Effect of the shear intensity on fouling in submerged membrane bioreactor for wastewater treatment

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Abstract

Air sparging is widely used to minimize membrane fouling within submerged membrane bioreactor (SMBR) applied to wastewater treatment. This paper discusses its effectiveness in hollow-fibre membrane modules and its relationship with permeate flux, backwashing frequency and duration and main biomass characteristics. The effect of air sparging is expressed as shear intensity $G$ which enables to describe the influence of several hydrodynamic parameters (viscosity, air sparging area and air flow-rate) on membrane fouling. The experimental study was carried out with sludge at four different biomass concentrations (MLSS = 4100–14,500 mg l$^{-1}$) filtered under a broad range of hydrodynamic conditions ($J = 20–63$ l h$^{-1}$ m$^{-2}$; $G = 0–375$ s$^{-n}$). Under constant filtration conditions, the slope of TMP against time, the fouling rate, is described by an exponential function of $G$: $r_f = (r_f)_0 \exp\left(-F_G G\right) + (r_f)_l$, where shear intensity sensitivity factor ($F_G$) enables quantification of effectiveness of air sparging and limit fouling rate ($r_f)_l$ describes the fouling caused by adsorption of micro-colloidal and soluble fractions over the external membrane surface. Also, it has been found that this sensitivity factor is a decreasing function of the imposed permeate flux and the biomass concentration.

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1. Introduction

Nowadays membrane bioreactor (MBR) has become a feasible option for the treatment and reuse of municipal wastewater. However, the economic viability of MBRs depends on a careful control of membrane fouling. This complex process is determined by the interaction among membrane, operating and hydrodynamic conditions and the microbial suspension characteristics including suspended particles and liquid phase. These biomass characteristics are often described as the mixed liquor suspended solid concentration (MLSS), viscosity, particle size and extracellular polymeric substances (EPS) [1].

Since the origin of the SMBR, air sparging has been widely used to mitigate fouling by constant scouring of the membrane surface [2] or by causing fibre lateral movement in hollow-fibre configuration [3]. While the membrane fouling has been studied and mathematically modelled in classic filtration regimes (cross-flow and dead-end) [4,5], the effect of turbulence induced by air sparging in SMBR systems is still being assessed [1,6]. An attempt to calculate cross-flow velocity along the membrane as a function of the aeration intensity has been done [7]. However, cross-flow velocity also depends on dimensional parameters of the bioreactor. More recently, aeration turbulent shear intensity [8] and shear stress over densely packed fibre [6] have been used in mathematical development of filtration models. In these cases, theoretical analysis did not consider non-Newtonian nature of sludge. Therefore, a deeper understanding is extremely important in order to minimize fouling problem and optimise aeration rate, which has been proven to be the major operation cost.

Rheological properties are of crucial importance due to its effect on hydrodynamic conditions near the membrane. Activated sludge has been reported as non-Newtonian fluid [9,10] which rheology can be described by the Bingham model, the Ostwald model and the Herschel–Bulkley model represented by Eqs. (1)–(3).

$$\mu_a = \frac{\tau_0}{dv/dr} + m$$ (1)
\[ \mu_a = m \left( \frac{dv}{dr} \right)^{n-1} \]  
\[ \mu_a = \frac{\tau_0}{dv/dr} + m \left( \frac{dv}{dr} \right)^{n-1} \]  

In these models \( \mu_a \) is the apparent viscosity, \( dv/dr \) is the shear rate and \( \tau_0, m \) and \( n \) are the models parameters. From the models it is deduced that the apparent viscosity can be described as a shear rate function. This behaviour can be explained by the sludge structure. As shear stress increases, the structure is opened and biological aggregates are reorganized resulting in a viscosity decrease. Also, a thixotropic nature is commonly accepted in microbial suspensions, which implies that viscosity decreases with time when samples are exposed to a constant shear rate. Finally, it has been reported that the sludge particle concentration has a strong influence on apparent viscosity. This behaviour has been modelled by expressing rheology models parameters as particle concentration functions \([9]\).

The aim of this work is to study the effectiveness of hydrodynamics in hollow-fibre membrane modules and its relationship with the permeate flux, the filtration mode (ratio between backwashing parameters: frequency and duration) and the main biomass characteristics including MLSS and viscosity.

