Suppression of brake squeal noise applying finite element brake and pad model enhanced by spectral-based assurance criteria

Yi Dai a, Teik C. Lim b,*

a Haldex Brake Products Inc., Research and Development Center, 103 Echlin Blvd., Prattville, AL 36067, USA
b University of Cincinnati, Department of Mechanical, Industrial and Nuclear Engineering, 598 Rhodes Hall, P.O. Box 210072, Cincinnati, OH 45221, USA

Received 22 September 2003; received in revised form 21 July 2006; accepted 28 September 2006
Available online 28 November 2006

Abstract

An enhanced dynamic finite element (FE) model with friction coupling is applied to analyze the design of disc brake pad structure for squeal noise reduction. The FE model is built-up from the individual brake component representations. Its interfacial structural connections and boundary conditions are determined by correlating to a set of measured frequency response functions using a spectral-based assurance criterion. The proposed friction coupling formulation produces an asymmetric system stiffness matrix that yields a set of complex conjugate eigenvalues. The analysis shows that eigenvalues possessing positive real parts tend to produce unstable modes with the propensity towards the generation of squeal noise. Using a proposed lumped parameter model and eigenvalue sensitivity study, beneficial pad design changes can be identified and implemented in the detailed FE model to determine the potential improvements in the dynamic stability of the system. Also, a selected set of parametric studies is performed to evaluate numerous design concepts using the proposed dynamic FE model. The best pad design attained, which produces the least amount of squeal response, is finally validated by comparison to a set of actual vehicle test results.

Keywords: Brake squeal; Brake pad design; Finite element brake model; Spectral-based assurance criteria

1. Introduction

Eliminating or substantially reducing braking related noises can prevent passenger complaints and significantly reduce warranty costs. In general, brake noise can be classified into numerous categories based on the occurring frequencies and excitation sources [1–4] as shown in Fig. 1. Among the different types of noise signatures, squeal noise, because of its higher frequency contents, persists to be one of the major concerns throughout the automotive industry in spite of much research and development work performed [5–24] in last few decades. One of the main theories behind the squeal occurrence, as summarized in Fig. 2, comprises of three distinct stages given by frictional vibration, triggering state, and resonance response [1–3,25].

When braking is applied, the frictional vibrations at the rotor–pad interface may be triggered given a set of favorable conditions. The excitation generated subsequently induces modal response in the brake system, which is the source of squeal noise. This phenomenon is known as mode coupling where two component modes, that are geometrically matched, will coalesce when friction contact is introduced. The resultant system mode can be triggered under suitable pressure, temperature, speed and humidity combinations. During the triggering state, two forms of friction mechanisms have been postulated, namely the stick-slip [5–7] and sprag-slip [8–10] theories. The actual occurrence may be due to either one of these mechanisms or a combination of both types. In both forms, the mechanism gives rise to an apparent negative damping effect acting along the tangential direction of rotor rotation, which can cause self-excited vibrations leading to squeal response.
To address the squeal noise problem, a significant number of previous studies have focused on examining brake system level design concepts, including investigations that have led to modifications made to the caliper stiffness \cite{11,12}, mounting bracket \cite{13,14}, and rotor geometry \cite{15,16}. In most cases, the analyses attempted to apply the modal participation factor of each component to assess their potential effect on squeal noise occurrences. Since the modal participation factors of larger size structures, such as the rotor and caliper, are typically higher than those of the smaller components such as the pad, and minor hinges and brackets, the effects of the lesser components have not received much attention even though they may be more economical and feasible to tackle in practice.

While modal participation factors do play a significant role in squeal noise, it may not be the only factor. Therefore, in contrast to previous work, this paper examines the tuning of selected brake pad parameters to determine the effectiveness in squeal reduction. In fact, most brake pad manufacturers currently rely on trial-and-error approaches to address brake noise concerns, which are not only time consuming and costly, but may not result in the best sets of remedies. The present study directly address this need by developing a sufficiently reliable brake dynamic modeling tool that can be applied to evaluate the effects of pad structural design parameters on brake system vibrations and subsequently squeal noise generations.

In recent years, the finite element (FE) method \cite{1,23,26–29} has gained wide acceptance for analyzing many varieties of complex problems in structural dynamics, including some of the more intricate brake vibrations and noise problems. Although successful cases studies have been reported from time to time, no real reliable brake noise prevention tool truly exist yet. In fact, recent literature reviews \cite{2,3} reported on the complexity and lack of understanding of the brake squeal problem. Furthermore, to the best of

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{m,n} )</td>
<td>damping coefficient</td>
</tr>
<tr>
<td>( K_{m,n} )</td>
<td>stiffness coefficient</td>
</tr>
<tr>
<td>( v_{m,n} )</td>
<td>displacement</td>
</tr>
<tr>
<td>( M_m )</td>
<td>mass</td>
</tr>
<tr>
<td>( I_m )</td>
<td>moments of inertia about pivotal center</td>
</tr>
<tr>
<td>( N_m )</td>
<td>braking force normal to disc face</td>
</tr>
<tr>
<td>( P_m )</td>
<td>pivotal center</td>
</tr>
<tr>
<td>( r_m )</td>
<td>radius of contact from pivotal center</td>
</tr>
<tr>
<td>( \theta_m )</td>
<td>angle of contact between pad and disc face</td>
</tr>
<tr>
<td>( \mu_m )</td>
<td>friction coefficient</td>
</tr>
<tr>
<td>( F_f )</td>
<td>generated friction force between pad and rotor</td>
</tr>
</tbody>
</table>

### Subscripts

- \( m \): a, b or d, indicates pad A, B or rotor
- \( n \): 1, 2 or 3, indicates normal, parallel or torsional direction

---

Fig. 1. Brake noise classification based on the frequency range of occurrence and excitation source.

