A multi-material level set-based topology and shape optimization method

Yiqiang Wang\textsuperscript{a}, Zhen Luo\textsuperscript{b}, Zhan Kang\textsuperscript{a,\textdagger}, Nong Zhang\textsuperscript{b}

\textsuperscript{a} State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China
\textsuperscript{b} School of Electrical, Mechanical and Mechatronic Systems, The University of Technology, Sydney, Sydney 2131, Australia

Received 7 August 2014; received in revised form 26 October 2014; accepted 3 November 2014
Available online 11 November 2014

Abstract

This paper proposes a new Multi-Material Level Set (MM-LS) topology description model for topology and shape optimization of structures involving multiple materials. Each phase is represented by a combined formulation of different level set functions. With a total number of $M$ level set functions, this approach provides a representation of $M$ materials and one void phase (totally $M + 1$ phases). The advantages of the proposed method include: (1) it can guarantee that each point contains exactly one phase, without overlaps between each two phases and redundant regions within the design domain; (2) it possesses an explicit mathematical expression, which greatly facilitates the design sensitivity analysis; and (3) it retains the merits of the level set method, including smooth boundary and distinct interface. A parametric level set method is applied to evolve the topology and shape of multi-material structures, with a high computational efficiency. Several numerical examples are presented to demonstrate the effectiveness of the proposed method.

\textcopyright 2014 Elsevier B.V. All rights reserved.

Keywords: Topology optimization; Shape optimization; Multiple materials; Level set; Topology description model

1. Introduction

Creating new structures containing different materials has attracted much attention. These design problems are usually to realize specific purposes that may be difficult to be attained by single-material structures [1], or to appropriately distribute different materials to achieve optimum performance [2]. As a powerful and advanced technology, topology optimization can inherently be used to design high-performance structures. Different topology optimization approaches have been proposed over the past two decades [3–11], and recent development of topology optimization can be found in review papers [12–14]. However, most of existing studies focus on single-material structural design, thus it is of great interest to develop topology optimization methods for multi-material structural design.

Compared with the single-material design case, topology optimization of multi-material structures faces more challenging issues. Firstly, an appropriate topology description model is required, which should be able to effectively indicate each distinct phase inside the design domain. An ideal representation model should also have an explicit and
continuous mathematical form, which is easy for sensitivity analysis. Secondly, overlaps between different phases should be avoided. Although this can be achieved by including some additional constraints to the optimization problem [15], a more natural way to achieve this purpose is using a proper representation model. Thirdly, the multi-material design domain is required to be completely covered by all the materials and void phase. It means that the design domain contains no redundant phases.

Up to date, a few topology optimization methods for multi-material structures have been developed. Based on the material distribution concept, Bendsøe and Sigmund [2] proposed a mixture rule of multi-material model in the SIMP (Solid Isotropic Material by Penalization) method. Such a mixture model has been extended to different optimization problems. For instance, Sigmund [16] used the method in the topology design of multi-physics compliant mechanisms. Luo and Kang [17] employed this model to achieve the layout design of multi-material reinforced concrete structures with yield stress constraints. Other multi-materials models are also developed for some specific design problems.

For instance, an extension of the SIMP method for the multi-materials problem is the recursive multiphase materials interpolation [18]. In the design of smart structures, various multi-material models were developed to seek the best distribution of the piezoelectric and elastic materials [19–22]. Zhu et al. [23] studied topology design of structures with multiple movable components, where the embedded components have different materials from the host structure. Wang et al. [24] proposed an integrated optimization method for multi-material design problems, which involve optimal design of piezoelectric materials as actuation elements and topology optimization of elastic materials as the host structure. Steigmann and Lund [25] developed the Discrete Material Optimization method to simultaneously optimize the topology and the layup of composites, and applied it to multi-material optimization [26]. In the framework of the phase field method, Zhou and Wang [27] and Tavakoli [28] achieved multi-materials optimization. By combining the binary phase algorithm and the block coordinate descent method, Tavakoli and Mohseni [29] solved multi-materials design problems.

