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This paper is concerned with multi-criteria and dynamic resource allocation problem in a naval engagement context. The scenario under investigation considers air threats directed towards a ship that has to plan its engagement by efficiently allocating the available weapons against the threats to maximize its survivability. This dynamic and multi-criteria decision-making problem is modeled using a multi-criteria decision tree and solved with two approaches: the multi-criteria decomposition approach and the multi-criteria myopic approach. We propose a novel metric for comparing two strategies within a multi-criteria decision tree and have developed a testbed in order to simulate the engagements. The results show that, when sufficient decomposition conditions are verified, the decomposition approach produces superior decision-making strategies compared to the myopic approach. Conversely, when the multi-criteria decision aid (MCDA) method does not satisfy the decomposition conditions (e.g., TOPSIS), there is no guarantee that decomposition will provide the best compromise strategies. From a military perspective, this work will help develop tactics, procedures and training packages for such a highly complex and dynamic decision-making problem. The plans generated by the approach presented here can also serve as a reference for assessment of the quality of the engagement plans yielded by real-time planning algorithms.

Keywords: Decision analysis; multiple criteria decision aid; dynamic planning; naval defense; decision tree; uncertainty modeling.
1. Introduction

This paper is concerned with the resource (weapon) allocation problem in the naval Command and Control (C2) process. This more general process includes target detection, tactical picture compilation, threat assessment, weapon allocation, engagement management and kill assessment. In particular, weapon allocation consists of assigning weapons including the assignment of support resources required for every engagement. The weapon-target assignment (WTA) problem is very complex. The dynamic and uncertainty aspects of the problem, the temporal constraints, as well as the variety of stakes, goals and objectives are some of the complex issues to be dealt with when trying to solve such a problem. In practice, military commanders have to act under tight temporal constraints and should provide a real-time reaction. Usually, the planning component uses a recovery mechanism (default strategy) in order to provide a solution. For instance, default procedures might consist of engaging weapons to prioritized targets (earliest intercept) based on maximum probability of kill. Solutions given by the default procedures are not guaranteed to be optimal but they could be satisfactory.

In real situations, many conflicting criteria should be taken into account when evaluating the performance of decisions in terms of risk (RK), effectiveness (Eff), and loss of opportunity (LO). Consideration of the dynamic aspect is also very important since the decision choices at each period should take into account not only the immediate consequences but also the impact of decisions on the whole horizon. It is then worth solving this problem looking at both its multi-criteria and dynamic aspects.

The weapon-target allocation problem has interested several researchers in the field of operations research. Previous approaches can be classified as two categories: exact and heuristic methods. The first category regroups those methods that aim to find the optimal solution ignoring the temporal constraints of the problem. The second category aims to provide sub-optimal solutions under temporal constraints. As reported in the literature review for this paper, the previous studies on weapon-target allocation problems did not simultaneously consider the multiple criteria and the dynamic aspects of the decision problem. In this paper, we propose a solution that solves weapon allocation problems by considering both their multiple criteria and dynamic aspects. The main objective of the paper consists of modeling and simulating defense scenarios for a single ship against a set of anti-ship missiles (ASM) and of providing the best compromise strategies while, at the same time, weighing the multiple criteria, and dynamic aspects of the problem. More specifically, the paper will

- Provide a model for the weapon-target allocation, decision-making problem using a multi-criteria decision tree.
- Propose a novel metric, which enables comparison of two strategies in the multi-criteria decision tree.
- Apply the Multiple Criteria Decomposition approach proposed in Ref. 1 and compare its results with the Multiple Criteria Myopic approach which does not take the dynamic aspect of the problem into account.
The Multiple Criteria Decomposition approach provides the best compromise solution on the whole horizon by considering the dynamic and multi-criteria aspects of the problem simultaneously. We chose to compare its results with the myopic approach because the myopic approach guarantees only local best compromise solution.

In this work, real-time constraint is not a limitation, given that the proposed approach is not intended to be implemented on ships or to support real-time decision-making. Rather, the best compromise strategies are intended to support the development of tactics, procedures and training packages. It could also be seen as a reference point for the assessment of the quality of decisions given by real-time algorithms.

This paper contains four sections. Section 2 presents the related works and the main literature review. Section 3 presents the problem resolution approach. It includes problem representation, the decomposition and myopic approaches, plus the novel metric for comparing two strategies in the decision tree. In Sec. 4, we provide the model of the naval engagement problem together with the testbed, an illustrative example and the results of the simulations.

2. Literature Review

The resource allocation problem in military context and more specifically the WTA problem has interested several researchers in the field of operational research. First research papers date from the 1950s to 1960s. Then, in the 1980s, several authors studied the weapon allocation problem in the particular case of defense of a naval vessel. Other studies considered this problem in a more general context as multiple-layer defense situations.

More recently in the last decade, several research works have been published on the weapon-target allocation problem. The literature review shows that the most used methods and procedures for resource allocation problems are linear and integer programming, compromise programming, ant colony optimization and genetic algorithms. More specifically, Li et al. and Karasakal formulate the problem as an integer programming problem with multiple knapsack constraints and multiple choice constraints. Ahuja et al. suggest solving the WTA problem with integer programming and network flow-based lower-bounding methods obtained from a branch-and-bound algorithm. Cha and Kim present a branch and bound algorithm with the objective of minimizing total threat of the targets. In addition, Bogdanowicz proposes an algorithm derived from the well-known auction algorithm.

Ant colony algorithm was used in Yanxia et al., Zhang et al., and Lee et al. propose an immunity-based ant colony optimization algorithm, which combines the ability to cooperatively explore the search space and to avoid premature convergence, and the ability to quickly retrieve good solutions. Zhang et al. built a reverse mutation ant colony algorithm model to solve the multiple targets assignment problem in cooperative air combat.