2. Theoretical approach

In order to mitigate membrane fouling, continuous power input is required. A relationship between power dissipation and velocity gradient was deduced by Camp and Stein \([11]\). In this section, shear intensity expression is deduced by analogy with the shear rate in a one-dimensional and laminar shear flow for a non-Newtonian fluid as follows.

Considering that power is force applied times velocity, the power dissipated \( P_w \) per unit volume \( V \) of an infinitesimal element can be expressed as:

\[ \frac{dP_w}{dV} = \tau \left( \frac{dv}{dr} \right) \]  

where \( \tau \) is the shear stress and \( v \) is the local velocity of fluid and \( r \) is the coordinate normal to velocity vector.

At this moment, power expended in a finite volume unit can be written as:

\[ \frac{P_w}{V} = \mu_a \left( \frac{dv}{dr} \right)^2 \]  

where \( \mu_a \) is the apparent viscosity and velocity gradient \( (dv/dr) \) is the same as Camp and Stein’s shear intensity \( G \), which can be rewritten for non-Newtonian fluid as:

\[ G = \left( \frac{P_w}{V \mu_a} \right)^{1/2} \]  

In submerged membrane bioreactors, the power supplied to the system can be due to different sources: air sparging, pumping or/and mixing. Since the aim of this work is to study the effect of air bubbling on membrane fouling it is only considered the power dissipated by the air flow-rate.

As the bubbles rise in the stagnant suspension, the liquid ahead of it is accelerated and moves sideways. Then, the power dissipated is given by:

\[ P_w = \rho_s Q_a gh \]  

where \( Q_a \) is the air flow-rate, \( \rho_s \) is the sludge density, \( g \) is the constant of gravity and \( h \) is the height of the liquid-phase covered by the bubbles.

Finally, the shear intensity linked to air-sparging can be expressed as:

\[ G = \left( \frac{\rho_s g Q_a}{A \mu_a} \right)^{1/2} \]  

where \( A \) is the cross-sectional air sparging area.

3. Material and methods

Two units were operated: a pilot SMBR unit (ZW10) and a laboratory unit (ZW1).

3.1. Pilot-SMBR system

A pilot SMBR plant (ZW10) was located in the wastewater treatment plant of Santa Cruz de Tenerife (Canary Island, Spain) and fed with screened (2.5 mm) municipal wastewater. The pilot consisted of a cylindrical 2201 SMBR equipped with a submerged hollow-fibre membrane of 0.03 \( \mu \)m rated pore diameter and 0.93 m\(^2\) filtering surface area supplied by Zenon Environmental. The effluent was extracted from the top header of the module under slight vacuum. Membrane fouling was mitigated by air sparging which was fixed at 3.7 N m\(^3\) h\(^{-1}\) m\(^{-2}\), expressed as aeration rate per membrane area (SAD\(_m\)). SMBR was operated at different sludge retention time (SRT) ranging from 8 to 33 days. In the bioreactor, air was supplied through the bottom at 8.4 N m\(^3\) h\(^{-1}\) providing oxygen and stirring for biological process. The dissolved oxygen concentration was always above 1.5 mg l\(^{-1}\) in both reactors operated at 23 ± 2℃. Samples of the microbial suspensions from pilot-scale SMBR treating municipal wastewater were taken and filtered in a laboratory filtration unit under different hydrodynamic conditions. Each suspension was characterized by particle size, viscosity and MLSS.

3.2. Bench filtration unit

The other experimental unit consists on an automatic filtration module ZW1, with a central hollow-fibre membrane (ZeeWeed\textsuperscript{®}, Zenon) of 93 mm in length, 0.03 \( \mu \)m rated pore diameter, 3.4 mm external diameter and 0.093 m\(^2\) of filtering area. ZeeWeed\textsuperscript{®} is a composite membrane with a polymeric external surface supported by a macroporous polymer. All the experiments are carried out at constant permeate flux, registering transmembrane pressure as a function of time. The unit operates in two modes: filtration and backwashing (Fig. 1). Each filtration cycle finishes when a pre-established transmembrane pressure is reached (42.6 kPa), beginning the backwash cycle immediately.
Table 1
Operating conditions and biological suspensions characteristics of pilot scale SMBR