Despite of the above-mentioned methods, another attractive manner to achieve topology design of multi-material structures is to make use of the level set model. The level set method has emerged as an advanced method for topology shape optimization of structures [8–10]. Generally, it employs higher-order surfaces to implicitly represent the structural boundary as zero level sets. Then, shape and possible topology changes of the structures are achieved by emerging and splitting of the moving boundaries, which can be tracked by solving the Hamilton–Jacobi equation or by updating the parameterization coefficients through mathematical programming algorithms [30,31]. Sethian and Wiegmann [8] first introduced the level set description into topology optimization. Wang et al. [9] and Allaire et al. [10] developed level-set based topology and shape optimization method using the shape-sensitivity. It has been shown that the level set method has advantages in terms of smooth boundary and distinct interface, as well as integrated topology and shape optimization. For the single-material design problems, some alternative level set methods have also been proposed to improve the optimization behavior, to save computational cost of the optimization process, or to achieve special objective functions [30–36]. Up to date, the level set-based approaches have been widely applied to various design problems, such as eigenvalue maximization [37], stress minimization [38], designs with geometrical uncertainties [39], design of photonic nanostructures [40] and optimization of functional graded material [41].

The level set methods have also been extended to the topology and shape optimization of multi-material structures. Wang and Wang [42] first proposed a ‘color’-level set method for the compliance minimization problem involving multiple materials. The method was further applied to the design of multi-material compliant mechanisms [43] and stress-related optimization problem [44]. One advantage of the ‘color’-level set method is that only $M$ level set functions are used to represent a total number of $2^M$ phases, saving much computational cost of the optimization process. However, in a general case where the number of the phases is not exactly equal to $2^M$, there will be redundant phases within the design domain, which requires certain interpretation for the extra phases. Furthermore, the ‘color’-level set method still needs re-initialization, velocity extension and lacks the mechanisms of nucleating new holes, which may influence the optimization performance. Another multi-material level set approach is to incorporate the piecewise constant level set model into topology optimization [15,45]. This method simplifies the representation model by using a single indicator function to identify all the material interfaces. However, one major issue is that overlaps between each two phases must be avoided by applying additional non-overlap constraints, making the optimization problem numerically difficult to solve [15]. Another variational multi-material level set method is developed for the optimization of heterogeneous objects [46]. Recently, Vermaak et al. [47] discussed the influence of interface between each two phases for multi-material topology optimization. All the above-mentioned issues motivate the development of a more advanced level set approach for topology optimization of multi-material structures.
This paper proposes a new Multi-Material Level Set (MM-LS) topology description model for topology and shape optimization of structures involving multiple materials. The basic idea of the proposed method is that the material property at any point in the design domain is calculated according to a combination rule of different level set functions. Here, the proposed MM-LS model employs a total number of $M$ level set functions to indicate $M + 1$ distinct phases, including $M$ materials and one void phase. The approach can naturally avoid overlaps between each two phases, and also contains no redundant regions in the design domain. Furthermore, the MM-LS model has an explicit mathematical expression, which facilitates the sensitivity analysis. Though the proposed MM-LS model can be incorporated into standard level set-based optimization frameworks, a parametric level set method [31] is employed to achieve the topology evolution of each phase. The underlying concept of the parametric level set method is to decouple the original Hamilton–Jacobi partial differential equation into a system of ordinary differential equations, and then the shape evolution of each phase is driven by updating its coefficients using a mathematical programming algorithm. It has shown that this method can retain the merits of the conventional level set approach, and can also effectively improve the computational efficiency of the optimization process [31]. Finally, several numerical examples will be used to demonstrate the effectiveness of the proposed method.

2. A new multi-material level set model

In this section, a new multi-material level set (MM-LS) model for topology and shape optimization of multi-material structures is proposed. In order to describe $M + 1$ distinct phases (including $M$ materials and one void phase), a total number of $M$ level set functions are used.