Fig. 2. Primary factors affecting brake squeal mechanism.
our knowledge, no publicly reported prior analysis that tackle the pad dynamics problem to determine suitable designs with lesser tendency towards squeal production has been performed. This paper attempts to fill the gap by addressing two important challenges in developing a feasible analysis tool: (i) validating the structural connections and boundary conditions critical to the true dynamic representation, and (ii) modeling the rotor–pad friction coupling mechanism essential to the self-excited excitation source. To tackle the first issue, a set of cross-point frequency response functions (FRF) is acquired to quantify the vibration transmissibilities across the relevant brake component interfaces. These response functions are then used in a spectral-based assurance criterion to evaluate the degree of model correlations achieved for the stationary case given a set of connectivity and boundary conditions.

For the second issue on rotor–pad friction model, a dynamic FE model with friction coupling that is incrementally more complex than the existing friction theories is proposed. This approach enables the effects of both structural modes and frictional excitation to be analyzed simultaneously. This modeling scheme differs from previous lumped parameter dynamic models [30–33] used mostly to examine the effects of global system parameters on self-excitation causing squeal, where the detailed pad design parameters that are of interest to the brake friction manufacturers are not typically analyzed.

To ease final FE modeling, the friction coupling formulation is first developed using a theoretical lumped parameter system similar to classical theories. This first order structural dynamic approximation is employed to determine the effects of system level modifications and also to guide initial pad design. The preliminary concept is then refined using the dynamic FE model. As a final step, vehicle level experiments are conducted to verify the final pad geometry and parameters that are least susceptible to squeal. The overall approach is summarized in Fig. 3. More detail discussions of each step shown in the flowchart will be given in the relevant parts of this paper below.

2. Stationary brake system

A stationary FE model of the brake assembly for a light truck application is developed using a dynamic finite element software package [34,35]. The model consists of rotor, caliper, mounting bracket, piston and brake pads as shown as an exploded view in Fig. 4. The usual process of FE modeling that involves generating the individual component models of the brake system and validating them by comparison to experimental modal analysis results under free boundary conditions, which was done in this study, will not be discussed in detail here. These validated FE component models are then used to construct the stationary system model. The final system assembly is accomplished by applying the appropriate structural couplings and boundary conditions. The nature of these constraints is determined based on the physical design, and tuned to match measured cross-point frequency response functions (FRF). The degree of correlation is verified by applying a spectral-based assurance criterion. Details of these analysis steps are discussed next.

2.1. Cross-point FRF experiment

A set of cross-point frequency response functions (FRF) are acquired to provide a basis to tune and correlate the disc brake structural coupling representations. The functions needed are measured using a multi-channel data acquisition setup as shown in Fig. 5a. The brake system of interest is set up on support stands with the front tire

---

**Fig. 3.** Proposed pad structural dynamic design evaluation and optimization flowchart.
removed to allow accessibility to the rotor-caliper assembly for application of the impact hammer excitation force. This test fixture is carefully designed to simulate the actual boundary conditions seen by each brake component. Nine measurement points scattered around the holding end of the disc assembly are tested. Table 1 list the specific FRF
terms measured, while the corresponding measurement points are illustrated in Fig. 5b. There are two types of transfer functions acquired. The ones labeled as ‘X’ are cross-point FRF used to guide structural coupling formulation, and those designated by ‘O’ represent FRF pairs confined within an individual component for use in determining structural damping and the nature of its boundary conditions.

2.2. Spectral-based assurance criterion

To assess the validity of the structural connectivity and boundary condition representations of the disc brake assembly, a spectral-based assurance criterion is proposed. The applicability of this scheme in the analysis of the coupling between different components in the brake system is intended to enforce the principle coupling mechanisms in the static brake assembly. For joints properties in operating conditions, the resultant connectivity may need to be refined. The connecting points between the adjacent components are made using a series of node-to-node infinitesimal translation linear spring elements. The equivalent damping is represented by employing a material damping formulation \([36]\). Their parameters are determined by correlating the coupled model results to the vibration transfer across the coupling elements that is represented indirectly by the measured cross-point FRF. Specifically, the number of connections, their locations and the stiffness values are altered iteratively until reasonable correlations are attained. The proposed spectral-based assurance criterion is actually derived from the classical modal assurance criterion (MAC) \([37]\):

\[
\text{MAC}(\{\phi_{\text{Test}}\}, \{\phi_{\text{FEM}}\}) = \frac{|\{\phi_{\text{Test}}\}^T \{\phi_{\text{FEM}}\}|^2}{\langle \{\phi_{\text{Test}}\}^T \{\phi_{\text{Test}}\} \rangle \langle \{\phi_{\text{FEM}}\}^T \{\phi_{\text{FEM}}\} \rangle},
\]