<table>
<thead>
<tr>
<th>Run</th>
<th>F/M (kg COD kg(^{-1}) MLSS day(^{-1}))</th>
<th>HRT (h)</th>
<th>SRT (days)</th>
<th>COD(^+) (mg l(^{-1}))</th>
<th>MLSS (mg l(^{-1}))</th>
<th>MLVSS (mg l(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.282</td>
<td>18.2</td>
<td>8</td>
<td>50</td>
<td>4075</td>
<td>3700–4440</td>
</tr>
<tr>
<td>2</td>
<td>0.202</td>
<td>15</td>
<td>20</td>
<td>52</td>
<td>7157</td>
<td>6470–8130</td>
</tr>
<tr>
<td>3</td>
<td>0.162</td>
<td>13.3</td>
<td>25</td>
<td>53</td>
<td>10050</td>
<td>9240–10820</td>
</tr>
<tr>
<td>4</td>
<td>0.180</td>
<td>12</td>
<td>33</td>
<td>56</td>
<td>14430</td>
<td>12780–17100</td>
</tr>
</tbody>
</table>

\(^+\) Samples were filtrated through filter paper with a nominal pore size of 0.45 \(\mu\)m.

Fig. 1. Experimental filtration unit, ZW1.

afterwards. All the filtration experiments were carried out within a period of a few hours (maximum 12 h).

3.3. Analytical methods

COD, MLSS, MLVSS were determined according to the Standard Methods (APHA, 1992). Proteins were determined as bovine albumin equivalent using the protein kit assay TP0300 supplied by Sigma, following the Lowry–Peterson method [12]. Polysaccharides were measured as glucose equivalent by the Dubois’ method [13]. Microbial floc size was measured by Coulter LS100 (Coulter, UK).

The sludge rheological properties were determined by using the concentric cylinder rotational viscosimeter Visco Star plus (FungiLab, Spain). The width of the annular gap was 1.0 mm. Measurements were done at 25 \(^\circ\)C.

4. Results and discussion

4.1. Biomass characterization

The biological suspensions which fed the filtration unit were obtained from a pilot-scale submerged membrane bioreactor (SMBR) that operated at different SRT values: 8, 20, 25 and 33 days. Regardless of operating conditions, a high organic matter removal (>96%, expressed as COD) and complete nitrification was obtained in the bioreactor. Table 1 lists operating conditions and each suspension characteristics at steady-state conditions for each experimental run.

Working at different SRT generates sludge with different biomass concentration, but in order to compare filtration behaviour, other floc characteristics should be taken into consideration. The important role of the granulometric distribution has been found in the literature [14]. To investigate floc size of biomass with different SRT, the particle size distribution of activated sludge was measured. No significant differences in particle size were observed. Mean size were 45 \(\mu\)m (SRT = 25 days), 57 \(\mu\)m (SRT = 20 days), 50 \(\mu\)m (SRT = 8 days) and 49 \(\mu\)m (SRT = 33 days). Those results seem to indicate that particle size does not depend on SRT in our operating conditions.

Also, sludge apparent viscosity has been measured. Fig. 2 shows one example of apparent viscosity reduction with the shear intensity, being detected decreases down to 75% when the shear varied from 13 to 130 s\(^{-1}\). Also, Fig. 2 shows the plots according to the Bingham, Ostwald and Herschel–Bulkley models. In general, the Ostwald model provided better results, probably due to its easier parameter evaluation under different biomass conditions. Recently, it has been reported that when fitting the Herschel–Bulkley model, the yield stress parameter varies over a wide range and its determination would require specific tests based on extrusion procedures [15]. Therefore, the Ostwald model was chosen as the most suitable for describing sludge rheology which is expressed as follows:

\[ \mu_a = m(G)^{n-1} \] (9)

where \(n\) is the flow behaviour index, \(m\) is the consistency index and \(G\) is the shear intensity.

The effect of particle concentration on the viscosity has been evaluated by fitting the specific parameters, flow behaviour index and consistency index, to the sludge concentration, measured as MLSS concentration (Fig. 3). The best results were obtained with non linear regression of both parameters to an exponential-

Fig. 2. Apparent viscosity against the shear intensity.
power law for the consistency index and a power law for the flow behaviour index, as reported elsewhere [9,10,15]. Therefore, the following equation (Eq. (10)) can estimate the apparent viscosity:

$$\mu_a = \exp(1.71\text{MLSS}^{0.45})G^{(-0.068\text{MLSS}^{0.81})}$$  \hspace{1cm} (10)

4.2. Membrane fouling characterization: TMP profiles

In SMBR with hollow-fibre module membrane, it is frequent to operate under consecutive filtration/backwashing cycles. Fig. 4 shows TMP evolution with elapsed time under consecutive cycles, which linearly increased during the filtration phase in all the runs. Then, TMP evolution can be described by the following equation:

$$\text{TMP} = \text{TMP}_0 + r_f t$$  \hspace{1cm} (11)

Slope of the straight line TMP against filtration time, fouling rate ($r_f$), has been used in many works as fouling quantification parameter in systems operated under constant permeate flux (for instance, refs. [16,17]). Typically, this fouling rate reaches a stationary value after initial cycles. This behaviour is due to a cake development mechanism against membrane wall.