2.1. Implicit boundary representation by multiple level sets

In the level set-based optimization, the structural boundary is implicitly represented by the zero level sets with Lipschitz continuity [9,10]. For the multi-material structures, multiple level set functions will be employed to denote different phases. These level set functions are used to define the following subdomains

\[
\begin{align*}
\phi_k(x) > 0 & \quad \forall x \in \Omega^k \setminus I^k \\
\phi_k(x) = 0 & \quad \forall x \in I^k, \quad k = 1, 2, \ldots, M \\
\phi_k(x) < 0 & \quad \forall x \in D \setminus (\Omega^k \cup I^k)
\end{align*}
\]

where the reference domain $D$ includes all admissible shapes of; $\Omega^k$ denotes the domain with positive value of the $k$th level set function $I^k$ is the boundary of the $k$th level set function, and $M$ is the number of the level set functions. An example for the design domain containing two level set functions is illustrated in Fig. 1.

Introducing a pseudo-time $t$ to enable the movement of the structural boundary and differentiating both sides of $\phi_k(x(t), t) = 0$ with respect to $t$, it can generate the following Hamilton–Jacobi equation

\[
\frac{\partial \phi_k(x, t)}{\partial t} - v_n^k \left| \nabla \phi_k(x, t) \right| = 0, \quad k = 1, 2, \ldots, M
\]

where $v_n^k$ is the normal velocity of the $k$th level set function, since only the normal component of the velocity contributes to the dynamic motion of the boundary [9,10]. The details for solving Eq. (2) can be found in Ref. [48].
2.2. A multi-material level set model: MM-LS

In this section, a new Multi-Material Level Set (MM-LS) topology description model-based method for topology and shape optimization of multi-material structures is proposed. Here, each material is represented by a combination of different level set functions, rather than by a particular level set function. For instance, for the cases with one, two or three materials, the elastic stiffness $D^{(i)}(x)$, $(i = 1, 2, 3)$ at a computational point $x$ can be calculated by:

One material: $D^{(1)}(x, \varphi) = H(\varphi)D^1$,

Two materials: $D^{(2)}(x, \varphi) = H(\varphi)\left[\left(1 - H(\varphi^2)\right)D^1 + H(\varphi^2)D^2\right]$,

Three materials: $D^{(3)}(x, \varphi) = H(\varphi)\left[\left(1 - H(\varphi^2)\right)D^1 + H(\varphi^2)\left((1 - H(\varphi^3))D^2 + H(\varphi^3)D^3\right)\right]$,

... where $H(\varphi)$ is the Heaviside function of the $i$th level set function [9,10], and $D^i$ is the elastic stiffness of the $i$th material. For the case with $k$ materials, the elastic stiffness $D^{(k)}(x)$ can be obtained according to similar rules as above.

The MM-LS approach given in Eq. (3) can be used to describe the distribution of multiple materials in the design domain. A schematic illustration is given in Fig. 2, in which three level set functions are combined together to describe three materials. The level set function $\varphi^1$ is firstly used to distinguish the solid materials (blue regions) and the void phase (gray regions), $\varphi^2$ is then applied to determine the first material (blue region) from the unified materials, and $\varphi^3$ is finally used to denote the second material (yellow region) and the third material (green region). In this case, the elastic stiffness at each point is calculated using the third equation in Eq. (3).

It is easily to see that the MM-LS exhibits some advantages: (1) it has an explicit mathematical expression, which benefits the sensitivity analysis; (2) it provides a unique and distinct description for each phase; and (3) it guarantees that any point inside the design domain indicates one and only one phase, without overlaps between different phases and without redundant regions.

Although it has a similar formulation compared with the mixture model in the density distribution methodology [1,2], the proposed MM-LS model is developed for the level set methodology. Thus, there are essential differences between the two approaches. For example, Eq. (3) employs continuous level set functions to describe each phase, thus the advantages for the level set method can be retained in the proposed approach.