The evolutionary approach was also used. Genetic algorithms were utilized in Erdem and Ozdemirel, Bayrak and Polat and Grant. Erdem and Ozdemirel
Table 1. Chief aspects of related work of the last decade.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Decision problem objective</th>
<th>Approach</th>
<th>Advantages/Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bayrak and Polat(^{29})</td>
<td>Prepare Air Tasking Orders, i.e., plans prepared to meet the objectives of air combat missions through optimizing the allocation of resources.</td>
<td>Genetic algorithm with novel crossover operations. Integer programming model</td>
<td>Compared to the integer programming model, the genetic algorithm is best in practical terms (computation time). Gain values of the solutions are not much worse than the reference values created by exhaustive search.</td>
</tr>
<tr>
<td>Zhang and Liu(^{26})</td>
<td>Solve the efficiency of the multiple target assignment problem in multi-aircraft cooperative air combat.</td>
<td>Reverse mutation ant colony algorithm model (RMACA)</td>
<td>Mono-objective optimization. Dynamic aspect not considered. The proposed algorithm enhanced the search efficiency of the target assignment problem. Shorter iteration time.</td>
</tr>
<tr>
<td>Cha and Kim(^{23})</td>
<td>Solve a fire scheduling problem with the aim of minimizing total threat of the targets, expressed by the destruction probabilities of the target.</td>
<td>Branch and bound algorithm</td>
<td>Mono-objective optimization. Dynamic aspect not considered. Optimal solutions for medium-size problems found in a reasonable CPU time.</td>
</tr>
<tr>
<td>Bogdanowicz(^{24})</td>
<td>Optimize the assignment of smart weapons to targets. A smart weapon is a weapon that relies on information provided by sensors.</td>
<td>Algorithm derived from auction algorithm</td>
<td>Algorithm generates optimal assignment. Mono-objective optimization. Dynamic aspect not considered.</td>
</tr>
<tr>
<td>Karassakal(^{21})</td>
<td>Allocate air defense missiles to maximize air defense effectiveness of a naval task group.</td>
<td>Integer programming</td>
<td>Mono-objective optimization; good computational time. Dynamic aspect not considered.</td>
</tr>
<tr>
<td>Yanxia et al.(^{25})</td>
<td>Assign weapons so target with greater threat degree has higher priority to be intercepted. The effect will be not to maximize damage probability but to satisfy the whole assignment result.</td>
<td>Ant colony algorithm</td>
<td>Mono-objective optimization. Dynamic aspect not considered. Effectiveness of model demonstrated.</td>
</tr>
<tr>
<td>Reference</td>
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<td>Advantages/Limitations</td>
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<tr>
<td>--------------------</td>
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</tr>
<tr>
<td>Ahuja et al. 22</td>
<td>Assign $n$ weapons to $m$ targets to minimize total expected survival value of targets.</td>
<td>Integer programming and network flow-based lower-bounding methods obtained from branch-and-bound algorithm.</td>
<td>Mono-objective optimization. Exact algorithm. Dynamic aspect not considered. Good computational time.</td>
</tr>
<tr>
<td>Li et al. 32</td>
<td>Weapon-target allocation considering dynamic characteristics of air defense operational command and decision of warship's formation.</td>
<td>Dynamic WTA model</td>
<td>Dynamic characteristics of the problem considered. Results concordant with intuitionistic tactical judgment. Mono-objective optimization.</td>
</tr>
<tr>
<td>Li et al. 20</td>
<td>Assign strike force assets to targets while maximizing target and threat damage values.</td>
<td>Integer programming with multiple knapsack constraints</td>
<td>Mono-objective optimization. Dynamic aspect not considered.</td>
</tr>
<tr>
<td>Erdem and Ozdemirel 26</td>
<td>Assign friendly military units to enemy units to minimize total weapon effectiveness index (value or cost) while attrition goals set for red units.</td>
<td>Evolutionary approach (genetic algorithm using a repair algorithm to ensure feasibility of solution). A local improvement algorithm used to improve solution quality.</td>
<td>Mono-objective optimization. Dynamic aspect not considered. Solutions with acceptable quality in reasonable computation time. Generation of feasible initial population may take a long time for tight problem.</td>
</tr>
<tr>
<td>Lee et al. 27</td>
<td>Find a proper assignment of weapons to targets with aim of minimizing expected damage of own force assets.</td>
<td>Immunity-based ant colony optimization algorithm.</td>
<td>Mono-objective optimization. Dynamic aspect not considered. Better search performances than other algorithms.</td>
</tr>
<tr>
<td>Kim and Choe 34</td>
<td>Maximize probability of hitting target and minimizing number of missiles launched by friendly forces in engagement.</td>
<td>Compromise programming</td>
<td>Multi-objective optimization. Dynamic aspect not considered.</td>
</tr>
</tbody>
</table>
developed a repair algorithm to ensure feasibility with respect to the attrition goal constraints. A tightness measure is proposed to determine the population size of the genetic algorithm and a local improvement algorithm is used to further improve the solution quality. In addition, Bayrak and Polat develop a genetic algorithm with customized encoding, crossover and fitness calculation mechanisms.

Benaskeur et al. and Benaskeur et al. propose a real-time planner, developed to support the command team of a naval force defending against multiple threats. The system uses a local planner to generate a set of local plans and then combines and coordinates them into a single conflict-free global plan. An iterative process of plan merging and conflict detection and resolution is used to perform coordination.

These research works do not address the multi-criteria and the dynamic aspects of the problem simultaneously. The WTA problem is usually viewed from a mono-objective perspective and the dynamic aspect is rarely addressed. Here, we report the recent publications that address either the dynamic or the multi-criteria aspect of the decision-problem. Li et al. establish a dynamic WTA model and the simulation results show that switch fire and repetition fire of anti-air weapon systems affect the result of the air defense operation remarkably and that the dynamic model is more satisfactory than static models. Naeem and mano discuss a new dynamic weapon scheduling algorithm that allows multiple engagements and uses a shoot-look-shoot (SLS) strategy to compute a near-optimal solution. Kim and Cho formulate the WTA problem as a multi-objective decision-making problem and solve it with compromise programming, with priorities assigned to the multiple conflicting objectives. Previously, goal programming was used in Green et al.

Table 1 summarizes the main aspects of the related works of the last decade. Most of the papers use static mono-objective optimization. Those which study the problem from a multi-objective perspective do not consider the dynamic aspect associated with the problem and vice versa. Since we find no related work which has studied the dynamic and multi-criteria aspects of the problem simultaneously, we believe that the research question we address in this paper is original.

In the next sections, we look ahead to solve weapon allocation problems considering both their multiple criteria and dynamic aspects. To do so, the methodology used will be based on the theoretical work of Frini et al. which generalizes the bellman’s principle of decomposition to the multi-criteria context. We will use their proposed methodology which combines the advantages of decomposing the problem with the application of multi-criteria decision aid (MCDA) methods. In this way, we will solve the dynamic multi-criteria problem without generating the set of non-dominated solutions. In addition, we will reduce the computation time and the cardinality of the solution set.

3. Problem Resolution Approach

The next sections include problem representation, the decomposition and myopic approaches, plus the novel metric for comparing two strategies in the decision tree.
The proposed metric in Sec. 3.2.3 will allow the comparison of the engagement strategies given by the decomposition and the myopic approaches based on the binary $\varepsilon$-indicator $I_\varepsilon(A,B)$ defined in Zitzler et al.\textsuperscript{35}

### 3.1. Problem representation

The scenario under investigation considers the defense of a naval platform (frigate) coming under attack by threats (ASM). The objective of the frigate is to allocate efficiently the limited weapons available against selected threats over a given period of time to maximize ship survivability. This is mainly a resource allocation problem which is sequential, dynamic and subject to uncertainty, with a decision-making process based on multiple conflicting criteria.

The resource allocation problem is subject to uncertainty because it is impossible to predict the evolution of the situation. Decision outcomes, such as kill, not kill, are in fact stochastic and usually modeled by probabilistic distributions. Models, such as possibility, fuzzy sets or belief theory could also model this uncertainty. Second, the resource allocation problem is sequential. Serial and interdependent decisions are made at different periods of time to deal with different threats. Outcomes of decisions made during a given period can influence subsequent decisions (e.g., using a given resource will impact the availability of that resource for subsequent engagements, engaging a threat will affect the list of threat alive, etc.). Because each sequence or stage depends on the time period, the resource allocation problem is not only sequential but also dynamic. Third, the decision-making process for the resource allocation is based on multiple conflicting criteria. For example, each decision presents a risk, has a level of expected effectiveness, and will imply a LO for future decisions. The risk is an important criteria connected to the fact that by choosing one decision, we can ignore certain threats that can damage the ship. Effectiveness and LO measure the expected success of the decision and the future utility of the used weapons, respectively. Such conflicting criteria should play a part in the decision process.

To model this decision-making problem, we will employ a multi-criteria decision tree to represent such a problem. This type of representation provides an interesting tool for modeling, analyzing, and communicating stochastic and sequential decision-making problems. This representation depicts all scenarios of a problem fully, including timing and events and provides a chronological and detailed view of the structure of the decision problem.\textsuperscript{36–38} We opted for this modeling tool after weighing its advantages in terms of exhaustively modeling all defense strategies for tactic development and training purposes, and its main limitation, namely tree explosion. Since we would be performing an offline analysis, we concluded that computational time was not an issue.