On the other hand, it is observed that initial transmembrane pressure $\text{TMP}_0$ increases after each backwash phase with time until reach a stationary value (Fig. 4). Also, $\text{TMP}_0$ can be related to hydraulic total resistance ($R_t$) using the Darcy’s law:

$$\text{TMP}_0 = J\mu R_t$$  \hspace{1cm} (12)

where $R_t$ can be considered as the sum of the initial membrane resistance ($R_m$) and an additional resistance associated with residual fouling phenomena after backwashing ($R_f$):

$$R_t = R_m + R_f$$  \hspace{1cm} (13)

similar as defined by Howell et al. [16] for intermittent permeation.

4.3. Residual fouling ($R_f$)

Shear intensity ($G$) and backwashing time ($t_r$) influence on residual fouling resistance ($R_f$) have been studied.

4.3.1. Effect of shear intensity

According to theoretical approach and assuming that apparent viscosity can be described by the Ostwald model, the shear intensity is defined as:

$$G = \left(\frac{\rho_s g Q_a}{Am}\right)^{1/(n+1)}$$  \hspace{1cm} (14)

Fig. 4. TMP vs. operation time under consecutive filtration/backwashing cycles.
Fig. 5. Residual fouling resistance against number of cycles; \( J = 321 \text{h}^{-1} \text{m}^{-2} \), \( t_r = 0.5 \text{ min} \), SRT = 20 days.

Also, \( m \) and \( n \) are the biomass concentration MLSS functions, according to the experimental equations (Fig. 3):

\[
m = \exp(1.71\text{MLSS}^{0.45})
\]

\[
n = 1 - 0.068\text{MLSS}^{0.81}
\]

Air sparging area is calculated assuming a cylindrical layout of vertical fibres which are fixed at both ends in the module. Air bubbles rise uniformly from the module bottom. Then, the sparging area is calculated:

\[
A = \frac{\pi}{4}d_s^2 - n_f \frac{\pi}{4}d_f^2
\]

where \( d_s \) is the module diameter, \( n_f \) is the number of fibres and \( d_f \) is the outer fibre diameter.

At different values of \( G \), \( R_f \) increases with cycles until reach a unique stationary value (Fig. 5), being this value shear intensity independent \((2.52 \times 10^{12} \text{m}^{-1})\). Also, as much higher is \( G \) fewer are the backwashing cycles required in order to reach that stationary value. This fact could be due to the effect of \( G \) values over frequency of cycles. On the experimental filtration set, backwashing cycles were automatically initiated once a transmembrane pressure was reached, and consequently, the frequency is a result of cake development rate.

From the present results it can be concluded that the hydrodynamics, measured as \( G \), seems to have not significant influence on stationary residual fouling \((R_f)\) in our experimental conditions.

4.3.2. Effect of backwashing time

Fig. 6 shows that when backwashing time \((t_r)\) increases, the stationary fouling resistance \((R_f)\) decreases. Consequently, for a given shear intensity, backwashing is a successful method to remove, at least partially, internal clogging caused by microcolloid and soluble matter. Nevertheless, even though more residual fouling is expected to be decreased when backwashing duration is increased, optimization of backwashing is needed in order to reduce energy and permeate flux consumption costs \([18]\).

Fig. 6. Residual fouling resistance against cycle number; \( J = 321 \text{h}^{-1} \text{m}^{-2} \), \( G = 167 \text{ s}^{-1} \), SRT = 25 days.

4.4. Cake fouling

Slope of the straight line TMP against time, which represents cake fouling rate \( r_c \), can be related with the shear intensity \( G \).