3. Topology and shape optimization of multi-material structures by MM-LS

3.1. Parametric level set method

A parametric level set method [30,31] is employed to evolve the topology of the multi-material structures. The basic idea of this approach is to use time- and space-independent terms to parameterize the level set function. In our approach, the level set function is parameterized by

$$\varphi^k(x, t) = \sum_{i=1}^{N} \omega_i(x) \alpha_i^k(t), \quad (k = 1, 2, \ldots, M)$$

(4)

where $N$ is the total number of the level set knots over the design domain; $\alpha_i^k$ is the expansion coefficient at the $i$th knot corresponding to the $k$th level set function; $\omega_i(x)$ is the CSRBF (Compactly-supported Radial Basis Function) located at the computational point $x$ [31]. In this paper, the following CSRBF is employed

$$\omega_i(x) = (1 - r_i(x))^4 \cdot (4r_i(x) + 1), \quad (i = 1, 2, \ldots, N)$$

(5)

where $r_i(x) = ||x - x_i|| / R$, $(i = 1, 2, \ldots, N)$, and $R$ is the radius of the support domain of point $x$; $(\cdot)^+$ is the truncated function. Here, only the knots within the support domain of the current point $x$ contribute to its function value. Substituting Eq. (4) into Eq. (2) leads to the decoupling of the time and space terms in the Hamilton–Jacobi equation:

$$\sum_{i=1}^{N} \omega_i(x) \alpha_i^k(t) - \nu_n \sum_{i=1}^{N} |\nabla \omega_i(x)| \alpha_i^k(t) = 0, \quad (k = 1, 2, \ldots, M).$$

(6)
Thus, the normal velocity of the $k$th ($k = 1, 2, \ldots, M$) level set function can be computed by

$$v_n^k = \frac{\sum_{i=1}^{N} \omega_i(x) \dot{\alpha}_i^k(t)}{\sum_{i=1}^{N} |\nabla \omega_i(x)| \alpha_i^k(t)}$$

(7)

where $\dot{\alpha}_i^k(t) = d\alpha_i^k/dt$. 

Fig. 2. MM-LS for representing a multi-material structure. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
3.2. Optimization model using MM-LS

In this paper, the mean compliance minimization problem for linear elastic structures is studied by using the proposed MM-LS method. The optimization problem with multiple materials is formulated as

\[
\begin{align*}
\text{find } & \alpha_k^i, \quad (k = 1, 2, \ldots, M; \ i = 1, 2, \ldots, N) \\
\text{min. } & J = \int_D \varepsilon(\mathbf{u}) : D(\varphi) : \varepsilon(\mathbf{u}) \, d\Omega \\
\text{s.t. } & a(\mathbf{u}, \mathbf{v}, \varphi) = l(\mathbf{v}, \varphi), \quad \forall \mathbf{v} \in U_{ad} \\
& V_f^k \leq f^k V^0, \quad (k = 1, 2, \ldots, M) \\
& \underline{\alpha}^k \leq \alpha_k^i \leq \bar{\alpha}^k, \quad (i = 1, 2, \ldots, N)
\end{align*}
\] (8)

where \(\mathbf{u}\) is the displacement field, \(\mathbf{v}\) is the virtual displacement and \(U_{ad} \in H^1(D)\) is the set of kinematically admissible displacements; \(\varepsilon(\mathbf{u})\) is the strain; \(D\) is the material elasticity tensor computed by the MM-LS model Eq. (3); \(\underline{\alpha}^k\) and \(\bar{\alpha}^k\) are the lower and upper bounds of the \(k\)th set of design variables; \(V^0\) is the volume of the design domain, and \(f^k\) denotes the prescribed volume fraction applied to the \(k\)th volume constraint.