Similar to a single criteria decision tree, the multi-criteria decision tree comprises decision nodes and uncertainty nodes. Branches stemming from each decision node correspond to feasible alternatives and those from uncertainty nodes to outcomes.
related to a specific occurrence of a state of nature. At each decision node, the
decision-maker observes the occurrence of the previous state (outcome of previous
decision) and is on the verge of making a new decision. The decision-maker monitors
the consequences of his/her previous decision at each uncertainty node. These con-
sequences generally depend on factors beyond the decision-maker’s control. The
decision tree is composed of many decision strategies. Each decision strategy is de-
\[ \text{Illustration.} \]
The decision tree in Fig. 1 is a simple illustration of some defense
strategies of a frigate attacked by 2 threats \((\text{th}_1, \text{th}_2)\). The alternatives will depend on
the threat range. In the first decision node, the frigate could use one or two missiles or
a gun, and, depending on the results (threat killed or not killed), it will choose
subsequent alternatives (gun or Close-in weapon system (CIWS)) since in the
meantime, the range threat has changed.

The remainder of this section addresses how we will solve and reach the
best compromise strategies of the WTA problem modeled with a multi-criteria
decision tree.

3.2. Approach to problem resolution
These sections present the decomposition and the myopic approaches. Next, we
propose a novel metric for comparing strategies within a multi-criteria decision tree.
This metric will enable comparison of the engagement strategies provided by the
decomposition and the myopic approaches.

![Fig. 1. Illustration of possible defense strategies.](image-url)
3.2.1. *The decomposition approach*

To solve the dynamic multi-criteria weapon-target allocation problem, we consider the decomposition approach introduced in Ref. 1. The proposed decomposition approach will allow us to solve the defense resource allocation problem taking into account both multiple criteria and the dynamic aspects of the problem. The decomposition approach solves the problem recursively by combining the use of a Multi-Criteria Decision Analysis (MCDA) method and the decomposition of the tree.

According to Frini et al., decomposition of the tree is possible when this decomposition principle is verified:

**Decomposition Principle** — Each best compromise strategy has the property that all its partial strategies are of best compromise.

Frini et al.\(^1\) stated and proved a theorem of decomposition which sets out four sufficient conditions under which applying the decomposition principle as defined actually provides the best compromise strategies (theorem of decomposition). Four conditions of the multi-criteria decision-making rule should be verified before any use of the decomposition principle. These conditions concern its preference order, temporal consistency, transitivity and the cardinality of the solution set. They are:

- The comparison between two strategies with the same initial conditions is done under a partial pre-order.
- The relational preference system verifies the temporal consistence (weak and strong versions).
- The preference and indifference relations are transitive.
- The set of best compromise solutions is not empty at each decision node.

The strong version (resp. weak) of temporal consistence means that if at a given period, the result of the multiple criteria decision-making rule indicates a strict preference for a given partial strategy compared to another (resp. indifference between two strategies), this result remains the same if we prolong the two strategies by the same action at a previous period.

With the decomposition approach and to ensure that the decomposition principle is valid and gives the best compromise strategies for the whole horizon, the decision-making rule used at decision nodes must verify sufficient conditions given by the theorem of decomposition. If this is not the case, application of a decision-making rule that violates any condition of the theorem does not guarantee to obtain the best compromise strategy as solution. Thus, the choice of the decision-making rule should be done carefully. The characterization of some decision-making rules shows that for instance, decision-making rules as dominance, lexicographic screening, and weighted sum could be used with decomposition, whereas TOPSIS, ELECTRE III, and PROMETHEE II could not (Ref. 1). In fact, according to Frini et al.\(^1\) the characterization of the dominance, lexicographic screening, and weighted sum shows that these methods respect the conditions stated in the theorem of decomposition while...
TOPSIS, ELECTRE III, and PROMETHEE II do not verify the temporal consist-
tence. When the decision-making rule verifies the decomposition condition, the
strategy that results from the decomposition approach is the best compromise one.
Moreover, this strategy is necessarily composed of partial strategies all of which are
of best compromise.

The proposed recursive approach consists of aggregating the distributional eval-
uations into punctual evaluations (at uncertainty nodes) and using an MCDA
method for selecting the best compromise strategy at each decision node. This de-
composition approach is valid if the MCDA method and the operator used for the
temporal aggregation at uncertainty nodes verify the theorem of decomposition
stated in Ref. 1. Based on this decomposition principle, the decomposition approach
consists of the following recursive steps:

**Step 1.** Model the decision-making problem with a multi-criteria decision tree.
**Step 2.** Choose the MCDA method to be used at decision nodes. Ensure that the
MCDA method verifies the conditions of the theorem of decomposition.
**Step 3.** Assign a probability distribution to states at each uncertainty node. Assign
the multi-criteria evaluations at each decision node.
**Step 4.** Evaluate each path in the tree according to each criterion.
**Step 5.** Perform temporal and preference aggregations for each period.

- Temporal aggregation: For each uncertainty node at each period, con-
  consider all partial strategies starting from this node. For each criterion,
  aggregate the distributional evaluations into a punctual evaluation using
  the expected value.
- Preference aggregation: For each decision node at period \( t \), apply the
  MCDA method to select the best compromise partial strategies. The
  considered strategies are composed of one immediate alternative, pro-
  longed by a partial strategy that starts at the next period \( t + 1 \) and was
  not eliminated at previous iterations. For each decision node at period \( t \),
  eliminate from further analysis the partial strategies that are not locally
  of best compromise.

The proposed recursive approach is illustrated in Fig. 2 and the algorithm pro-
vided in Appendix B.

In this paper, the expected value operator (resp. a MCDA method) will be used
for the aggregation of the evaluations at uncertainty nodes (resp. decision nodes).
The characterization results show that the expected value operator verifies the
theorem of decomposition. However, as mentioned before, not all MCDA methods
verify the same conditions stated in this theorem (see Ref. 1).

### 3.2.2. The myopic approach

The multi-criteria myopic approach consists in solving the defense resource alloca-
tion problem as a multiple criteria decision-making problem while the dynamic
aspect of the problem is not considered. It involves planning only one step ahead and considering information about the current decision period only. At the beginning of each period, the choice of each decision is carried out among the set of feasible actions. A multi-criteria decision-making rule is applied to choose from among these decisions. After the engagement is executed, results of the kill assessment are given. Then, a set of feasible decisions, from among which the next decision will be selected, is identified depending on the outcome measured. Then the decision-making rule is applied once more at the beginning of next period, and so on. The myopic approach provides a strategy corresponding to a sub-tree that describes the decisions to be made for each period. Each decision in the strategy is locally of best compromise. Nevertheless, there is no guarantee that the whole strategy is of best compromise. The myopic approach is illustrated in Fig. 3.
3.2.3. A novel metric for comparing two strategies in a multi-criteria decision tree

In order to compare results obtained with the decomposition and the myopic approaches, a metric for comparison of two strategies in the tree is proposed below. This metric consists of the following three steps:

**Step 1. Identifying the best and worst paths of each strategy**

The best paths are defined as those that dominate at least one path and are not dominated by any other path of the strategy. The worst paths are defined as paths dominated by at least one path and that do not dominate any other path of the strategy. The sets of best and worst paths are necessarily composed of paths that are nondominated among them.

**Step 2. Comparing the best and worst paths of each strategy**

To compare the set of worst and best paths of each strategy, the binary $\varepsilon$-indicator $I_\varepsilon(A,B)$ defined in Zitzler\(^{35}\) is used. This indicator concludes to strong dominance, weak dominance or incomparability of one set with another.

