Heat Transfer Characteristics Between Inner and Outer Rings of an Angular Ball Bearing

Keiji Mizuta, Takayoshi Inoue, Yasuro Takahashi, Shuwei Huang, Kouji Ueda, and Hidefumi Omokawa
Mitsubishi Heavy Industries, Ltd., Hiroshima Machinery Works, Hiroshima, 733-8553 Japan

Heat transfer between the inner and the outer rings of an angular ball bearing is investigated experimentally and heat transport by balls is analyzed theoretically. The bearing used is lubricated by oil and rotated in the range from 600 to 4000 rpm. Considering heat generation by friction, the net heat flow between the rings is evaluated. The results show that balls are the dominant heat carrier and their conductance depends on rotational speed and thrust force. The other heat transfer route is supposed mainly to be between the rings based on the fact that its heat flow rate depends on the rotational speed. © 2002 Wiley Periodicals, Inc. Heat Trans Asian Res, 32(1): 42–57, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/htj.10070

Key words: heat transfer, heat conduction, bearing, rotational motion, EHL, elastic contact, thermal conductance, ball bearing

1. Introduction

The performance of many machines depends mainly on speed and stiffness of the rotating shaft such as motors, engines, turbines, and tooling machines. If the driving condition becomes hard, its temperature usually goes up and becomes one of the causes of seizure or extraordinary wear. In order to predict critical conditions or try to improve its capacity, accurate prediction of the bearing temperature is necessary. The bearing temperature depends on ① heat generation rate from the bearing and other heat sources on the shaft, ② thermal conductance from the inner ring to the housing, ③ heat transfer condition at the housing outer surface. Among them, ① is already being investigated in the field of tribology [e.g., 1–6]. ③ is also becoming fairly predictable through general heat transfer investigation. The heat transfer mechanism of ② has not yet been clarified.

This paper studies the angular ball bearing used as a standard type for a rotational machine requiring high accuracy and high stiffness of the rotating shaft. First, the heat transfer characteristics between the inner and the outer ring are measured experimentally and then the effect of balls as a heat carrier is calculated theoretically.

© 2002 Wiley Periodicals, Inc.
Nomenclature

- $a$: radius of major axis of contact oval area (m)
- $a_c$: thermal diffusivity ($m^2/s$)
- $b$: radius of minor axis of contact oval area (m)
- $b_r$: width of bearing ring (m)
- $C$: thermal conductance (W/K)
- $c$: specific heat (J/(kgK))
- $d$: diameter (m)
- $d_m$: diameter of pitch circle (m)
- $E$: Young’s modulus (Pa)
- $F$: force (N)
- $F_a$: thrust force (N)
- $g$: gravitational acceleration ($m/s^2$)
- $h_m$: oil film thickness (m)
- $N$: rotational speed ($r/s$, $r/min$)
- $n_b$: number of balls (—)
- $Q$: heat flow rate/heat generation rate (W)
- $R$: thermal resistance (K/W)
- $r$: radius of curvature (m)
- $T$: temperature (K, °C)
- $v$: velocity (m/s)
- $\alpha$: heat transfer coefficient ($W/(m^2K)$)
- $\beta$: thermal expansion rate ($1/K$)
- $\Delta$: gap between contact tracks on ball surface (m)
- $\delta$: sunk depth of ball center contacting with ring (m)
- $\theta$: angle (rad, deg)
- $\lambda$: thermal conductivity ($W/(m-K)$)
- $\rho$: density ($kg/m^3$)
- $\tau$: time (s)
- $\omega$: angular velocity (rad/s)

Subscripts

- $a$: direction in rotational axis
- $b$: ball/auto rotation axis
- $c$: centrifugal force/center of revolutional track/contact
- $H$: external heating
- $i$: inner ring
- $o$: outer ring
- $r$: radius direction/both inner and outer ring
- $t$: tangential direction
- $1$: (external heating) + (friction heat)/inner side
- $2$: (friction heat)/outer side
2. Experimental Method

Figure 1 shows representative dimensions of a tested bearing cross section. Nineteen steel balls are supported by steel rings and a polyimide retainer. Figure 2 shows the structure of the experimental equipment. The inner ring rotates and the outer ring is fixed. Thrust force is loaded on the outer ring, and is measured by load cell. The outer ring is pressed into a cooling ring which has water cooling grooves. A thermocouple is situated at the center of the outer surface of the outer ring. The inner ring is also pressed into a cooling support ring placing a thermocouple on the center of the inner surface. The inner ring support has a groove-shaped cone. The heating water is fed to the end of the groove and flows out through the cone surface by centrifugal force and is collected into the

Fig. 1. The representative dimensions of tested bearing.