Here, we only consider the case with fixed traction force on the external boundaries \(\Gamma_t\) of the design domain. In this sense, the energy bilinear form \(a(\mathbf{u}, \mathbf{v}, \varphi)\) and the load linear form \(l(\mathbf{v}, \varphi)\) are given by

\[
a(\mathbf{u}, \mathbf{v}, \varphi) = \int_D \varepsilon(\mathbf{u}) : D(\varphi) : \varepsilon(\mathbf{v}) \, d\Omega
\]
\[
l(\mathbf{v}, \varphi) = \int_{\Gamma_t} t \cdot \mathbf{v} \, d\Gamma.
\] (9)

In the optimization problem (8), the volume constraints are given to restrict the allowable material usage in an implicit manner. Generally, the number of the volume constraints is equal to the number of the materials included. In order to achieve a better convergence process, we choose the following volume constraints:

\[
V_f^k = \int_D \prod_{i=1}^k H_i \, d\Omega, \quad (k = 1, 2, \ldots, M). \tag{10}
\]

The above volume constraints impose a restriction to the overall material usage, as well as the volume of each individual material. For the case with three materials, the volume fractions can be expressed by

\[
V_f^1 = \int_D H_1 \, d\Omega, \quad V_f^2 = \int_D H_1 H_2 \, d\Omega, \quad V_f^3 = \int_D H_1 H_2 H_3 \, d\Omega \tag{11}
\]

where the first constraint refers to the overall usage of the total materials, the second one restricts the usage of the materials except material one, and finally the third one restricts the volume ratio of material three.

3.3. Sensitivity analysis

In the proposed method, the parameterization coefficients of the level set functions are to be optimized with a gradient-based optimization algorithm, e.g., the Method of Moving Asymptotes (MMA) [49]. Thus, the shape derivatives [50] of the objective function with respect to these design variables are required.

Considering a variation of the moving boundary due to the pseudo-time \(t\), the material derivative [50] of the objective function is computed by the adjoint sensitivity analysis, as

\[
\frac{dJ}{dt} \bigg|_{\varphi^k} = \sum_{i=1}^N \left( \int_D \beta^k \omega_i(\mathbf{x}) \, d\Omega \frac{d\alpha_k^i(t)}{dt} \right), \quad (k = 1, 2, \ldots, M) \tag{12}
\]

where

\[
\beta^k = -\varepsilon(\mathbf{u}) : \frac{\partial D(\mathbf{x}, \varphi)}{\partial \varphi^k} : \varepsilon(\mathbf{u}). \tag{13}
\]
The derivatives of the objective function with respect to $t$ can also be obtained using the chain-rule in terms of the coefficients as

$$
\frac{dJ}{dt} = \sum_{i=1}^{N} \frac{\partial J}{\partial \alpha_i^k} \frac{d\alpha_i^k}{dt}, \quad (k = 1, 2, \ldots, M).
$$

(Eq. 14)

Eqs. (12) and (14) are satisfied for any time variation of $\alpha_i^j$, $(i = 1, 2, \ldots, N; \ k = 1, 2, \ldots, M)$. Therefore, by comparing the corresponding terms, the design sensitivities of the objective function with respect to the design variables is obtained by

$$
\frac{\partial J}{\partial \alpha_i^k} = \int_D b_i^k \omega_i(x) \ d\Omega, \quad (i = 1, 2, \ldots, N; \ k = 1, 2, \ldots, M).
$$

(Eq. 15)

The term $\partial D (x, \psi) / \partial \psi^k$ in Eq. (13) can be easily calculated from Eq. (3).

Similarly, the derivative of the volume constraints is computed by

$$
\frac{dV^k}{d\alpha_i^k} = \int_D \left( \prod_{p=1,\ p \neq j}^{k} H (\psi^p) \right) \delta (\psi^j) \omega_i(x) \ d\Omega, \quad (i = 1, 2, \ldots, N; \ j = 1, 2, \ldots, M; \ k = 1, 2, \ldots, M)
$$

(Eq. 16)

where $\delta (\psi^j) = \partial H (\psi^j) / \partial \psi^j$.