For an imposed permeate flux, fouling rate decreases as shear intensity \( G \) increases. It is observed an increment on the fouling rate in the first three or four cycles, being this behaviour more pronounced for lower \( G \) values \((0–76 \text{ s}^{-1})\). Then, the fouling rate becomes stable in a value that depends on hydrodynamics.

Fig. 7. Stationary residual fouling resistance against \( t_r \); \( J = 321 \text{h}^{-1} \text{m}^{-2} \), \( G = 157 \text{ s}^{-1} \), SRT = 25 days.
where \((r_f)_0\) is the fouling rate in absence of shear intensity, \(G\) is the shear intensity, \(F_G\) is the shear intensity sensitivity factor and \((r_f)_l\) is the limit fouling rate.

\(F_G\) describes the effectiveness of the shear intensity on the cake fouling rate. In addition, this factor can be related to other operating conditions. The higher is \(F_G\), the stronger is the impact of these conditions on the effectiveness. \((r_f)_l\) is the fouling rate value where fouling can not be decreased by increasing shear intensity. Finally, \((r_f)_0\) represents the subtraction between the fouling rate in absence of turbulence \((G = 0\text{ s }^{-1})\) and the limit fouling rate. In the following sections it is presented the effect of permeate flux and biomass concentration over these coefficients.

4.4.1. Effect of the permeate flux imposed

Fig. 8a shows the fouling rate as a function of the shear intensity at different permeate fluxes. As expected, higher fluxes result in higher fouling rates. Also, it has been found that the permeate flux produces a significant effect on the shear intensity sensitivity factor. Similar results have been found with different biomass concentrations (Fig. 9). These results indicate that the effectiveness of the shear intensity decreases with flux (34% from 20 to 421 h\(^{-1}\) m\(^{-2}\)) until being shear intensity independent.

On the other hand, it has been found that \((r_f)_l\) exponentially depends on permeate flux (Fig. 10). For high shear intensity values \((G > 160\text{ s }^{-1})\) it is expected that the fouling rate is mainly caused by adsorption of micro-colloidal and soluble fraction over the external membrane surface. For this reason, these results were compared, at the same experimental conditions, with those obtained while filtering liquid-phase obtained by solids separation from the sludge (through 0.45 \(\mu\)m filter paper). Main characteristics (COD and polymeric extracellular substances EPS) of the liquid-phase and permeate are presented in Table 2, where a considerable membrane rejection which justifies the fouling is observed. Fig. 10 shows a similar behaviour between limit fouling rate \((r_f)_l\) (black squares in figure) of sludge suspension and the fouling caused by the same suspension previously filtered \((r_f)_l\), (white squares) which confirms the physical significance of the \((r_f)_l\) parameter.
<table>
<thead>
<tr>
<th></th>
<th>COD (mg l(^{-1}))</th>
<th>Polysaccharide (mg l(^{-1}))</th>
<th>Protein (mg l(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Filtrate Permeate</td>
<td>Filtrate Permeate</td>
<td>Filtrate Permeate</td>
</tr>
<tr>
<td>Mean</td>
<td>53 27</td>
<td>7.9 4.3</td>
<td>17.1 8.8</td>
</tr>
<tr>
<td>Minimum</td>
<td>42 16</td>
<td>5.3 2.6</td>
<td>15.5 4.8</td>
</tr>
<tr>
<td>Maximum</td>
<td>64 53</td>
<td>10.8 5.3</td>
<td>19.3 10.9</td>
</tr>
</tbody>
</table>

It is generally accepted that even for very low fluxes, complete absence of fouling rate may be never obtained when filtering real or synthetic sewage [19]. From the results of fouling rate at different fluxes (Fig. 10), it can be distinguished flux value where over it the fouling increases at unacceptable rate. This value, whose determination is indeed quite subjective, has been defined as sustainable flux and it is related to the economic and operational sustainability of the process [20]. Therefore, an “apparent” sustainable flux can be determined (in the range of 32–351 l h\(^{-1}\) m\(^{-2}\)) based on short-term experiments. Obviously, a deeper investigation in long-term experiments at a pilot-scale should be done in order to confirm the evaluated sustainable flux.

Over a similar range of permeate fluxes (22.3–33.3 l h\(^{-1}\) m\(^{-2}\)) cake fouling rates were lower than those of 139–444 Pa min\(^{-1}\) (1.39–4.44 mbar min\(^{-1}\)) reported from a pilot-scale hollow-fibre SMBR in a long-term experiments during the sustainable filtration time [21].