(1)
where \( \{ \Phi_{\text{Test}} \} \) and \( \{ \Phi_{\text{FEM}} \} \) stand for measured and predicted mode shapes, respectively. The MAC value can be used to compare the degree of similarity between any two pairs of mode shapes. For two identical mode shape vectors, the value of MAC is identically 1.0, whereas for two unrelated mode shapes, a MAC value close to 0.0 is expected.

From this fundamental MAC theory, the spectral-based assurance criterion is formulated. This is accomplished by replacing the mode shape \( \{ \Phi \} \) with a response function \( \{ H \} \) in Eq. (1) since \( \{ H \} \) is in fact dependent on \( \{ \Phi \} \). This substitution yields the individual spectral-based assurance criterion (ISAC) given by,

\[
\text{ISAC}(\{ H_{T,k} \}, \{ H_{F,k} \}) = \frac{|\{ H_{T,k} \}^T \{ H_{F,k} \}|^2}{|\{ H_{T,k} \}^T \{ H_{T,k} \} | |\{ H_{F,k} \}^T \{ H_{F,k} \} |},
\]

(2)

where \( \{ H_{T,k} \} \) and \( \{ H_{F,k} \} \) are the measured and FE predicted \( k \)th cross-point response functions. Eq. (2) essentially provides a means to evaluate the correlation level of a specific pair of cross-point response functions. However, in the model updating process, change in any structural parameter will have an influence over all cross-point response functions. Hence, an overall measure of the correlation levels accomplished for a sufficiently large set of cross-point functions is needed to better assess the accuracy of the complete assembly model. In our analysis, 26 cross-point FRF functions given in Table 1 are utilized. Using these FRF functions, a system response vector \( \{ \psi(\omega) \} \) can be formed as

\[
\{ \psi(\omega) \} = \{ H^1(\omega), H^2(\omega), H^k(\omega), \ldots, H^{26}(\omega) \},
\]

(3)

where \( H^k(\omega) \) is the \( k \)th cross-point FRF response at a specified frequency \( \omega \). By further substituting the FRF vector \( \{ H \} \) in Eq. (2) with this system response vector \( \{ \psi \} \), an overall spectral-based assurance criterion (OSAC) can be formulated as,

\[
\text{OSAC}(\{ \psi_{\text{FEM}}(\omega) \}, \{ \psi_{\text{Test}}(\omega) \})
\]

\[
= \frac{|\{ \psi_{\text{FEM}}(\omega) \}^T \{ \psi_{\text{Test}}(\omega) \}|^2}{|\{ \psi_{\text{Test}}(\omega) \}^T \{ \psi_{\text{Test}}(\omega) \}| \{ \psi_{\text{FEM}}(\omega) \}^T \{ \psi_{\text{FEM}}(\omega) \}},
\]

(4)

where \( \{ \psi_{\text{FEM}}(\omega) \} \) and \( \{ \psi_{\text{Test}}(\omega) \} \) are predicted and measured system response vectors, respectively. Note that the proposed OSAC function is frequency dependent. Hence, it provides a means to evaluate the correlation level for a given set of transfer functions in frequency domain.

\[\text{Experimental brake system setup}\]

\[\text{Original FE system model}\]

\[\text{measured cross-point FRF}\]

\[\text{Assessment using ISAC criterion applied to individual FRF pair}\]

\[\text{FE predicted cross-point FRF}\]

\[\text{Assessment using OSAC criterion applied to a set of FRF functions}\]

\[\text{Structural coupling model refinement}\]

\[\text{Correlated FE system model}\]

Fig. 6. Proposed procedure for refining structural coupling representation to achieve correlation between predicted and test results.
Based on these proposed assurance criteria that employ FRF as the input function, the FE parameters including coupling stiffnesses, coordinates and positions are revised iteratively until satisfactory fit is obtained between measured and predicted response. The overall structural coupling correlation step is summarized in Fig. 6. The assumption used in the correlation process here is that the measured data is exact and that any difference between predicted and measured data is due to discrepancies in the FE model. The experimental data, which is measured from a brake assembly with 56 mm twin piston caliper for a 8000-lb gross weight truck, becomes the target for the FE model response. The predicted FRF functions are recomputed each time the FE model is updated in the effort to achieve match with the experimental target functions. Fig. 7 illustrates the typical level of final correlation.

Fig. 7. Sampled cross-point FRF between the bracket and caliper: (a) \( b_1 - y/c2 - z \) and (b) \( b_2 - z/c1 - x \) (solid line, measured; dashed line, original FE model; dot-dashed line, correlated FE model).
attained for a couple of cross-point FRF functions between the bracket and caliper. The percentage improvements in the ISAC and OSAC functions for the bracket–pad, caliper–pad and caliper–bracket models are given in Table 2. Overall, reasonably good correlation is observed. For the ISAC and OSAC functions, the improvements are more than 20% in most cases. The resultant structural coupling representations are summarized in Table 3 with the coupling points illustrated in Fig. 8. This correlated FE model is later combined with a friction coupling representation to analyze squeal behavior.