3.4. Numerical implementation

The finite element method is employed in the displacement analysis. The ‘ersatz material’ model [10] is used to compute element stiffness matrices, as well as the integrations given in Eqs. (15) and (16). For the cases with one, two or three materials in Eq. (3), the elastic stiffness $D_e (x)$, $(e = 1, 2, \ldots, N_e)$ of the $e$th element is calculated by

$$
D_e^{(1)} = \left[ \frac{1}{A_e} \int_{\Omega_e} H^1 \ d\Omega \right] D^1,
$$

$$
D_e^{(2)} = \left[ \frac{1}{A_e} \int_{\Omega_e} H^1 (1 - H^2) \ d\Omega \right] D^1 + \left[ \frac{1}{A_e} \int_{\Omega_e} H^1 H^2 \ d\Omega \right] D^2,
$$

$$
D_e^{(3)} = \left[ \frac{1}{A_e} \int_{\Omega_e} H^1 (1 - H^2) \ d\Omega \right] D^1 + \left[ \frac{1}{A_e} \int_{\Omega_e} H^1 H^2 (1 - H^3) \ d\Omega \right] D^2 + \left[ \frac{1}{A_e} \int_{\Omega_e} H^1 H^2 H^3 \ d\Omega \right] D^3
$$

(Eq. 17)

where $H^1 = H (\psi^e)$, $\Omega_e$ is the region covered by the $e$th element, $A_e$ is the area of the $e$th element, and $N_e$ is the number of elements.

In the numerical implementation of the optimization process, the following smoothed approximation of $H (\psi^k)$ and $\delta (\psi^k)$, $(i = 1, 2, \ldots, M)$ is used [9],

$$
H (\psi^k) = \begin{cases} 
\alpha & \psi^k \leq -\Delta \\
\frac{3 (1 - \alpha) \psi^k}{4 \Delta} & \psi^k = \frac{1}{3} \left( \psi^k \leq \Delta \right) \\
1 & \psi^k \geq \Delta 
\end{cases}
$$

(Eq. 18)

$$
\delta (\psi^k) = \begin{cases} 
\frac{3 (1 - \alpha) \psi^k}{4 \Delta} & \psi^k \leq \psi^k \\
\gamma & \psi^k \geq \psi^k
\end{cases}
$$

where $\alpha$ and $\gamma$ are small positive numbers to ensure that the overall stiffness of the structure is nonsingular, and $\Delta$ describes the width of numerical approximation.
4. Numerical examples

In this section, several numerical examples are presented to demonstrate the effectiveness of the proposed method for topology and shape optimization of multi-material structures. Here, 2D linear elastic planer structures are considered. The design domain is discretized with four-node quadrilateral (Q4) square finite elements with unit edge length. The level set knots are positioned at the finite element nodes. The design problems involve two materials and one void phase.

In all the examples, the lower and upper bounds of the design variables are chosen to be $\alpha^k = 6 \times \min_i \{\alpha_i^{k,0}\}$ and $\bar{\alpha}^k = 6 \times \max_i \{\alpha_i^{k,0}\}$, where $\alpha_i^{k,0}$ denotes the initial values for the $k$th set of coefficients. The support radii for all the computational points are fixed as $R = 2.5$; the parameters in Eq. (18) are $\alpha = 0.001$, $\gamma = 0.0005$ and $\Delta = 1$. The optimization is terminated if the relative difference of two successive objective function values falls below 0.0001.

In real-world design problems, the volume constraints for each material phase can be prescribed as a result of compromise between the material cost and the structural stiffness. In the second example, we will show that the optimal distribution of different materials can be also obtained by restricting the total material cost rather than the volume ratio of each phase.

4.1. Cantilever beam

The first example considers the topology optimization of a cantilever beam with two materials. The dimension of the rectangle design domain is 100 × 50, as shown in Fig. 3. The beam is fixed at the left side, and a unit concentrated force is applied at the lower right corner. The Young’s moduli of material 1 and material 2 are $E_1 = 20$ and $E_2 = 1$, respectively. Both materials have the same Poisson’s ratio $\nu = 0.3$. The volume fractions are given by $f^1_v = 0.5$ and $f^2_v = 0.1$, which means that the maximum volume usage for material 1 and material 2 are 0.4 and 0.1, respectively.