Let $A$ and $B$ be two sets of nondominated paths.

**Strong dominance**: $A \gg \varepsilon B \iff (I_\varepsilon(A,B) < 1)$, \hfill (1)

**Weak dominance**: $A > \varepsilon B \iff (I_\varepsilon(B,A) > 1 \text{ and } I_\varepsilon(A,B) \leq 1)$, \hfill (2)

**Incomparability**: $A ? B \iff (I_\varepsilon(A,B) > 1 \text{ and } I_\varepsilon(B,A) > 1)$, \hfill (3)

with

$$I_\varepsilon(A,B) = \max_{x_B \in B} \min_{x_A \in A} \max_{1 \leq m \leq M} \left\{ \frac{g_m(x_A)}{g_m(x_B)} \right\},$$ \hfill (4)

where $g_m$ for $m = 1, \ldots, M$ are the set of criteria, $x_A$ and $x_B$ are two non-dominated paths in $A$ and $B$.

**Step 3. Using results of Step 2 to compare two different strategies**

Let us consider two different strategies: strategy 1 and strategy 2. Let $W1$ (resp. $W2$) be the set of worst paths of strategy 1 (resp. strategy 2), $B1$ (resp. $B2$) be the set of best paths of strategy 1 (resp. strategy 2). $Card1$ is the number of paths of strategy 1 that dominates the set of best paths of strategy 2. $Card2$ is the number of paths of Strategy 1 that are dominated by the set of worst paths of Strategy 2.

Depending on the results of Step 2, one of the following conclusions are stated: a strong dominance ($D^F$), a weak dominance of level 1 ($D^f$) or level 2 ($D^{ff}$), incomparability (R) of one strategy against another, equivalence or equality of the two strategies.

\begin{align*}
\text{Strategy 1 } & D^F \text{ Strategy 2 } \iff \quad W1 \gg \varepsilon \text{ (or } > \varepsilon \text{) } B2, \quad \hfill (5) \\
\text{Strategy 1 } & D^f \text{ Strategy 2 } \iff \quad B1 \gg \varepsilon \text{ (or } > \varepsilon \text{) } B2 \text{ and } W1 \gg \varepsilon \text{ (or } > \varepsilon \text{) } W2, \quad \hfill (6) \\
\text{Strategy 1 } & D^{ff} \text{ Strategy 2 } \iff \quad B1 \gg \varepsilon \text{ (or } > \varepsilon \text{) } B2 \text{ and } W2 \gg \varepsilon \text{ (or } > \varepsilon \text{) } W1 \text{ and Card1 } > \text{ Card2}, \quad \hfill (7)
\end{align*}
Strategy 1 \( \equiv \) Strategy 2 \( \iff \) \( W_1 \equiv W_2 \) or \( and B_1 \equiv B_2 \), \( (8) \)
Strategy 1 \( \sim \) Strategy 2 \( \iff \) \( W_1 = W_2 \) and \( B_1 = B_2 \), \( (9) \)
Strategy 1 \( = \) Strategy 2 \( \iff \) Strategy 1 and Strategy 2 are identical. \( (10) \)

**Table 2. Strategies.**

<table>
<thead>
<tr>
<th></th>
<th>Strategy 1</th>
<th>Strategy 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best path</td>
<td>((0, 0.65, 0.485))</td>
<td>((0, 0.325, 0.97))</td>
</tr>
<tr>
<td>Worst path</td>
<td>((0.347, 0, 0.541))</td>
<td>((0.352, 0, 1))</td>
</tr>
</tbody>
</table>

**Illustration.** Let us compare strategies 1 and 2 in Table 2. Let \( W_1 \) (resp. \( W_2 \)) be the set of worst paths of strategy 1 (resp. strategy 2), \( B_1 \) (resp. \( B_2 \)) be the set of best paths of strategy 1 (resp. strategy 2). Each of these sets is composed of only one path. The evaluations of these paths are given in Table 2.

The computation of \( \varepsilon \)-indicator gives the following results:

\[
I_{\varepsilon}(B_1, B_2) = \max_{x_B \in B_2} \min_{x_A \in A} \max_{1 \leq m \leq M} \left( \frac{g_m(x_A)}{g_m(x_B)} \right) = \max \{0.51; 1; 0.5\} = 1,
\]
\[
I_{\varepsilon}(B_2, B_1) = \max \{1.92; 1; 2\} = 2,
\]
\[
I_{\varepsilon}(W_1, W_2) = \max \{0.98; 1; 0.51\} = 1,
\]
\[
I_{\varepsilon}(W_2, W_1) = \max \{1.01; 1; 1.94\} = 1.94.
\]
So, \( B_1 > \varepsilon B_2 \) and \( W_1 > \varepsilon W_2 \).
Then, Strategy 1 \( D^f \) Strategy 2.

**4. Problem Modeling and Results**

This section presents the model, gives an illustrative example and discusses the main results. Section 4.1 details the model including the assumptions, the set of actions, the state of the nature and the decision criteria. Then, we present the testbed in Sec. 4.2, followed by an illustrative example in Sec. 4.3, and the discussion of the results in Sec. 4.4.

**4.1. The model**

Let us assume a set of air threats, namely ASM emerging from hostile platforms and directed towards a defending ship (a Frigate). In the operations room, the members of the combat team perceive and interpret the information available to them from sensors and then plan and conduct operations. The operators’ reactions are based on years of experience and training, and follow doctrines and standard operating procedures. In this paper, we will consider a warship equipped with the following type of
hardkill weapons: Surface-to-Air Missile (SAM), medium caliber gun systems, and CIWS. These resources can be fired only after a Fire Control Radar (FCR) has locked onto the target. The SAM and the gun share an independent FCR (such as STIR: Separate Tracking and Illuminator Radar) while the CIWS has its own FCR.

4.1.1. Assumptions

It is assumed that multiple sequential, simultaneous or overlapping random attacks appear from any direction towards the ship. These threats are originating randomly from any azimuth (between $0^\circ$ and $360^\circ$) within a time interval. Threats are supposed to have constant speed and deterministic trajectories. The cruising speed for each threat is a random constant between minimum and maximum speed thresholds ($V_{\text{min}} \leq V \leq V_{\text{max}}$). Threats are launched at any distance from the vessel. The ship is considered at rest or moving at slow speed compared to ASMs. Its geometry and configuration are constant and remain unchanged during engagement. We assume that only hardkill weapons (SAM, GUN, CIWS) will be engaged against threats. The ship is equipped with two SAM launchers, one GUN system, one CIWS and two FCR (STIR-A and STIR-B). GUN and SAM necessitate a FCR to operate, while the CIWS has its self-contained FCR. The defensive weapons are reliable and have constant speed and a straight-line trajectory between the ship and the intercept point. The engagement strategy considered is the SLS. This means that the outcome of a target engagement must be known before another weapon is used against the same target. Meteorological conditions, which are not under control, are supposed ideal and have no effect on the scenario. It will be assumed that the slew time to move the weapon into fire position is negligible and there is no delay between the time the fire order is issued and the weapon starts shooting. After engagement, kill assessment enables knowing which threats have been destroyed in order to cease or continue to allocate weapons to those threats. Probability of kill is assumed constant and independent of target position.