### 3. Friction coupling model

In this section, the friction coupling mechanism is first analyzed using a simplified lumped parameter model to determine the most feasible form of friction parameters. The refined friction terms are subsequently added to the correlated FE model for simulating the dynamic phenomenon causing brake squeal. These processes are described next.

#### 3.1. Lumped parameter model

Since the friction force does not portray reciprocal behavior, that is the displacement in the normal direction generates a tangential force (friction force), but the tangential displacement does not generate a normal force (contact force), a set of time-invariant coefficient, non-reciprocal stiffness elements $K_f$ is proposed. These spring elements, uniformly distributed across the entire region of rotor–pad surface in contact as shown in Fig. 9, represent the friction coupling. Although an evenly distributed spring coupling model is assumed, the actual load distribution is typically a non-linear (hyperbolic) function. This effect was measured by Tumbrink using the ball pressure method [38]. Due to this nonlinear load distribution, a contact angle $\theta$ exists at the pad and rotor interface as illustrated in Fig. 10. This pad–rotor pin contact representation is formulated into a lumped parameter model as shown in Fig. 11 for use to investigate the roles of the friction coupling mechanism and damping levels on system response. The relevant system parameters shown in Fig. 11 are defined in the Nomenclature list and will be described in the formulation given later. It may be noted that a physical description of the friction mechanism theorized as the sprag-slip action that controls the pad and disc dynamic interaction was initially given by Spurr [8]. More recently, a number of other investigations [6,7,9,10,24] have suggested a kinematic constraint or geometrically induced concept as the primary mechanism of vibration instability causing the disc brake to squeal. Our proposed theory based on a double-pin on disc formulation as shown in Fig. 11 is more sophisticated compared to both existing theories as it includes a contact model based on the nonlin-
ear load distribution and fairly detail pad–rotor representation. In this proposed contact model, the pads are pivoted against a pair of rigid contact pins designated as $P_a$ and $P_b$ that is enforced by a pair of unknown constraint normal force given by $N_a$ and $N_b$. Note that, these pivots are only able to transfer normal force exclusively. The friction coupling springs $K_f$ are evenly distributed over the entire contact surface as described earlier, and the resultant contact forces, $N_{f(a)}$ and $N_{f(b)}$, are located under the pad centers $O_a$ and $O_b$. The friction coupling sets are capable of transferring both normal and lateral forces. The equations of motion for this coupled system can be shown to be

$$m_a \ddot{x}_a + C_a \dot{x}_a + K_a x_a + N_{f(a)} = -N_a,$$

$$m_b \ddot{x}_b + C_b \dot{x}_b + K_b x_b = \Delta F_{f(b)},$$

$$I_a \ddot{\theta}_a + C_a \dot{\theta}_a + K_a \theta_a - \Delta F_{f(a)} r_a \cos(\theta_a) = N_a r_a \sin(\theta_a),$$

$$m_b \ddot{\theta}_b + C_b \dot{\theta}_b + K_b \theta_b = \Delta F_{f(b)} r_b \cos(\theta_b),$$

$$m_d \ddot{x}_d + C_d \dot{x}_d + K_d x_d - N_{f(a)} + N_{f(b)} = N_a - N_b.$$  

Note that the variation of the friction force $\Delta F_f$ is proportional to the variation of the normal force $\Delta N_f$, and the friction coefficient $\mu$ is given by $\Delta F_f = \mu \Delta N_f$. Moreover,
Fig. 10. Schematic of contact angle \( \theta \) due to non-linear (hyperbolic) load distribution: (a) pad without chamfer, and (b) pad with chamfer. Here, \( r \) is the distance between the centroids of pressure load and brake force.

Fig. 11. Schematic of the proposed lumped parameter model of the rotor–pad assembly for analyzing the effect of friction excitation (– linear spring; — linear damper; — rotary spring; — rotary damper).
the variation of the normal force \( \Delta N_f \) is linearly dependent on the deformation of the springs represented by the product of \( K_f \) and the relative displacement \( \Delta X \) of the pad with respect to the rotor:

\[
\Delta N_f = K_f \Delta X = K_f (v_{a,1} - v_{d,1}), \tag{12}
\]

\[
\Delta N_f = K_f \Delta X = K_f (v_{a,1} - v_{b,1}). \tag{13}
\]

Accordingly, the variation of friction forces can be written as

\[
\Delta F_f = \mu K_f (v_{d,1} - v_{b,1}), \tag{14}
\]

\[
\Delta F_f = \mu K_f (v_{d,1} - v_{b,1}). \tag{15}
\]

Assuming constant pin and disc contact at points \( P_a \) and \( P_b \), the following approximations can be derived,

\[
v_{d,1} = v_{a,1} - v_{a,3} r_s \sin \theta_s, \tag{16}
\]

\[
v_{d,1} = v_{b,1} - v_{b,3} r_b \sin \theta_b. \tag{17}
\]