Fig. 2. Structure of the experimental equipment.
jacket around the brim. The bearing housing is covered with insulator. An air gap is kept between the support rings and housing to maintain an insulated condition. An oil–air system is used as a lubrication method to supply a certain amount of oil carried with air flow. The oil has a kinematic viscosity of $22 \times 10^{-6} \text{ m}^2/\text{s}$ at 37.8 °C. The oil feed rate is changed as one of the test conditions. Both the heat flow rate to the outer ring and the temperature of rings are measured; the inner ring is heated and insulated. Table 1 shows the thermal properties of the bearings parts and oil used in the experiments. These are also used in the further theoretical analysis.

### 3. Experimental Results

Figure 3 shows the heat flow rate to the outer ring without heating the inner ring. This heat is considered to be generated by friction between balls and rings. The following approximate equation is obtained by the least squares method applied to these plots:

$$Q_b = 0.126 (4.5N^{0.1} + 33 \times 10^{-6}N^{5/3})F_a^{0.3}$$

Here the exponent in the second term in parentheses on the right-hand side is nearly equal to 1.658 which is calculated by multiplying the rotational speed N to Aihara’s value of 0.658 [6] in his experimental work for torque and rotational speed.
Thermal conductance is defined as the heat flow rate at a unit temperature difference between the inner surface of the inner ring and the outer surface of the outer ring. This thermal conductance is analyzed by extracting pure heat flow rate from the inner ring to the outer ring measuring when the inner ring is heated and thereby reducing the friction heat in Eq. (1). This conductance is shown by the following equation:

\[
C_{io} = \frac{1}{R_o + R_{ob} + R_b + R_{ib} + R_i} = \frac{Q_o - Q_b}{(T_{i1} - T_{o1}) - (T_{i2} - T_{o2})}
\]

Here the following assumptions are used. (1) Friction heat is generated equally both at the ring and the ball surface. (2) Difference of heat generation between the case where the inner ring is heated and insulated is neglected. Figures 5 to 8 plot the relationships between rotational speed and \(C_{io}\). Thrust force is varied in each figure. The right-hand longitudinal axis shows dimensionless conductance obtained by dividing \(C_{io}\) by the conductance of the imaginary thick ring where the inner and the outer

---

**Fig. 4.** The conceptual view of friction heat and thermal resistance model.

**Fig. 5.** Thermal conductance and rotational speed (\(F_a = 50 \text{ N}\)).

**Fig. 6.** Thermal conductance and rotational speed (\(F_a = 150 \text{ N}\)).

46
radius are equal to those of the actual inner and outer ring, respectively, and made of the same material. This dimensionless conductance is obtained as

\[ C_{\text{io}} = C_{\text{io}} / (2 \pi b_1 / \ln(d_0/d_1)) \]  

(3)

Figures 5 to 8 show that \( C_{\text{io}} \) increases with rotational speed, but its inclination decreases gradually. \( C_{\text{io}} \) also increases with thrust force. From Fig. 6, the oil feed rate does not show any clear effect on \( C_{\text{io}} \) within the standard range.

4. Theoretical Evaluation

4.1 Assumption of heat transfer route

The balls and the air or the oil are presumed to be the representative carrier of heat from inner to outer ring. The role of the balls is (1) at first to catch heat from the inner ring at their contact surface, (2) next to roll about a half revolution, and (3) then to release the heat to the outer ring at their contact surface. However, the contact point of the ball with the inner ring may not necessarily contact the outer ring. In other words, two separated contact tracks for inner and outer rings can exist. In this case, thermal conduction resistance between the two contact tracks must be considered on the ball.