4.1.2. The set of actions

Given that a weapon cannot be used without being guided by FCR, an engagement is considered to be a combination of a weapon type and FCR locking onto a given threat. Each engagement towards threat $th$ is expressed by $[(w, \text{FCR}), th]$ where $w \in \{\text{Missile, Gun, CIWS}\}$, FCR $\in \{\text{STIR-A, STIR-B, CIWS-FC}\}$. There are five possible combinations of weapons with FCR: (Missile, STIR-A), (Missile, STIR-B), (Gun, STIR-A), (Gun, STIR-B), and (CIWS, CIWS-FC). An action comprises one or many simultaneous engagements (performed at the same time) against the same or different threats. A feasible action is an action that verifies all the constraints (ammunition availability, FCR availability, weapon range constraints, blind zones, etc.). In other words, if at least one of these constraints is not satisfied, the alternative is not feasible and will not be considered in the analysis. The blind zones that we considered for the different equipment combinations are presented in Fig. 4.
4.1.3. The state of the nature

After each engagement, a kill assessment is executed to evaluate the consequences. The outcome of kill assessment operation is assumed to be binary, i.e., a total success or a total failure. No partial damage is treated in this study. Consequently, after each engagement, threats are either killed or not killed. The probability of the killed state is calculated by

$$P_K = 1 - P_{NK} = 1 - \prod_{w \in W} (1 - P_w),$$

where $W$ is the set of weapons used simultaneously toward the threat, $P_w$ is the probability of successful engagement of weapon $w$, $P_K$, and $P_{NK}$ are the probability of the outcome “successfully engaged” and “not successfully engaged”, respectively.

4.1.4. The decision criteria

In order to make a decision, feasible actions should be evaluated. To this end, several conflicting decision criteria are relevant. By way of illustration, we will consider three conflicting criteria: risk (RK), effectiveness (Ef), and loss of opportunity (LO). These criteria do not represent necessarily the real decision-making criteria that commanders take into account. They are chosen for academic purposes and not for the realism of the problem formulation.

Criteria #1: Risk (to be minimized)

The risk is associated with the fact that by choosing one decision, we ignore some threats that can damage the ship. This criterion quantifies the risk that the operator takes by not engaging one threat.

The risk of decision $D$ is computed with Eq. (15) where $J$ is the set of threats that are not engaged with decision $D$; $R_{th}$ and $L_{th}$ are, respectively, the range and threat...
level of threat \( th \) when decision \( D \) is chosen (at \( t = \tau \)). The threat level measures the degree of severity of the damage that the threat could cause if not engaged.

The global risk of one threat \( th \) along the whole horizon is computed with Eq. (14) where \( \delta = 1 \) if threat \( th \) is not engaged at period \( \tau \), \( \delta = 0 \) elsewhere, \( R_{th}^\tau \) is the position of threat \( th \) at the beginning of period \( \tau \), \( R_{FC} \) the range of the FCR and \( L_{th}^\tau \) is the threat level at period \( \tau \).

\[
RK(0) = L_0, \quad RK(+\infty) = 0, \quad \overline{RK}(th) = \max_{\tau=1,...,T} \{ \delta \times L_{th}^\tau \times e^{-R_{th}^\tau/R_{FC}} \}, \quad RK(D) = 1 - \prod_{th \in J} (1 - L_{th} \times e^{-R_{th}^\tau/R_{FC}}).
\]

The risk of one path in the tree is computed with Eq. (16) where \( P \) is a path in the tree and \( n \) is the number of threats at period 1.

\[
\overline{RK}(P) = 1 - \prod_{th \in \{t_1,...,t_n\}} (1 - \overline{RK}(th)).
\]

Criteria #2: Effectiveness (to be maximized)

Effectiveness measures the success of a given decision. If the combination of \( N \) weapons (\( w \in W \)) is attacking the same threat, the effectiveness of decision \( D \) is given by Eq. (17):

\[
\text{Eff}(D) = 1 - \prod_{w \in W} (1 - P_w),
\]

where \( P_w \) is the probability of successful engagement of weapons \( w \) and \( n \) the number of threats.

If the combination of \( N \) weapons is attacking different threats, the effectiveness of decision \( D \) is given by Eq. (18):

\[
\text{Eff}(D) = \left( \sum_{w \in W} P_w \right)/N.
\]

The effectiveness of the path \( P \) in engaging the threat \( th \) is calculated by Eqs. (19) and (20).

If threat \( th \) hits the vessel, \( \text{Eff}(P, th) = 0 \); (19)

Else,

\[
\text{Eff}(P, th) = \frac{1}{P_{\text{missile}} + P_{\text{gun}} + P_{\text{CIWS}}},
\]

where \( N_1, N_2 \) and \( N_3 \) are, respectively, the number of missiles, gun, and CIWS used against threat; \( P_{\text{missile}}, P_{\text{gun}}, \) and \( P_{\text{CIWS}} \) are the probability of kill of missiles, gun and CIWS, respectively.
The global path effectiveness (considering all threats) is given by Eq. (21):

\[ \text{Eff}(P) = \sum_{th=t_1,...,t_n} \text{Eff}(P, th). \]  

(21)

Criteria #3: Loss of opportunity (to be minimized)

The LO is measured by the utility the weapons engaged in the current decision could have for future decisions. This utility depends on the rate of resource depletion and the cost of resource replacement. We assumed that the resources are limited with no possible replacement. If a resource is used, it is deducted from the resources available for the next decision.

The LO derived by decision \( D \) is computed using Eq. (22):

\[ \text{LO}(D) = \sum_{w \in W} U(w), \] 

(22)

where \( W \) is the set of weapons used for decision \( D \), \( U(w) \) is the utility of weapon \( w \).

The LO of path \( P \) is given by Eq. (23):

\[ \text{LO}(P) = \sum_{D \in P} \text{LO}(D). \] 

(23)

The weights that were considered for the criteria are 0.4, 0.5, and 0.1 for risk, effectiveness and LO, respectively.

Those three criteria are conflicting. Risk quantifies the risk that the operator incurs by not engaging one threat. The closer the threat and the higher its threat level, the greater the risk we are taking by not engaging it. The greater the risk, the lower the likelihood (expected probability) of being effective, and the less important the future LO (by taking risk, we do not engage and so LO is lower).

4.2. Testbed

In order to simulate the naval defense environment, a testbed was developed with Matlab (see Fig. 5). The interface consists of five parts. Part 1 (top right) illustrates the detected threats positioned around the ship. Part 2 (top left) illustrates the decision tree with all feasible decisions and outcomes. Part 3 (bottom center) and Part 4 (bottom right) provide support to the navigation process within the tree. Part 3 highlights the action (resp. outcome), which is pointed at the decision tree by the black arrow and Part 4 gives the details of this related action (resp. outcome). In particular, Part 4 specifies which FCR is used for the alternative; the estimated time including the kill assessment; the number of munitions remaining; the time when weapons are launched; damage to the ship before the execution of this action; the decision criteria values (RK, Eff and LO) of the given action; and the estimated decision criteria values of the entire strategy, if this action is chosen. Part 5 of the interface (bottom left) illustrates the best compromise strategy obtained by applying either the multi-criteria myopic approach or the multi-criteria decomposition
approach. Both approaches were implemented using the weighted sum, lexico-graphic, TOPSIS and dominance.

4.3. Illustrative example

We consider a frigate equipped with detection radar, two missile launching systems, rapid fire gun, CIWS, two distinct FCR (STIR A and STIR B). Tables 3–5 present the parameters used for all these equipments. Note that table data are not real data but hypothetical assumptions that we made for testing purposes.