Substituting Eqs. (14)–(17) into Eqs. (5)–(11) yields a set of equations of dynamic motion that can be rewritten more compactly as

\[
[M] \{\dot{X}\} + [C] \{\ddot{X}\} + [K] \{X\} = 0, \tag{18}
\]

where the vectors \( \{\dot{X}\}, \{\ddot{X}\} \) and \( \{X\} \) are

\[
\{\dot{X}\} = \begin{bmatrix} \dot{v}_{a,2} \\ \dot{v}_{b,2} \\ \dot{v}_{d,1} \\ \dot{v}_{a,3} \\ \dot{v}_{b,3} \end{bmatrix}, \quad \{\ddot{X}\} = \begin{bmatrix} \ddot{v}_{a,2} \\ \ddot{v}_{b,2} \\ \ddot{v}_{d,1} \\ \ddot{v}_{a,3} \\ \ddot{v}_{b,3} \end{bmatrix}, \quad \{X\} = \begin{bmatrix} v_{a,2} \\ v_{b,2} \\ v_{d,1} \\ v_{a,3} \\ v_{b,3} \end{bmatrix}. \tag{19}
\]

Note that the detail forms of the system mass, damping and stiffness matrices are given in Appendix A.

### 3.2. Coupled FE model

The proposed groups of infinitesimal translation linear spring elements employed in the first order structural dynamic approximation alone are further incorporated into the correlated FE model. The combined model is shown in Fig. 12. Using the assumed non-reciprocal friction coupling term \( \Delta F_f \), the corresponding equations of vibratory motions for the FE model can be formulated as

\[
[M] \{\dot{X}\} + [C] \{\ddot{X}\} + [K_1 + K_c] \{X\} = \Delta F_f, \tag{20}
\]

where \( [K_1] \) and \( [K_c] \) are stiffness matrices associated with system structure and component coupling. Substituting the friction coupling term \( \Delta F_f = \mu K_f \Delta X \) used earlier in the lumped parameter model into the above equation yields

\[
[M] \{\dot{X}\} + [C] \{\ddot{X}\} + [K_1 + K_c - \mu K_f] \{X\} = 0, \tag{21}
\]

which lead to an asymmetric form of system stiffness matrix. This asymmetry essentially gives rise to a complex eigenvalue problem, as also reported by many other previous studies [1–3], from which unstable system modes are computed as discussed next.

### 4. Complex eigenvalue analysis

The Laplace transform is applied to Eq. (18) that is in time-domain to obtain the equivalent frequency domain eigenvalue problem in the form of \( [B(s)] [X(s)] = [0] \) where \( [X(s)] \) is the Laplace transform of displacement vector, \( s \) is the Laplace variable, and \( [B(s)] = s^2 [M] + [C] + [K] \) is the system characteristic matrix. Due to the friction coupling mechanism, the resultant form of \( [B(s)] \) becomes asymmetric, which yield eigenvalues in complex conjugate pairs, \( p_k = \sigma_k \pm j \omega_k, \quad (k = 1, \ldots, m) \), containing the modal frequency \( \omega_k \) and damping value \( \sigma_k \). Some of them are in fact responsible for the dynamic instability in the system, which leads to squeal. For each eigenvalue, there is a corresponding eigenvector \( \{X_k\} \) that represents the vibration mode shape. These complex eigenvalues are usually plotted in the \( s \)-plane as defined in Fig. 13, where the imaginary part or frequency is plotted versus the real part or damping coefficient. Eigenvalues with negative damping coefficient present will produce decaying oscillations that is characteristic of a stable vibratory system. On the other hand, a positive damping coefficient will cause the amplitude of the oscillation to grow with time without bounds (unstable). Therefore, eigenvalues with \( \sigma_k > 0 \) will have greater tendency to cause squeal. Using the eigenvalue results, the disc brake systems are then categorized according to the influence of modal damping present. Any mode placed in the right half of the \( s \)-plane (positive damping coefficient) is assumed unstable and hence regarded as undesirable. In fact, our test data as well as other experimental
results [39] have shown that modes with higher positive damping coefficient are more likely to produce audible brake squeal. The resulting complex eigenvalue analysis is also applied to Eq. (21) of the dynamic FE model to determine the extent of the unstable modes present. Fig. 14 illustrates some of unstable modes that are found to produce squeal.

In order to confirm the trends predicted by the complex eigensolutions, a 90-stop squeal noise matrix test as shown in Appendix B is used. The matrix consists of various combinations of prescribed driving conditions including lining temperature, pressure and vehicle speed. These prescribed stopping conditions and sequences are applied to each brake design of interest. The specific operating parameters are carefully selected to encompass as wide range of driving conditions as possible, which are prone to squeal occurrence. The interior sound pressure data for each stopping event is needed for the comparative analysis. The measured acoustic noise data for each stop test is first converted into a frequency spectrum where the peaks are associated with the squeal occurrences. Using a peak detector program that utilizes the slope concept defined by the ratio of the rise to run in frequency domain to pinpoint squeal tones above 40 dB, these peaks are identified to produce a scattered plot as shown in Fig. 15b. Here, each point represents a single squeal occurrence with specific frequency and amplitude values acquired during the braking test. For further information about this vehicle noise matrix test and its data processing procedures, reader can refer to an earlier study [40].