The contact area, oil film thickness at the contact area, time interval for the contact, and properties for thermal diffusion at the contact track are considered the dominant parameters for heat transfer. Thermal conductance from the inner to the outer ring is expressed by estimating these parameters respectively from former experiments or numerical analysis and by connecting them in series.

4.2 Theoretical model

The following assumptions of the forces loaded on the balls are used. (3) Traction force is ignored and the reaction force at the contact surface and the centrifugal force on the balls are
considered. (4) Thrust force is loaded evenly on all balls and radial force is ignored. In this environment the balance of the forces loaded on the balls are expressed as follows using the symbols shown in Fig. 9.

In radial direction: \[ F_{or} = F_c + F_{ir} \] (4)

In axial direction: \[ F_a = F_{ir} \tan \theta_i = F_{or} \tan \theta_o \] (5)

From Eqs. (4) and (5), the relationship between two contact angles \( \theta_i \) and \( \theta_o \) is expressed as follows:

\[ \tan \theta_o = \frac{F_a}{\tan \theta_i} = \frac{F_{or} + F_c \tan \theta_i}{F_{or}} \] (6)

Another restriction for \( \theta_i \) and \( \theta_o \) is a geometric relationship. \( \theta_o \) is expressed as follows since the normal directions of the ball and the outer ring surface must coincide with each other:

\[ \tan \theta_o = \frac{(x_{bc} - x_{oc})}{(y_{bc} - y_{oc})} \] (7)

The coordinates of the ball center are expressed as follows using \( \theta_i \) since the ball must inscribe to the inner ring surface curvature.

\[(x_{bc}, y_{bc}) = \left\{ -\left( r_{ia} - \frac{d_b}{2} \right) \sin \theta_i, r_{ia} + \left( r_{ia} - \frac{d_b}{2} \right) \cos \theta_i \right\} \] (8)

The coordinate of the y component of the curvature center in the axial direction of the outer ring is expressed as follows using the symbols in Fig. 9:

\[ y_{oc} = r_{oa} - r_{oa} \] (9)

Fig. 9. Symbols used in the analysis.
The $x$ component is expressed as follows considering that the distance between the ball center and the curvature center in the axial direction of the outer ring must coincide with the difference of each radius length:

$$x_{oc} = x_{bc} - \sqrt{\left(r_{oa} - \frac{d_b}{2}\right) - (y_{oc} - y_{bc})^2}$$  \hspace{1cm} (10)

The relation between $\theta_i$ and $\theta_o$ is expressed as follows substituting Eqs. (8) to (10) for Eq. (7):

$$\tan \theta_o = \sqrt{\left(\frac{r_{oa} - d_b}{2}\right)^2 - \left[r_{ia} + r_{ia} - r_{oa} - \left(r_{ia} - \frac{d_b}{2}\right) \cos \theta_i \right]^2}
\hspace{1cm} (11)$$

$\theta_i$ and $\theta_o$, which satisfy Eqs. (6) and (11) are the actual contact angles. These solutions are calculated numerically by the Newton–Raphson method giving each force $F_a$ and $F_c$. The normal components of the contact force are expressed as follows using $\theta_i$ and $\theta_o$:

$$F_i = F_a / \sin \theta_i$$ \hspace{1cm} (12)

$$F_o = F_a / \sin \theta_o$$ \hspace{1cm} (13)

The ball revolution speed $\omega_{bc}$ and the auto rotation speed $\omega_{bb}$ are expressed as follows [8] using symbols in Figs. 9 and 11 supposing (5) that both $\theta_i$ and $\theta_o$ are fairly close to the gradient of auto rotation axis:

$$\omega_{bc} = \{1 - (d_b/d_m) \cos q_b\} w_c/2$$ \hspace{1cm} (14)

$$\omega_{bb} = \{(d_m/d_b) - (d_b/d_m) \cos^2 q_b\} w_c/2$$ \hspace{1cm} (15)

The balls and rings contact each other elastically, and the contact area could be considered as oval. The radius length of the oval $2a$, $2b$ and the sunk depth of the ball center $\delta$ are estimated using Hertz’s formula [9, 10].

