)^k p_w, \quad m = 1, 3, 5, \ldots, \infty; \quad k = 1, 2$$

where variable $k$ has been introduced to differentiate between the various asynchronous space harmonics which are associated with each harmonic of the magnetic field produced by the permanent magnets. The angular velocity of the flux density space harmonics is given by

$$\Omega_{m,k} = p_w \omega_h + (-1)^k mp \omega_r$$

and it can be seen that the velocity of the space harmonics differs from the velocity of both the high-speed rotor (wave generator) and the low-speed rotor. Since the highest asynchronous space harmonics occur for $m = 1$, the number of pole pairs on the stator must be equal to $p + p_w$ or $p - p_w$ in order to maximize the torque transmission between the stator and the low-speed rotor. Furthermore, since the asynchronous space harmonic, which transmits the torque, couples with the static field produced by the stator, its velocity is zero ($\Omega_{1,k} = 0$), and the gear ratio $G_r$ can be deduced from (4) as

$$G_r = (-1)^{k+1} \frac{p_w}{p}.$$  \hspace{1cm} (5)

Finally, it should be noted that $\omega_r$ represents the average angular velocity of the low-speed rotor, since the individual magnets rotate at different instantaneous speeds as a result of their different relative radial positions around the sinusoidally shaped outer surface of the low-speed rotor. Fig. 2 shows the rotation of both the high-speed and low-speed rotors.

III. SIMULATION STUDIES

Fig. 3 shows schematics of four magnetic harmonic gears, which differ from each other by the number of pole pairs on
the low-speed rotor and the number of cycles in the air gap. The gears have been analyzed using 2-D magnetostatic finite-element analysis. For all four gears, the outer radius is 85 mm, 6 mm, and \( p_{w} = 2 \).

\[ \frac{\mu_{r}}{\mu_{0}} = 1.05 \]

A. Flux Density Waveforms

Figs. 4–7 show the variation of the radial component of flux density which results at the center of the stator magnets due to the low-speed rotor magnets, and their corresponding space harmonic spectra, for the different gears. It is clear from the figures that \( m = 1 \) and \( k = 2 \) result in the highest asynchronous space harmonic, such that a gear achieves the greatest torque transmission capability if the number of stator pole pairs equals \( p + p_{w} \), as is the case for the four gears shown in Fig. 3. It can also be seen that, contrary to (2), the space harmonic, which corresponds to \( m = 1 \) and \( k = 1 \), is significantly lower than the one which corresponds to \( m = 1 \) and \( k = 2 \). Unfortunately, a simple explanation of this phenomenon is yet to be proposed. Finally, although the flux density of the active asynchronous harmonic is relatively low, the transmitted torque is high since it is proportional to the product of the air-gap flux density and the number of pole pairs of the active harmonic.

B. Torque Prediction

Two-dimensional finite-element analysis has been employed to compute the torque on each component in the gear. As shown in Fig. 8, the total force \( F_{r} \), which is exerted on each magnet
Fig. 7. Radial flux density waveform due to low-speed rotor permanent magnets, and space harmonic spectrum, for $p = 30$, $g_{\text{max}} = 6 \text{ mm}$, and $p_w = 1$.

Fig. 8. Components of force on the magnets of the low-speed rotor.

of the low-speed rotor, has components $F_n$ and $F_t$, which are normal and tangential to the bearing outer circumference, respectively. The tangential component $F_t$ enables the low-speed rotor to slide over the high-speed rotor and therefore contributes to the torque on the low-speed rotor. The normal component $F_n$ is transmitted through the bearing and contributes to a torque on the high-speed rotor, because $F_n$ is generally not applied through the center of rotation of the high-speed rotor. Hence, a Maxwell stress calculation around an integration path which encloses all the low-speed rotor magnets will predict the combined torque which is exerted on both rotors.