In Fig. 15, the frequency versus damping plot derived from the dynamic FE model with friction coupling is compared qualitatively to the summary plot from vehicle noise matrix test. Some of the complex modes, especially those around 9.1 and 14 kHz, show higher positive damping coefficients. The cluster of modes with the largest damping coefficients appear to coincide with the frequencies where squeal is observed to occur the most often in the vehicle. This implies that the damping coefficients $\sigma_k$ appear to be useful for assessing the likelihood of squeal occurrences in all potential unstable modes. In subsequent analysis, $\sigma_k$ is defined as the squeal propensity factor. In the subsequent parametric studies, only damping coefficients falling in the unstable zone are examined, and used for squeal prediction.
5. Design studies

First, a series of parametric studies is performed using the lower order lumped parameter model to determine initial pad prototype concepts that will yield higher number of stable system modes. The design parameters of interest include: (1) radii of pin contacts from pivotal center given by \( r_a, r_b \); (2) angles of contact between pin and disc face denoted by \( \theta_a, \theta_b \); (3) coefficients of friction, \( \mu_a, \mu_b \); and (4) pad damping in the out-of-plane direction symbolized by \( C_{a,1}, C_{b,1} \). The effects of varying each pair of these parameters one at a time on the magnitude of the squeal propensity factor derived from \( \sigma_k \) are examined. From the trends of these parametric studies, a strategy is devised for refining the modified design of the pad to reduce the tendency of causing brake squeal.

5.1. Effect of radii of pin contacts from pivotal centers

The radii of pin contacts from pivotal centers, \( r_a, r_b \), is normally determined by the lining length, lining and shoe thickness, and geometry of chamfer. Fig. 16a illustrates the effect of these radii on the degree of vibration instability in the brake system. In general, increasing the radius increases the degree of instability, which actually makes physical sense since the asymmetry of the predicted motion increases as the radii are lengthened. In practice, a shorter lining will shift the centroid of the load distribution towards the center line of the pad, and thereby reducing the pin contact radius \( (r_{a2} < r_{a1}) \) as seen in Fig. 10. Thus, a shorter lining will reduce the likelihood of squeal occurrences.

5.2. Effect of contact angles between pins and disc surface

The contact angles between the pin and disc face, \( \theta_a, \theta_b \), are essentially determined by the locations of centroid of the load distribution, \( P_a, P_b \), between pad and rotor. This centroidal point of load distribution is affected by various factors, including length of lining, chamfer geometry and nature of load distribution. Again, using \( \sigma_k \) to indicate the severity of squeal, the oscillatory instability regions can be observed over a range of inner and outer pad contact angles as shown in Fig. 16b. The plot reveals a strip of heighten indices, where one or both of the pins has a negative (digging-in) angle of orientation relative to the disc face. Stable motion is obtained when both pins have positive angles of orientation relative to the disc face, or when one pin has a large negative \((<-28^\circ)\) angle of orientation and the other angle is not within the range \(0^\circ\) to \(-28^\circ\). Note that the unstable region is symmetrical about the axes \( \theta_a = \theta_b = -14^\circ \), along which the maximum values of \( \sigma_k \) are seen. In light of this fact, a chamfer is introduced to shift the centroidal point of the load distribution closer to the geometric center as shown in Fig. 10b, which resulted in a smaller contact angle. In the final design development phase, the chamfer geometry is thoroughly re-examined using a detailed FE representation to maximize its effectiveness.

![Fig. 16. Variation of squeal propensity index \( \sigma_k \) with respect to: (a) radii of pin contacts from pivotal center; (b) angles of contact between pin and disc surface; (c) lining friction coefficient; (d) damping in the out-of-plane direction.](image-url)
5.3. Effect of pad coefficient of friction

The effect of friction coefficient of the lining-rotor interface is investigated for both the inner and outer pads. Usually, the analysis is performed for multiple friction coefficients. As friction increases some of the adjacent modal frequencies start to merge towards each other. At the critical friction coefficient, the merged modes have identical imaginary part, i.e. identical frequency, as depicted in Fig. 17a. The real parts of this merged modes have the same magnitude but opposite sign as shown in Fig. 17b. Hence, the merging process gives rise to an unstable mode. Fig. 16c plots the severity of the instability versus friction coefficients of the inner and outer pads. As expected, the propensity for squeal increases with higher coefficients of friction as shown in Fig. 16c, which is consistent with previously reported results [11–16]. This is because the higher coefficient of friction causes the variable frictional forces to be higher resulting in the tendency to excite greater number of unstable modes. In the past, a friction coefficient of 0.35 was typical. However, brake compounds today possess coefficients of friction that is 0.45 or higher [41], which increases the likelihood of squeal. This poses a greater challenge for brake designer to develop a quiet brake system.