As for the oil film thickness at the contact area, the representative film thickness $h_m$ is expressed by using the experimental equation for elastohydrodynamic lubrication (EHL) based on Westlake–Cameron [11]. Here (6) the oil is a Newtonian fluid (except for grease). Thermal resistance at the contact area is expressed assuming (7) the thickness profile of the oil film is flat over all of the contact area. $\delta$ and $h_m$ are calculated numerically using $F_i$ and $F_o$ and are used to correct the actual curvature radius of the contact track as follows:

$$r_{it}' = r_{it} - \delta_i + h_m$$  \hspace{1cm} (16)
This correction is iterated until the radius is converged.

The heat flow rate and thermal conductance as averages over time during the contact interval are based on the previous contact area, oil film thickness, and rotational speed. Initially, contact time between the inner ring and the ball is expressed using contact length in revolution direction and rotational speed as follows:

\[ t_{ci} = 2b'/\left\{ r_{i} + (1 - \cos q_i) r_{i}' \right\} D_{ib} \]  \hspace{1cm} (20)

Here \( \Delta \omega_{ib} \) denotes the difference of rotational speed between the inner ring and the ball revolution and is expressed using Eq. (14) as follows:

\[ \Delta \omega_{ib} = \omega_c - \omega_{bc} = \frac{\omega_c}{2} \left( 1 + \frac{d_b}{d_m} \cos \theta_b \right) \]  \hspace{1cm} (21)

Interval time for a contact is expressed as

\[ \tau_{cic} = \frac{2\pi}{n_b \Delta \omega_{ib}} \]  \hspace{1cm} (22)

Interval times for the outer ring and ball are expressed in the same way as

\[ \tau_{co} = 2b_o'/\left\{ r_{o} - (1 - \cos q_o) r_{oa}' \right\} w_{bc} \]  \hspace{1cm} (23)

\[ \tau_{coc} = \frac{2\pi}{n_b \omega_{bc}} \]  \hspace{1cm} (24)

Concerning the heat flow rate between ball and rings, the following assumptions are used. Initially, as the thermal diffusion depth during the contact interval is less than 1 ms as a practical condition, it is usually thinner than the thickness of the rings. (8) The contact wall thickness is assumed to be semi-infinite. The thermal conductivity of oil is generally extremely less than that of the rings and the ball since the oil film thickness is less than the depth of thermal diffusion. (9) Therefore, the thermal resistance of oil film is considered and its heat capacity is ignored. (10) Contact cycle time is sufficiently longer than contact time. Under these assumptions, contact heat flow is estimated as an unsteady thermal conduction problem in a semi-infinite thickness body when the surface is heated or cooled under steady contact heat transfer conditions. Therefore, the heat flux concerning the inner ring and ball \( q_{ib} \) is expressed as follows [12]:

\[ q_{ib} = \frac{1_i}{\sqrt{n_i \alpha_i T_{ci} + 1_i/a_{bii}}} (T_i - T_{bi}) \]  \hspace{1cm} (25)
Here

\[ T_{bi} = \frac{r_1 C_1 T_i + r_b C_b T_b}{(r_1 C_1 + r_b C_b)} \]  

(26)

\[ a_i = \frac{\lambda_i}{(\rho_i C_i)} \]  

(27)

\[ \alpha_{bii} = \left( \frac{l_i}{d_n} \right) \left( \frac{(T_i - T_b)}{(T_i - T_{bi})} \right) \]  

(28)

The transferred amount of heat \( Q_i \) during contact time \( \tau_{ci} \) is calculated as follows by integrating \( q_{ib} \) for \( \tau \) and multiplying by the contact area and the number of balls:

\[ Q_i = \frac{S_i n_b}{\tau_{ci}} \frac{21_i(T_i - T_{bi})}{\pi a_{ci}} \times \left\{ \frac{l_i}{\sqrt{\frac{p a_{ci} l_i}{a_{bii}}} - \frac{1}{a_{bii}}} \right\} \left( 1 + \frac{a_{bii} \sqrt{p a_{ci} l_i}}{l_i} \right) \]  

(29)