However, by using the virtual work method, the torque on each rotor can be calculated separately. By way of example,

Table I shows the predicted values of stored magnetic energy in an 18 : 1 harmonic gear with $g_{\text{max}} = 8 \text{ mm}$ and $p_w = 1$ for three different positions of the rotors, i.e., no rotation of both rotors, a rotation of 1° of the high-speed rotor with no rotation of the low-speed rotor magnets but with a change in shape of the low-speed rotor to accommodate the rotation of the high-speed rotor, and no rotation of the high-speed rotor with the low-speed rotor rotated over 0.0556°. From these results, it can be shown that the torques on the high-speed and low-speed rotors equal 198 and 3576 N·m, respectively, which is in agreement with the gear ratio of 18 : 1.

C. Torque Transmission

Fig. 9 compares the variation of the maximum torque density of the four harmonic gears shown in Fig. 3 with the maximum air-gap length $g_{\text{max}}$. It can be seen that the optimal value of $g_{\text{max}}$ to maximize the torque transmission capability is dependent on the gear ratio.

Fig. 10 shows the variation of the transmitted torque with the rotation angle of the high-speed rotor for each of the four gears, based on the value of $g_{\text{max}}$ that corresponds to the maximum torque transmission capability for each gear ratio. It can be seen that there is no ripple in the transmitted torque.

IV. PRACTICAL IMPLEMENTATION

The practical realization of a magnetic harmonic gear having $p_w > 1$ is complicated by the need for a flexible permanent-magnet low-speed rotor assembly and a coupling with which to interface to an external load via a rigid shaft. Whereas a flexible rotor can be realized using the same technology that is employed in mechanical harmonic gears, a simplified construction for gears with $p_w = 1$ is discussed. Furthermore, a magnetic coupling to connect the low-speed output rotor to an external load is also proposed.
Fig. 10. Variation of maximum torque with rotation of high-speed rotor.

A. Rotor Structure of a Harmonic Gear With $p_w = 1$

For harmonic gears which exhibit only one sinusoidal cycle in the air gap between the low-speed rotor and the stator ($p_w = 1$), the flexible rotor can be replaced by a conventional solid permanent-magnet rotor, which is positioned via a bearing on an eccentricity of the high-speed rotor, as shown in Fig. 11, such that the gear is akin to a magnetic version of the mechanical cycloid gear and may exhibit higher level of mechanically and magnetically generated acoustic noise. However, since the low-speed rotor is mounted eccentrically to the high-speed rotor, a mechanical system is required to connect the low-speed eccentric rotor to an external concentric shaft [10].

B. Magnetic Torque Transmission Between Output Rotor and External Load

The requirement for a coupling between the flexible structure and an output shaft can be overcome if a dual-stage harmonic gear is employed, as shown in Fig. 12 [11]. This configuration consists of a first stage, which is implemented as discussed previously, and a second stage, in which the outer concentric magnet array is allowed to rotate and constitutes the output rotor. The high-speed and flexible rotors are common to both stages, although the flexible rotor carries a different number of magnet poles in each of the two stages. As discussed previously, the rotation of the high-speed rotor results in a geared rotation of the flexible rotor in the first stage, which, in turn, results in a geared rotation of the output rotor in the second stage. However, because the intermediate flexible rotor is no longer connected to an external load, the total torque, which is applied to it, is equal to zero under steady-state operating conditions. This arrangement can also be employed in the gear, which is shown in Fig. 11, in order to transmit the low-speed output rotation to the load without transmitting the eccentric motion of the low-speed rotor.

In order to simplify the discussion, it is assumed that a magnet combination is adopted in each of the two stages, which results in maximum torque transmission, as demonstrated previously, i.e.,

$$q_{1,2} = p_{1,2} + p_w$$

where $p_1$ and $p_2$ represent the number of pole pairs on the intermediate rotor in the first and second stages, respectively, and $q_1$ and $q_2$ represent the number of pole pairs on the stator and low-speed rotor, respectively.