5.4. Effect of pad damping in the out-of-plane direction

Another pad parameter that may have an important effect on squeal tendency is the pad structural damping that is represented in the form of a viscous damper in the proposed model. The desired pad damping values can be tuned via structure or material modifications, or bonded damping shims. For example, low-density damping insulator bonded adjacent to pad backing-steel form a very effective dissipation mechanism. The damping in this case is mostly due to multi-elastic layers in the shim, which is most effective against bending waves in the pad. Fig. 16d demonstrates the effect of increased viscous damping on squeal propensity. The results appear to show that certain squeal modes can be made stable by addition of damping, while others remain unstable. A related case study [42] concluded that the unaffected modes may be associated with in-plane

![Fig. 17. Effect of friction coefficient on the predicted complex eigenvalues associated with the 7.12 kHz coupling mode: (a) frequency versus friction coefficient, (b) frequency versus real part.](image-url)
vibrations. If the theory is right, then the out-of-plane damping, $C_{a,1}$, $C_{b,1}$ introduced, cannot completely eliminate the occurrences of these other modes. To add damping that is effective for both directions of vibratory motion, one possibility is to embed isotropic elastic granular in the friction lining to allow for diffusion of the direction of vibratory forces. This could then simultaneously tackle both classes of out-of-plane and in-plane vibration modes.

5.5. Design modification and verification

From the individual parametric studies, the effects of various critical parameters on system vibratory response leading to squeal noise occurrences are documented. These design guidelines are employed to refine the initial design concept using the proposed dynamic FE model with friction coupling. To carry out this process, the detailed features affecting system vibration stability are modified including chamfer and slot geometry, and pad structural and constraint reconfigurations. Pad chamfer and slot design determines the actual pin radius and contact angle, which we know influence the squeal noise occurrences significantly as discovered in the parametric study results. In our analysis, several commonly used chamfer and slot configurations, as shown in Fig. 18, are evaluated. The predicted unstable complex eigenvalues for these chamfer and slot configurations applying the FE model are shown in Fig. 19. The squeal propensity factors are obtained using a complex eigenvalue solver available in a finite element software package [36]. The results show that the pad design with a radial chamfer possesses the least number of unstable modes, which implies lesser tendency towards squeal. This radial chamfer design is further verified experimentally based on the proposed braking noise matrix test. The measured squeal response for this new design is compared to the previous pad with no chamfer in Fig. 20. The results show significantly less squeal noise occurrence for the proposed chamfer configuration as expected. Also,

![Fig. 18. Chamfer and slot configurations analyzed: (a) vertical chamfer, (b) radial chamfer, (c) inverse chamfer with central slot, and (d) diamond chamfer with central slot.](image)
the observed squeal occurrences qualitatively match the trend of the squeal propensity plots predicted by the complex eigenvalue analysis in Fig. 19b. This implies that the proposed model is capable of tracking the vibration modes responsible for squeal reasonably well. This comparison indirectly verifies the validity of the dynamic FE model for use in squeal noise assessment. To achieve further squeal reduction by stabilizing more modes, two modularly tuned damping layers are applied to the pad structure, where corresponding physical properties such as Young's modulus, Poisson's ratio, etc. are adjusted systematically. The first one is an added damping layer between the friction material and steel (that is a modularly tuned, isotropic, elastic granular layer) to allow for diffusion of vibratory forces in the attempt to eliminate in-plane and out-of-plane classes of vibration modes. The second application involves bonding a layer of noise insulator to the back of the steel, which provides extra damping in the out-of-plane direction for the most frequently occurring bending modes. In addition, various constraint and load patterns are evaluated to seek out a set of suitable configurations by again applying the complex eigenvalue analysis using the proposed dynamic FE model. Combining the best modifications attained yield the final design as shown in Fig. 21, which is then verified experimentally using a vehicle level test setup. Vehicle braking experimental results given in Fig. 22 show virtually no squeal generated as compared to the un-modified pad structure.
6. Conclusions

An enhanced dynamic finite element (FE) model with friction coupling is developed to evaluate brake pad structure design modifications for squeal noise reduction. In the development of the FE model, a pair of new spectral-based assurance criteria for modeling and validating the structural coupling and boundary conditions of the rotor–pad assembly is applied successfully. The corresponding friction coupling formulation that gives rise to a set of asymmetric stiffness matrix is first developed for a low order lumped parameter model and then later used in the proposed FE model. The lumped theory is utilized to assess initial pad prototypes, while the FE representation is applied to tune and refine the pad design. The asymmetry in the stiffness matrix leads to a complex eigenvalue problem as expected. From the eigensolution results, a squeal propensity factor derived from eigenvalues with positive real parts is identified. This proposed index is shown to provide a fairly reliable means to evaluate the effectiveness of various design improvements in suppressing squeal response. The friction coupling study yields the following specific conclusions:

(a) Shorter lining reduces the likelihood of squeal.
(b) Higher coefficient of friction increase squeal occurrences.
(c) A range of pin-disc face contact angles most susceptible to squeal is identified.
(d) Higher damping in the out-of-plane direction tends to decrease squeal.