Generation of threats is made using the Poisson distribution. Time at which threats appear within the engagement area is generated by Poisson distribution with mean $\lambda = 2$. Then, speed, azimuth, and threat level are randomly generated according to the uniform probability distribution in their respective intervals

---

**Table 3. Parameters of detection radars.**

<table>
<thead>
<tr>
<th>Radar parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{1s}$: Maximum radar range (m)</td>
<td>250,000</td>
</tr>
<tr>
<td>$R_{1s}$: Range at which detection probability is equal to 1 (m)</td>
<td>150,000</td>
</tr>
</tbody>
</table>
\[ V_{\text{min}}, V_{\text{max}}, [0, 360^\circ] \text{ et } [0, 0.5]. \text{ Table 6 presents 3 threats with the parameters generated randomly.} \]

Threat 2 is in the blind zone of STIR A, Threat 3 is in the blind zone of the gun. The tree is generated according to the strategy act-observe-act considering all the constraints. Some paths of the tree are represented in Fig. 6. For instance, the path in the upper part of the tree corresponds to the situation where the 3 threats are killed after the engagements.

The decision tree is then solved using the myopic and the decomposition approach. Figure 7 (resp. Fig. 8) shows the strategy that results from applying the decomposition approach (resp. myopic approach) with the weighted sum as decision rule. Comparison of the strategies provided by the decomposition and the myopic approaches (where the weighted sum is used for decision nodes) indicates \( s_d \cdot D \cdot s_m \) where \( s_d \) and \( s_m \) are the strategies provided by the decomposition and the myopic approaches, respectively.

### 4.4. Results, discussions and implications

Sixty different scenarios were randomly generated (20 scenarios with each of 2, 3, and 4 threats). For each scenario, all feasible strategies are generated. Next, to identify the best compromise strategy, the decomposition (resp. myopic) approach is applied,
Fig. 6. Some paths of the tree with 3 threats.
Fig. 7. Some paths of the strategy resulting from the decomposition approach using the weighted sum.
Fig. 8. Some paths from the strategy resulting from the myopic approach using the weighted sum.
taking the multiple criteria and dynamic aspects (resp. only the multiple criteria aspects) of the decision-making problem into consideration. Then, the strategies yielded by each of them are assessed and compared using the metric proposed in Sec. 3.2.3.

Based on the MCDA characterization done in Ref. 1, we have chosen to use the decomposition and myopic approaches with the following decision-making rules: weighted sum, lexicographic screening, dominance and TOPSIS. We decided on the weighted sum, lexicographic screening and dominance because they verify the four conditions of the theorem of decomposition, which guarantee that decomposition will provide the best compromise strategy. Of those methods, dominance in particular, was taken into account so as to try the decomposition approach with a MCDA method necessitating higher computational time. However, TOPSIS was chosen based on the fact that it does not verify the temporal consistence condition. Our aim here is to compare the decomposition approach with a method that does not verify the sufficient conditions of decomposition and to assess its results compared to the myopic approach.

Let $s_d$ and $s_m$ be the strategies obtained, respectively, with the myopic and decomposition approaches. Table 7 provides the results of the comparison of these two strategies.

### Results obtained with weighted sum, dominance or lexicographic methods:

When weighted sum, dominance or lexicographic methods are used, strategy $s_d$ given by the decomposition approach is always at least as good as strategy $s_m$ given by the myopic approach. Results also show that there is no strong dominance between these two strategies for any scenario. Strategy $s_m$ is either weak dominated, equivalent, equal or incomparable, compared with $s_d$. Results show too that with those specific

<table>
<thead>
<tr>
<th></th>
<th>$s_d = s_m$</th>
<th>$s_d \succeq s_m$</th>
<th>$s_d \succeq s_m^{D^F}$</th>
<th>$s_d \succeq s_m^{D^T}$</th>
<th>$s_d \succeq s_m^{D^U}$</th>
<th>$s_d \succeq s_m^{R}$</th>
<th>$s_d \succeq s_m^{R^T}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 threats</td>
<td>5%</td>
<td>0%</td>
<td>0%</td>
<td>65%</td>
<td>5%</td>
<td>25%</td>
<td></td>
</tr>
<tr>
<td>3 threats</td>
<td>5%</td>
<td>5%</td>
<td>0%</td>
<td>60%</td>
<td>0%</td>
<td>30%</td>
<td></td>
</tr>
<tr>
<td>4 threats</td>
<td>5%</td>
<td>0%</td>
<td>0%</td>
<td>45%</td>
<td>0%</td>
<td>50%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dominance</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 threats</td>
<td>10%</td>
<td>0%</td>
<td>0%</td>
<td>70%</td>
<td>0%</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>3 threats</td>
<td>5%</td>
<td>5%</td>
<td>0%</td>
<td>60%</td>
<td>0%</td>
<td>30%</td>
<td></td>
</tr>
<tr>
<td>4 threats</td>
<td>20%</td>
<td>0%</td>
<td>0%</td>
<td>30%</td>
<td>0%</td>
<td>50%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lexicographic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 threats</td>
<td>5%</td>
<td>0%</td>
<td>0%</td>
<td>70%</td>
<td>0%</td>
<td>25%</td>
<td></td>
</tr>
<tr>
<td>3 threats</td>
<td>5%</td>
<td>5%</td>
<td>0%</td>
<td>60%</td>
<td>0%</td>
<td>30%</td>
<td></td>
</tr>
<tr>
<td>4 threats</td>
<td>20%</td>
<td>0%</td>
<td>0%</td>
<td>30%</td>
<td>0%</td>
<td>50%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TOPSIS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 threats</td>
<td>5%</td>
<td>0%</td>
<td>0%</td>
<td>65%</td>
<td>0%</td>
<td>25%</td>
<td></td>
</tr>
<tr>
<td>3 threats</td>
<td>5%</td>
<td>5%</td>
<td>0%</td>
<td>65%</td>
<td>0%</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>4 threats</td>
<td>30%</td>
<td>0%</td>
<td>0%</td>
<td>20%</td>
<td>0%</td>
<td>50%</td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Comparison of strategies given with multi-criteria decomposition and myopic approaches.
multiple criteria decision-making rules, \( s_m \) never dominates \( s_d \). All these results are consistent with the theoretical results in Ref. 1. In fact, those three decision-making rules verify the decomposition conditions that ensure the decomposition principle is applicable. On the other hand, the myopic approach is not guaranteed to give more than a strategy that is locally of best compromise. And so it can never dominate the decomposition approach which gives a global best compromise strategy.

**Results obtained with TOPSIS:** When TOPSIS is used as multiple criteria decision-making rule, results show that for one scenario with 2 threats and one scenario with 3 threats, strategy \( s_m \) given by the myopic approach dominates strategy \( s_d \) given by the decomposition approach. This is because TOPSIS does not verify the decomposition conditions stated in Ref. 1. So, in this case, the decomposition approach is not guaranteed to provide the best compromise strategy.

**Results obtained with dominance:** When dominance is used as decision-making rule, we conducted only the simulations with 2 threat scenarios to show some results. We have not conducted much simulation with dominance because the objective of our study was to solve the dynamic multi-criteria problem without generating the set of nondominated solutions. Therefore, dominance is not one of the MCDA methods that is interesting to use. In addition, those simulations with dominance are too time-consuming due to the high cardinality of the set of nondominated solutions at each decision node. The limit of number of threats for the simulation is about 10 before an explosion of the tree.

Given all the above-mentioned decision-making rules, one general comment concerns the incomparability results. The results in Table 7 show that the incomparability rate between strategies given by the decomposition and the myopic approaches is relatively high and this rate increases with the number of threats. This is due to the \( \varepsilon \)-indicator used to compare the set of best and worst paths of each strategy when computing the metric proposed in Sec. 4.3. In fact, as the number of threats increases, the cardinality of sets of best and worst paths increases and the likelihood that the \( \varepsilon \)-indicator ends in incomparability increases. By the same token, the results of comparison of these two sets given by the \( \varepsilon \)-indicator are actually more likely to lead to incomparability for big size sets.