Optimal designs for two cases of different initial configurations of the two-material structure are shown in Fig. 4, and the iteration history of the objective function of case 1, together with several intermediate results, are given in Fig. 5. The topology evolution of both materials is achieved by moving the level sets, thus a relatively smooth structural boundary with distinct material interface can be well retained. Although different optimal solutions are obtained under different initial configurations, both results in Fig. 4 provide reasonable distribution of both materials, where the strong material is mainly distributed in the force transmission path and the weak material acts as a reinforcement phase. From our experience, a reasonable number of initial holes should be included to guarantee a desired design resolution. The objective function values of the two cases are $J_{\text{case 1}}^{\text{opt}} = 4.0712$ after 114 iterations (see Fig. 5) and $J_{\text{case 2}}^{\text{opt}} = 4.1002$ after 79 iterations. This shows that the proposed approach possesses a high computational efficiency compared with conventional level set methods. Fig. 6 presents the changes of the volume fractions of each material during the optimization process. It is found that the MM-LS method well preserves the volumes of each material.
Fig. 4. Topology design of two-material cantilever beam with different initial configurations (blue $E^1 = 20$; red $E^2 = 1$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

For comparison, the designs obtained with four different stiffness ratios between the two materials are given in Fig. 7. Here, the Young’s modulus of the weak material (material 2) is kept unchanged at $E^2 = 1$, and that of the strong material (material 1) is given as $E^1 = 4, 10, 100$ and 200 (Fig. 7(a)–(d)). The results show that the optimal design is dependent on the stiffness ratio of the two materials. For the case where the stiffness of the strong and weak materials are comparable (Fig. 7(a) and (b)), the two materials form independent load-bearing structural members. However, if material 1 is much stronger than material 2 (Fig. 7(c) and (d)), material 1 will be used in the main load-bearing components, while material 2 is mostly attached to these components as an enhancing material. This tendency is in accordance with the SIMP solutions for multi-material structures [51].

Further, the optimal solutions with different material volume fractions are compared. Here, the Young’s Moduli for the two materials are set to be $E^1 = 4$ and $E^2 = 1$. The total material usage is fixed as $f^1_v = 0.5$, and the second constraint is chosen as $f^2_v = 0.35, 0.25$ and 0.2, which means the strong material occupies 15%, 25% and 30% of the design domain. The optimal designs are presented in Fig. 8. It is seen that all the optimal results provide quite similar
topologies, only with different localized details. Here, the weak materials tend to form independent members along the force transmission path.
4.2. Two-bar bracket

The second example is to design a two-bar bracket structure with two materials. As shown in Fig. 9, the geometrical dimension of the design domain is $40 \times 80$. The left edge of the structure is fixed, and a unit concentrated force is applied at the middle point of the right edge. The well-known optimal solution of this structure consists of two $45^\circ$ arranged members [42]. The Young’s moduli of the two materials are $E_1 = 5$ and $E_2 = 1$, and the Poisson’s ratios are both $\mu = 0.3$. The volume fractions are set to be $f_{v1} = 0.3$ and $f_{v2} = 0.15$, which means the maximum volume ratios of both materials are 15%.

The optimization results obtained with two different initial designs are plotted in Fig. 10. For both cases, the iteration histories of the objective function and the material usage are given in Figs. 11 and 12, respectively. Even with quite different initial designs, the present approach generates almost identical optimal results, with the strong material in the inner core areas surrounding by the weak one. Both results contain two $45^\circ$ inclined bars. The optimized basic topological features are consistent with the solution of the ‘color’-level set method (Fig. 8 in [42]), but exhibit subtle differences in the material layout caused by local optimality of the optimal result. The objective function for the two cases converge to $J_{\text{case } 1}^{\text{opt}} = 2.4157$ and $J_{\text{case } 2}^{\text{opt}} = 2.4174$, respectively. In both cases, the volume fractions of each material are well preserved during the optimization process.