Finally, through Eq. (18) and fitted non-linear coefficients \((r_f)_{fb}, F_G\) and \((r_f)_{h}\), the fouling rate performance under a broad range of hydrodynamic conditions can be globally assessed (Fig. 8b).

### 4.4.2. Effect of biomass concentration

Biomass concentration influence on fouling was also studied by measuring the fouling rate under different aeration conditions (Fig. 11a). For lower values \((G < 150 \text{ s}^{-n})\) the fouling rate increases with suspended solid concentration. However, as turbulence becomes higher, fouling rate decreases until become independent of the solid concentration and \((r_f)\) takes values between 1.5–3.0 Pa s\(^{-1}\). Tardieu et al. [22] reported similar behaviour by microscopic observation of the membrane surface. At low circulation velocity (0.5 m s\(^{-1}\)) biological flocs were deposited, while at high recirculation conditions (4 m s\(^{-1}\)) the flocs are not deposited onto the membrane and the fouling is caused by soluble and colloidal fractions. Fig. 12 shows a higher sensitivity factor of shear intensity at low biomass concentration (4000–7000 mg l\(^{-1}\)). However, this effectiveness decreases with increasing the particle concentration.

As it is done with the permeate flux, through Eq. (18) and the fitted coefficients, the effect of both biomass concentration and shear intensity on fouling rate performance can be assessed simultaneously (Fig. 11b).
5. Conclusions

- Air sparging enables to reduce significantly the cake fouling rate (slopes of TMP against time), but no considerable effect on the residual fouling resistance (TMP) has been appreciated. However, a pronounced effect of backwashing time on residual fouling has been observed.
- In order to generalize the system analysis, it seems interesting to introduce the shear intensity $G$, which takes in account the effect of several parameters on the different hydrodynamic conditions as air sparging flow-rate, viscosity (considered as non-Newtonian fluid and biomass concentration function) or area of air sparging. Moreover, mean fouling rate at steady-state can be well expressed by an exponential equation: $r_f = (r_f)_0 \exp(-(F_G) + (r_f))$.
- Under high turbulence conditions ($G > 160$ s$^{-n}$) membrane fouling is only caused by the micro-colloidal and soluble fraction regardless of the biomass concentration or the permeate flux imposed.
- The shear intensity sensitivity factor $F_G$ can be used to quantify the effectiveness of hydrodynamic conditions over fouling rate.

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\[
\begin{align*}
R_m & : \text{membrane resistance (m}^{-1}) \\
R_t & : \text{hydraulic total resistance (m}^{-1}) \\
t & : \text{time (s)} \\
\text{TMP} & : \text{transmembrane pressure (Pa)} \\
\text{TMP}_0 & : \text{initial transmembrane pressure (Pa)} \\
V & : \text{volume of the liquid phase (m}^3) \\
\end{align*}
\]

\textbf{Greek letters}

- $\mu$: solvent viscosity in Eq. (13) (Pa s)
- $\mu_a$: apparent viscosity (mPa s)
- $\rho_s$: suspension density (kg m$^{-3}$)
- $\tau$: shear stress (mPa)
- $\tau_0$: yield stress (mPa)

\textbf{References}


\textbf{Nomenclature}

- $A$: air sparging area (m$^2$)
- $d_l$: outer fibre diameter (m)
- $d_k$: module diameter (m)
- $dv/dr$: shear rate (s$^{-1}$)
- $F_G$: shear intensity sensitivity factor (s$^n$)
- $g$: constant of gravity (m s$^{-2}$)
- $G$: shear intensity (s$^{-n}$)
- $I$: permeate flux (l h$^{-1}$ m$^{-2}$)
- $m$: consistency index (mPa s$^n$)
- MLSS: biomass concentration (mg l$^{-1}$)
- $n$: flow behaviour index
- $n_t$: number of fibres
- $p$: pressure (Pa)
- $P_w$: power dissipated (W)
- $Q_a$: air flow-rate (m$^3$ s$^{-1}$)
- $r_f$: fouling rate (Pa s$^{-1}$)
- $(r_f)_0$: substitution of fouling rate in absence of shear intensity and limit fouling rate (Pa s$^{-1}$)
- $(r_f)_1$: limit fouling rate (Pa s$^{-1}$)
- $R_f$: residual fouling (m$^{-1}$)
- $(R_f)_s$: stationary fouling resistance (m$^{-1}$)