Let \( s_{WS} \) and \( s_{Lex} \) be the strategies given by the decomposition approach with the weighted sum and lexicographic screening, respectively. Table 8 summarizes the

<table>
<thead>
<tr>
<th>Threats</th>
<th>( s_{WS} )</th>
<th>( s_{Lex} )</th>
<th>( s_{WS} \leq s_{Lex} )</th>
<th>( s_{WS} D!^1s_{Lex} )</th>
<th>( s_{WS} D!^2s_{Lex} )</th>
<th>( s_{WS} R!^1s_{Lex} )</th>
<th>( s_{Lex} D!^2s_{WS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 threats</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>3 threats</td>
<td>80%</td>
<td>0%</td>
<td>0%</td>
<td>5%</td>
<td>0%</td>
<td>10%</td>
<td>5%</td>
</tr>
<tr>
<td>4 threats</td>
<td>65%</td>
<td>0%</td>
<td>0%</td>
<td>15%</td>
<td>0%</td>
<td>15%</td>
<td>5%</td>
</tr>
</tbody>
</table>
results obtained with the decomposition approach when using the weighted sum and
the lexicographic screening. It shows that the results are not always the same when
using these decision-making rules. Results are the same: 100% (for 2 threats), 80%
(for 3 threats) and 65% (for 4 threats).

Results differ when the decomposition approach is used with different decision-
making rules. There is no tendency. Sometimes, strategy given with weighted sum
weakly dominates strategy given with lexicographic screening and sometimes the
contrary occurs. Therefore, we cannot draw any conclusion as to the superiority of
using the decomposition approach with one decision-making rule over another. The
characterization of these two decision-making rules indicates that each one has good
properties (Ref. 1): neutrality, anonymity, fidelity, dominance and independence.
This difference in the results given by different decision-making rules is a problem
frequently encountered in multiple criteria decision-aid. In fact, whatever be the
decision-making problem, there is no guarantee that you will obtain same results
with two different decision-making rules. Decision-making rules differ in the concepts
they use, the assumptions they make, and in limitations, informational needs, etc.
For example, the methods used here have different compensatory levels, use different
aggregation principles and suppose different assumptions, all of which make it more
likely to reach different results. Therefore, it is most important to choose carefully
what decision-making rule to use.

Table 9 shows that the decomposition approach is producing better decision-
making strategies overall, when compared with the myopic approach. In fact, in only
two of the 60 scenarios, did the myopic approach produce a better decision strategy.
These cases are related to employing TOPSIS at the decision nodes. However, it
must be remembered that TOPSIS does not verify the decomposition conditions
given in Ref. 1.

The computation results in Table 9 are consistent with the theoretical bases of the
decomposition approach as discussed in Sec. 3.2.1. In fact, the weighted sum, dom-
inance and lexicographic methods verify the decomposition conditions and then give
as results strategies that are globally of best compromise when the decomposition
approach is used. However, when TOPSIS is used, the strategy obtained is not
always of best compromise, which is consistent with the theoretical results.

<table>
<thead>
<tr>
<th></th>
<th>Decomposition is better</th>
<th>Myopic is better</th>
<th>Not conclusive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 threats</td>
<td>3 threats</td>
<td>4 threats</td>
</tr>
<tr>
<td>Weighted Sum</td>
<td>75%</td>
<td>70%</td>
<td>50%</td>
</tr>
<tr>
<td>Dominance</td>
<td>80%</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Lexicographic</td>
<td>75%</td>
<td>70%</td>
<td>50%</td>
</tr>
<tr>
<td>TOPSIS</td>
<td>70%</td>
<td>75%</td>
<td>50%</td>
</tr>
</tbody>
</table>
The results presented above can be explained as follows:

— Compared with the myopic approach, the decomposition considers all the information over time not only the evaluation of decisions at the current stage (as the myopic approach does) but also the evaluation of all possible future decisions starting with this current decision.

— The decomposition approach identifies the best compromise strategy for the whole decision horizon, whereas the myopic method does not guarantee to determine the best compromise strategy. This conclusion is driven by theoretical results showed with the theorem of decomposition.

It is important to mention that the approach we present does not take into consideration real-time aspects. Since the planning operation is performed offline, the computational resources are not viewed as a limitation. Also, the results presented were obtained under several assumptions. These results should be validated with more realistic models, and ultimately with real-world data. In future works, comparing the strategy given by the decomposition approach with the strategy given by a human military agent can strengthen conclusions regarding the decomposition approach.

From a military perspective, this work will help in developing tactics for highly complex and dynamic decision-making problems, such as weapon management of a Frigate facing multiple threats. In fact, modeling and simulating multiple high intense scenarios will enable identification of a set of best decision strategies dealing with resource management. These strategies should be validated with subject matter experts and maritime warfare centers. In this way, these strategies might be used to review and update existing tactics. The real-time constraint is, therefore, not a limitation since the proposed approach is not intended for implementation on ships and as support for real-time decision-making. When strategies are derived using the synthetic environment, rule-based or heuristics-based systems might be designed to deal with real-time requirements.

5. Conclusions
This paper addresses dynamic decision-making in a naval engagement context. The objective is to identify strategies of best compromise for weapons allocation against a set of incoming threats over a given period of time, in order to maximize the survivability and security of the ship. This problem has been represented with multiple criteria decision tree. The authors considered two different approaches to solve such a problem: (1) the multi-criteria myopic approach, which takes into consideration the multiple criteria aspect of the problem but not the dynamic one; and (2) the multi-criteria decomposition approach, which considers both aspects.

Sixty different scenarios were randomly generated (20 scenarios, each with 2, 3 and 4 threats). Results show that the decomposition approach is always superior to the myopic approach when sufficient conditions related to the decision-making rule
are verified. These results are predictable since it is guaranteed that the decision strategy identified by the decomposition approach is always of best compromise.

When combined with good synthetic environment models, this work is expected to develop tactics for highly complex and dynamic decision-making problems like weapon management of a Frigate facing multiple threats. Consequently, these decision strategies will be used to review and update existing tactics. In future works, experiments will be undertaken to measure operator (human) performance in addition to algorithm performance. The comparison of the strategy provided by the multi-criteria decomposition approach with the strategy given by a human military agent can strengthen conclusions regarding the decomposition of the tree. We propose to introduce a human agent and to observe his/her choice at each decision period depending on the outcomes of previous decisions. Then, the strategy proposed by the agent will be compared with the strategy of the decomposition approach.

Acknowledgment

This work was funded by Defense Research and Development Canada — Valcartier (DRDC Valcartier).

Appendix A

The weighted sum: consists of computing for each action $a_i$, its overall performance using:

$$V(a_i) = \sum_{j=1}^{n} \pi_j \cdot g_j(a_i),$$  \hspace{1cm} (A.1)

where $\pi_j$ is the weight of criteria $j$ and $g_j(a_i)$ is the evaluation of action $i$ on the criteria $j$.

Lexicographic method

The lexicographic method assumes that we have elucidated the relative importance of the criteria and consists of comparing the alternatives on the most important criterion. If this criterion does not discriminate between the alternatives, then we consider the second most important criteria, the third most important criteria, etc. The method consists of determining the subset $A(1)$ of actions which have the best performance on the most important criteria. Then we determine the subset $A(2)$ of $A(1)$ which has the best performance on the second most important criteria, and so on until the subset has only one element.