Using the surface temperature just before the contact, thermal resistance and conductance between the contact surfaces are expressed as follows:

\[ C_{ib} = \varepsilon \frac{Q_i}{(T_i - T_b)} \]  

(30)

\[ R_{ib} = 1/C_{ib} \]  

(31)

Here \( \varepsilon \) is the correction factor considering the error of \( Q \) caused by ignoring the initial temperature gradient in the wall; \( \varepsilon \) is calculated numerically under representative conditions where the initial temperature gradient is given. An example of the calculation is shown in Fig. 10. In this case \( \varepsilon \) is nearly 0.8 which is almost constant throughout the experimental range. They are identical for the outer ring.

Next, the thermal resistance between two contact tracks on the ball has to be considered. First the gap between tracks on the ball is calculated and then the thermal resistance is calculated.

Fig. 10. An example of temperature profile calculation.
numerically by FEM. Because the contact surface is curved, the edge and the center in a contact area have a relative velocity gap and slip past each other. As the slip direction is reverse at both sides of the major axis in contact oval as shown in Fig. 11, it generates the friction moment. This moment acts on the ball to accelerate rotation velocity at the plus side of the x axis in 2ωc direction, and to decelerate at the minus side. Therefore, it acts on the auto rotation axis to incline in the revolution axis direction. Similar force acts on the reverse direction in the inner ring contact area. The angle of the auto rotation axis is the balance point of these forces. To calculate this angle, the contact area is approximated to a rectangular figure with a’ × b’ side lengths the area of which is equal to that of the oval one:

\[ a' = (\sqrt{p}/2)a \]  
\[ b' = (\sqrt{p}/2)b \]  

Then the friction moment is expressed as follows:

\[ M = 4b' \int_{a}^{a'} \eta f h_m (v_b(x) - v_i(x)) \, dx \]  

Here the circumferential speed of the ball and the inner ring are expressed as follows when \( v_b = v_i \) where \( x = 0 \):

\[ v_b(x) = \frac{d_b}{2} \cos(\theta_i - \theta_b + \frac{x}{d_b}/2)w_{bb} \]  
\[ v_i(x) = \left[ r_{ia} + r_{ia} \left\{ 1 - \cos \left( \theta_i + \frac{x}{r_{ia}} \right) \right\} \right] w_i \]  

Here \( \omega_i \) is the inner ring circumferential speed based on the ball center, and is expressed as follows when \( v_b = v_i \) where \( x = 0 \):

Fig. 11. Symbols used in the contact orbit analysis.
where \( \theta_b \) is the solution for inclination of the auto rotation axis against the revolution axis. \( \theta_b \) is solved numerically. From the contact angles \( \theta_o, \theta_i \) and the inclination of the auto rotation axis, the gap \( \Delta x \) between contact track on the ball for the inner ring and the outer ring is expressed as

\[
\Delta x = (2\theta_b - \theta_i - \theta_o)db / 2
\]
Fig. 12. FEM model for thermal conductance analysis of rings.

Fig. 13. FEM model for thermal resistance analysis of ball.

Fig. 14. Results of FEM analysis for ring thermal conductance.
boundary condition, and those results are used for comparison with the experimental results. Figure 15 shows the analytical results for the balls. The horizontal axis is the dimensionless gap between two contact tracks made by contact track width. The longitudinal axis is the dimensionless thermal resistance made by \( \Delta x/(\pi db\Delta x\lambda_b) \) where \( \pi db\Delta x \) is the heat transfer area, \( \Delta x \) denotes heat transfer thickness, and \( \lambda_b \) denotes thermal conductivity. Thermal resistance is shown to change depending on the gap of contact tracks and on the contact heat transfer coefficient.