These design guidelines are then refined further using the dynamic FE model to obtain a final pad design. The resultant design is further verified using a set of vehicle level braking experiments.
Appendix A. System matrices of the lumped parameter brake model

The system mass \([M]\), damping \([C]\) and stiffness \([K]\) of Eq. (18) are,

\[
[M] = \begin{bmatrix}
    m_a & 0 & 0 & 0 & 0 & 0 \\
    0 & m_b & 0 & 0 & 0 & 0 \\
    0 & 0 & r_s \sin \theta m_a & r_s^2 \sin \theta^2 m_a + I_s & 0 & 0 \\
    0 & 0 & r_s \sin \theta m_b & 0 & r_s^2 \sin \theta^2 m_b + I_b & 0 \\
    0 & m_a - m_b & m_a r_s \sin \theta & -m_b r_s \sin \theta & 0 & 0 \\
    0 & 0 & 0 & 0 & r_s & 0 \\
\end{bmatrix}
\]  \hspace{1cm} (A1)

\[
[C] = \begin{bmatrix}
    C_{a,2} & 0 & 0 & 0 & 0 & 0 \\
    0 & C_{b,2} & 0 & 0 & 0 & 0 \\
    0 & 0 & r_s \sin \theta C_{a,1} & r_s^2 \sin \theta^2 C_{a,1} + C_{a,3} & 0 & 0 \\
    0 & 0 & r_s \sin \theta C_{b,1} & 0 & r_s^2 \sin \theta^2 C_{b,1} + C_{b,3} & 0 \\
    0 & 0 & C_{a,1} - C_{b,1} + C_{a,3} & C_{a,1} r_s \sin \theta & -C_{b,1} r_s \sin \theta & 0 \\
\end{bmatrix}
\]  \hspace{1cm} (A2)

\[
[K] = \begin{bmatrix}
    K_{1,1} & 0 & 0 & K_{1,4} & 0 & 0 \\
    0 & K_{2,2} & 0 & 0 & K_{2,5} & 0 \\
    0 & 0 & K_{3,3} & K_{4,4} & 0 & 0 \\
    0 & 0 & K_{3,3} & K_{4,4} & K_{5,5} & 0 \\
\end{bmatrix}
\]  \hspace{1cm} (A3)

where

\[ K_{1,1} = K_{a,2} \] \hspace{1cm} (A4)

\[ K_{1,4} = -\mu K r_r \sin \theta_a \] \hspace{1cm} (A5)

\[ K_{2,2} = K_{b,2} \] \hspace{1cm} (A6)

\[ K_{2,5} = \mu K r_b \sin \theta_b \] \hspace{1cm} (A7)

\[ K_{3,3} = r_s \sin \theta_a K_{a,1} \] \hspace{1cm} (A8)

\[ K_{3,4} = r_s^2 \sin \theta_a \cos \theta_a \] \hspace{1cm} (A9)

\[ K_{4,5} = K_{b,3} + r_s^2 \sin \theta_b K_{b,1} + r_s^2 \sin \theta_b K_f \] \hspace{1cm} (A10)

\[ -\mu K r_b^2 \sin \theta_b \cos \theta_b \] \hspace{1cm} (A11)

\[ K_{5,4} = K_{a,1} r_r \sin \theta_a \] \hspace{1cm} (A12)

\[ K_{5,5} = -K_{b,1} r_b \sin \theta_b - 2K_{r_r} \sin \theta_b \] \hspace{1cm} (A13)

Appendix B. A 90-stop vehicle interior noise evaluation matrix

![Squeal Noise Evaluation Sheet](image)

where

\[ K_{1,1} = K_{a,2} \]

\[ K_{1,4} = -\mu K r_r \sin \theta_a \]

\[ K_{2,2} = K_{b,2} \]

\[ K_{2,5} = \mu K r_b \sin \theta_b \]

\[ K_{3,3} = r_s \sin \theta_a K_{a,1} \]

\[ K_{3,4} = r_s^2 \sin \theta_a \cos \theta_a \]

\[ K_{4,5} = K_{b,3} + r_s^2 \sin \theta_b K_{b,1} + r_s^2 \sin \theta_b K_f \]

\[ -\mu K r_b^2 \sin \theta_b \cos \theta_b \]

\[ K_{5,4} = K_{a,1} r_r \sin \theta_a \]

\[ K_{5,5} = -K_{b,1} r_b \sin \theta_b - 2K_{r_r} \sin \theta_b \]
References


[34] Solidworks Corporation. SOLIDWORKS user’s guide; 2000.


学霸图书馆
www.xuebalib.com

本文献由“学霸图书馆-文献云下载”收集自网络，仅供学习交流使用。

学霸图书馆（www.xuebalib.com）是一个“整合众多图书馆数据库资源，
提供一站式文献检索和下载服务”的24小时在线不限IP图书馆。
图书馆致力于便利、促进学习与科研，提供最强文献下载服务。

图书馆导航：
图书馆首页 文献云下载 图书馆入口 外文数据库大全 疑难文献辅助工具