In each stage, the radial component of flux density distribution due to the magnets on the intermediate rotor is described by (2) and (4). The average rotational speed of the intermediate rotor, i.e., $\omega_i$, is determined by the interaction of the magnets in the first stage and, by following (5), can be written as

$$\omega_i = -\frac{p_w}{p_1} \omega_h.$$  

Furthermore, the magnetic field due to the magnets on the intermediate rotor in the second stage couples with the output rotor. It follows from (4), therefore, that

$$\omega_l(p_2 + p_w) = p_w \omega_h + p_2 \omega_i$$

where $\omega_l$ is the low rotor speed. Combining (7) and (8) results in

$$\omega_l = \frac{p_w}{p_2 + p_w} \left(1 - \frac{p_2}{p_1}\right) \omega_h$$

from which the gear ratio of the dual-stage harmonic gear can be determined. It can be seen that a range of gear ratios
can be attained by appropriate choice of $p_1$ and $p_2$, whereas $p_2 = (p_1 + 1)$ results in the highest attainable gear ratio for a given value of $p_1$. For example, Table II lists a number of different gear ratios that can be attained. As can be seen, the overall gear ratio $G_r$ of the dual-stage harmonic gear is greater than the product of the ratios, $g_{r1}$ and $g_{r2}$, that each stage would achieve with the given pole-pair combination, for gears with $p_w = 1$, for example.

In order to illustrate the realization of such a high gear ratio, the internal torque balance in the dual-stage gear is evaluated. As discussed previously, the intermediate rotor has no external connection. Thus

$$T_{i1} + T_{i2} = -T_{h1}g_{r1} - T_{h2}g_{r2} = 0$$ (10)

where $T_{i1}$ and $T_{h1}$ are the torques, which are exerted on the intermediate and high-speed rotors, respectively, and suffixes 1 and 2 refer to the respective gear stages where these torques are applied. The overall gear ratio $G_r$ of the dual-stage gear can then be written as a function of the individual gear ratios as

$$G_r = \frac{T_l}{T_h} = \frac{-T_{i2} - T_{h2}}{-T_{h1} - T_{k2}} = \frac{1 - g_{r2}}{1 - g_{r1}}$$ (11)

where $T_l$ and $T_h$ are the torque on the low-speed rotor in the second stage and the externally applied torque on the high-speed rotor, respectively.

Fig. 13 shows the variation of the torque on the low-speed rotor (output rotor) as the high-speed rotor is rotated, for a gear with $p_1 = 18$, $p_2 = 19$, and $g_{max} = 8$ mm. In the analysis, the angular positions of the intermediate rotor and the low-speed rotor were varied with the high-speed rotor according to (6) and (8).

V. EXPERIMENTAL VALIDATION

A prototype dual-stage magnetic harmonic gear was built with $p_w = 1$, $p_1 = 18$, $p_2 = 19$, and an overall gear ratio of $-360:1$. The gear has an outside diameter of 140 mm, back-iron thicknesses of 5 mm, an inner magnet thickness of 6 mm, an outer magnet thickness of 5 mm, and an active length of 50 mm per stage. Fig. 14 shows a cross-sectional view of the prototype, where it can be seen that the gear is constructed with a rigid intermediate rotor which can rotate around an axis that is eccentric to the high-speed input rotor. Fig. 15(a) shows the intermediate rotor, on which the two stages of the gear can be recognized. The low-speed output rotor of the gear is shown in Fig. 15(b). The measured gear ratio is consistent with the gear ratio given by (10), while at room temperature, the measured pull-out torque of the gear is 115 N·m, which occurs at a load angle of 4.5° (mechanical), and compares favorably with the predicted peak torque of 122 N·m. Furthermore, since the pull-out torque is proportional to the square of the remanence of the permanent magnets, its dependence on temperature would be more significant than the torque produced by permanent-magnet brushless machines. Finally, the operation of the gear is similar to other magnetic couplings/gears, for which disengagement will occur if the load angle reaches the maximum under transient or steady-state conditions.

VI. CONCLUSION

The principle of operation and the torque transmission capability of a novel topology of magnetic gear have been presented. It has been shown that, at relatively high gear ratios, an active torque density of up to $150$ kN·m/m$^3$ can be achieved in a...
single stage. It has also been shown that the transmitted torque exhibits no ripple. A practical dual-stage implementation of the magnetic harmonic gear, which exhibits a gear ratio that is greater than the product of the ratios of the individual stages and a torque density of up to 75 kN·m/m³, has been described. Finally, measurement results on a prototype gear have been reported.

REFERENCES