4.3 Analytical results for overall thermal resistance between rings

Figures 16 to 19 show analytical results for thermal resistance between the inner and the outer rings. Figure 16 shows thermal resistance where the thrust force is 150 N. \( R_{ib} \) and \( R_{bo} \) are seen to be dominant regardless of rotational speed. These decrease with rotational speed up to 2000 rpm. This is caused by the increasing contact heat flow rate depending on \( \tau_{ci}^{-1/2} \) as in Eq. (29). At higher rotational speed, \( R_{ib} \) increases gradually. This is caused by the increase of oil film thickness and decrease of contact width as shown in Fig. 17. Thermal resistance \( R_b \) in the ball is comparatively small, but the gap of two contact tracks shows a complex curve profile. This is caused by the fluctuation of inclination of the auto rotation axis which causes replacement of two contact tracks to the opposite sides of each other. Figure 18 shows that the thermal resistance is less than that of Fig. 16 where the thrust force is lower. This is caused by the increase of contact width and the decrease of oil film thickness caused by higher contact force. Thermal resistance in the ball is comparatively small for these conditions. Here contact heat transfer coefficient and contact width for the inner and the outer rings are supposed to be equal in the analytical model shown in Fig. 13. These assumptions are considered to be proper from the calculation results for oil film thickness and contact width shown in Figs. 17 and 19 as long as the rotational speed is less than 4000 rpm. However, it is noteworthy that the difference of the contact width increases gradually with the rotational speed where speed is over 4000 rpm and the error of the analytical model increases.

4.4 Comparison of experimental and analytical results

Dependence on the oil feed rate is not noticeable from Fig. 6 within the experimental range from 0.02 to 1.0 cc/h. It shows that it is proper to use the EHL model assuming that the film thickness
does not depend on the feed rate. It also shows that the oil film adhering and rotating with the ball does not affect the heat transfer measurably.

This can also be presumed from the geometrical estimation for contact angle at the edge of the contact area. That is, when the ball is sinking in the ring by Hertz contact pressure, the angle takes a relatively high value, and as the gap is filled, oil becomes the dominant thermal resistance because of its low thermal conductivity, and the area which can contribute to heat transfer is limited in such a narrow ring area.

The analytical results for thermal conductance $C_{io}$ which is the inverse of thermal resistance are shown as curves in Figs. 5 to 8. The results calculated considering only balls as the heat carrier show a constant difference versus experimental plots in the figures. This means that the cause of the difference does not depend on the thrust force. However, the difference increases with rotational speed gradually. Thus, the difference could be caused mainly by air between the rings which is stirred by the balls and the retainer. As an example, the analytical results for the stationary condition are shown in Figs. 5 to 8 by an arrow on the longitudinal axis. These are calculated by FEM considering heat conduction in the ball’s diameter direction and ignoring oil film resistance which is comparatively

![Analytical results for contact conditions (Fa = 150 N).](image1)

![Components of thermal resistance (Fa = 1000 N).](image2)

![Analytical results for contact conditions (Fa = 1000 N).](image3)
small. These values fit well with the experimental plots. These results show that the cause of the
difference during rotation does not act on the stationary bearing.

5. Conclusions

Thermal conductance between the inner and the outer ring of an angular ball bearing is clarified
experimentally by excluding friction heat flow from total heat transfer to the outer ring and compared
with analytical results considering only balls as the heat carrier.

The following conclusions are obtained.

1. Thermal conductance depends mainly on rotational speed and thrust force. It goes up with
rotational speed, but inclination of the curves decreases gradually. However, the conductance for the
stationary condition is higher than what is extrapolated from plots for the rotating condition.

2. Balls are considered to be the most dominant heat carrier under the oil lubrication
condition, and the most dominant thermal resistance is the contact area between the ring and the ball.

Adding to the above results, air stirred between the rings is considered another heat carrier
since the thermal conductance depends on the rotational speed.

Precedent investigations in this field are rarely seen, but thermal conductance of a bearing is
always required when designing a high-grade rotation machine. Therefore, extended investigations
are needed under such conditions as higher speed level, bearing under radial load, roller bearing,
sliding bearing, conductance between gears, and couplings.

Literature Cited

5. Eguchi M, Yamamoto K. Proceedings of Spring Meeting of Japan Lubrication Society, 1981,
p. 137.

Translated by Keiji Mizuta, Mitsubishi Heavy Industries, Ltd., Hiroshima Machinery Works, 6-22, 4-chome, Kan-on-shin-machi, Nishiku, Hiroshima, Hiroshima, 733-8553 Japan.