Dominance

The dominance concludes that alternative $a_i$ dominates $a_k$ if

$$\begin{align*}
\text{If } a_i &\succeq_{g_m} a_k \quad \forall g_m \in F \\
\text{and } \exists g_l \in F/ a_i &\succ_{g_l} a_k \quad \Rightarrow a_i D a_k,
\end{align*}$$

(A.2)
where \( \succ g_m \) is the relational preference according to criterion \( g_m \) and \( F \) the set of criteria.

**TOPSIS**

The TOPSIS method consists of the following steps

**Step 1.** Construct normalized decision-matrix

\[
\forall i \in [1, \ldots, m], \quad \forall j \in [1, \ldots, n], \quad r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}},
\]

(A.3)

**Step 2.** Construct the weighted normalized decision-matrix

\[
v_{ij} = w_j r_{ij}, \quad i = 1, \ldots, m; \quad j = 1, \ldots, n,
\]

(A.4)

where \( w_j \) is the weight of criteria \( j \).

**Step 3.** Determine the positive ideal \( A^* \) and the negative ideal solutions \( A^- \)

\[
A^* = \{ v_{i1}^*, v_{i2}^*, \ldots, v_{in}^* \}
\]

\[
= \{(\max_i v_{ij}|j \in J_1), (\min_i v_{ij}|j \in J_2)|i = 1, \ldots, m\},
\]

\[
A^- = \{ v_{11}^-, v_{12}^-, \ldots, v_{n1}^- \}
\]

\[
= \{(\min_i v_{ij}|j \in J_1), (\max_i v_{ij}|j \in J_2)|i = 1, \ldots, m\},
\]

where \( J_1 \) is the set of criteria to be maximized and \( J_2 \) is the set of criteria to be minimized.

**Step 4.** Calculate the separation measures for each alternative \( i \):

- The separation from positive ideal alternative,

\[
S_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij}^* - v_{ij})^2}.
\]

(A.5)

- The separation from negative ideal alternative,

\[
S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_{ij}^-)^2}.
\]

(A.6)

**Step 5.** Calculate the relative closeness to the ideal solution,

\[
C_i^* = \frac{S_i^-}{S_i^* + S_i^-}.
\]

(A.7)

Select the alternative with the highest \( C_i^* \).

**Appendix B**

**Notations**

Let us note the following:

- \( n_t \) the total number of decision nodes at period \( t \) and let us use \( k \) as the index associated to these nodes.
- $n(k)$ the total number of uncertainty nodes from decision node $k$ at period $t$ and let us consider $i$ the index associated to these nodes.
- $M$ : the number of decision criteria and $F^T = \{g_1, \ldots, g_m, \ldots, g_M\}$ is the set of decision criteria.
- $f^t(k, i, j)$ is the index of the decision node of period $t + 1$ to which we arrive by passing by decision node $k$, uncertainty node $i$ and state of the nature $j$ of period $t$.
- $A^t_k = \{a^t_{i(1(k))}, \ldots, a^t_{i(n(k))}\}$ is the set of possible actions starting at decision node $k$ of period $t$.
- $E^t_{i,k} = \{e^t_{i(1(k))}, \ldots, e^t_{j(i,k)}, \ldots, e^t_{n(i,k)}\}$ is the set of state of the nature starting from decision $a^t_{i(k)}$.
- $p^t_j$ the probability of state of the nature $j$.
- $V^t_{j(i,k),m}$ is the evaluation of action $a^t_{i(k)}$ according to criteria $m$ when the state of the nature $j$ occurs.
- $\otimes$ is an operator that extends an action with a partial strategy starting at next period.
- $\Gamma^t_k$ is the set of strategies starting at decision node $k$ of period $t$ and which are decomposable into sub-strategies which all of them are of best compromise.
- $\Gamma^t_k$ is the set of strategies of best compromise which result from applying an MCDA method on the set $\Gamma^t_k$.

**Algorithm**

**For** $t = T$

```
For k = 1 to k = n_T do
  For i = 1 to i = n(k) do
    For m = 1 to m = M do
      Compute $g^t_m(a^T_{i(k)}) = \sum_{j=1}^{n(i,k)} p^t_j V^T_{j(m)}$, where $V^T_{j(m)}$ is the evaluation according to criteria $g_m$ at final node $f^T(k, i, j)$ of the tree.
  End For
End For
```

Define $\Gamma^T_k = \{a^T_{i(k)}, i \in \{1, 2, \ldots, n(k)\}\}$,
Define $F^T = \{g_1, \ldots, g_m, \ldots, g_M\}$,
Define $V^T_k = \{g^t_m(a^T_{i(k)}), i \in \{1, 2, \ldots, n(k)\}, m \in \{1, 2, \ldots, M\}\}$,
Compute $\Gamma^T_k = \text{MCDA}[\Gamma^T_k, F^T, V^T_k]$ where $\text{MCDA}[\Gamma^T_k, F^T, V^T_k]$ is the result obtained by the MCDA method when applied on the set of strategies $\Gamma^T_k$, considering the decision criteria $F^T$ and the evaluations $V^T_k$. All strategies which do not belong to $\Gamma^T_k$ are not of best compromise and then will be eliminated from further analysis.

**End For**

(2) For $t = T - 1$ to $t = 1$ do
```
For k = 1 to k = n_t do
  For i = 1 to i = n(k) do
```


For $m = 1$ to $m = M$

$$\gamma_k^t = a_{i(k)}^t \otimes \gamma_{i(k)}^{t+1}$$

where $\gamma_{i(k)}^{t+1} = \begin{cases} 
\gamma_{f^t(k,i,1)}^{t+1} \\
\vdots \\
\gamma_{f^t(k,i,j)}^{t+1} \\
\vdots \\
\gamma_{f^t(k,i,n(k))}^{t+1}
\end{cases}$ and

$$\gamma_{f^t(k,i,j)}^{t+1} \in \Gamma_{f^t(k,i,j)}^{t+1} \forall j.$$ 

$$g_m(\gamma_k^t) = g_m(a_{i(k)}^t) + \sum_{j=1}^{n(i,k)} p_j^t \cdot g_m(\gamma_{f^t(k,i,j)}^{t+1}),$$

where $g_m(\gamma_{f^t(k,i,j)}^{t+1})$ is the evaluation according to criteria $g_m$ of partial strategy $\gamma_{f^t(k,i,j)}^{t+1}$ and $p_j^t$ is the probability of state of nature $j$.

End For

Define $\Gamma_k^t = \{ a_{i(k)}^t \otimes \gamma_{i(k)}^{t+1} | a_{i(k)}^t \in A_{i(k)}^t, \gamma_{i(k)}^{t+1} \in \Gamma_{f^t(k,i,1)}^{t+1} \times \cdots \times \Gamma_{f^t(k,i,n(k))}^{t+1} \}$

Define $F^t = \{ g_1, \ldots, g_m, \ldots, g_M \}$.

Define $V_k^t = \{ g_m(\gamma_k^t), \gamma_k^t \in \Gamma_k^t, m = 1, \ldots, M \}$.

Compute $\Gamma_k^T = \text{MCDA}[\Gamma_k^t, F^T, V_k^T]$, where MCDA[\Gamma_k^T, F^T, V_k^T] is the result obtained by the MCDA method when applied on the set of strategies $\Gamma_k^T$, considering the decision criteria $F^T$ and the evaluations $V_k^T$. All strategies not belonging to $\Gamma_k^T$ are not of best compromise and then will be eliminated from further analysis.

End For

(3) The results obtained by applying the MCDA method at the unique decision node of period 1 on the set $\Gamma_1^1$ is the set of best compromise strategy which consists of the solution of the problem.

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