To further examine the performance of the proposed MM-LS method in problems involving multiple materials, we now consider the bracket design with three materials. The design domain and the applied force are the same as in Fig. 9. The Young’s moduli of the materials are $E_1 = 100$ (blue regions), $E_2 = 20$ (red regions) and $E_3 = 1$ (green regions). The volume constraints are $f_{v1} = 0.4$, $f_{v2} = 0.2$ and $f_{v3} = 0.1$, which means that the material volume fractions of the three materials are specified as 20%, 10% and 10%. A set of intermediate optimization results are shown in Fig. 13. The optimal result contains three layers of materials, with the strongest material as the inner core and the weaker two around it. This design indicates a similar tendency as in the case with two materials (Fig. 10).

It is also noted that the optimized structural layout is different from the result presented by Fig. 11 in [42]. This is due to the fact that considered optimization problem is not a convex one and usually only local optima can be achieved.

We have also performed the optimization to illustrate the trade-off relationship of material usage. In the optimization problem, only the total material volume and material cost are constrained. The Young’s moduli for the two materials are $E_1 = 5$ and $E_2 = 4$. The total volume constraint is $f_{v1} = 0.4$, while the total material cost is restricted by $5V^{M1} + 2V^{M2} \leq 1.7V^0$. Here $V^{M1}$ and $V^{M2}$ are the volume, and 5 and 2 are the cost per volume for the strong and weak materials, respectively. The optimal result is plotted in Fig. 14. Clearly, this result reflects a trade-off between the structural stiffness and the material cost. Here, the volume fractions of the strong and weak materials are 0.2970 and 0.1023, respectively.
Fig. 10. Optimization results of the two-bar bracket from different initial designs (blue $E^1 = 5$; red $E^2 = 1$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 11. Iteration history of the objective function for different cases of initial design.
In the last example, the design of a structure subject to multiple external forces is considered. The geometrical dimensions of the design domain are $100 \times 50$, as shown in Fig. 15. Here, the structure has three concentrated forces with $F = -1$ at its bottom edge. The Young’s moduli of the two materials are $E^1 = 20$ and $E^2 = 1$, and the Poisson’s ratio is $\mu = 0.3$. The volume fractions are $f^1_v = 0.5$ and $f^2_v = 0.20$, which means that the material usage of the strong and weak materials are restricted to be 30% and 20% of the design domain, respectively.

After 102 iterations, the structural compliance converges to $J^{\text{opt}} = 2.873$. The volume constraints of two materials are well satisfied during the whole course of the optimization. The initial and optimal designs are given in Fig. 16. The iteration history of the compliance and selected intermediate solutions are plotted in Fig. 17, and the changes of the volume fractions of each material are shown in Fig. 18. It is seen that the proposed method generates a reasonable layout of both materials inside the design domain, and relatively smooth structural boundaries.
Fig. 13. Optimization results of the two-bar bracket with three materials (blue $E^1 = 100$; red $E^2 = 20$; green $E^3 = 1$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 14. Optimization results of the two-bar bracket under a total volume constraint and a material cost constraint.

Fig. 15. The design domain of three point-loaded beam.

5. Conclusions

This paper proposes a novel MM-LS topology description approach for topology and shape optimization of multi-material structures. The proposed representation method describes each phase by a combination of different level
set functions, and also possesses the following advantages: (1) it can effectively indicate each distinct phase; (2) it has an explicit mathematical formulation, which facilitates the sensitivity analysis; (3) it naturally avoids overlaps between each two phases, and guarantees that the design domain contains no redundant phases. In the numerical
implementation, an efficient parametric level set method is applied to achieve the topology and shape optimization of the multi-material structures. However, it is straightforward to extend the proposed method to other level set-based optimization approaches. Numerical examples demonstrate the efficiency and validity of the proposed method.

One major issue of the proposed approach is that the number of the design variables increases along with the increase of the materials included. How to improve the computational efficiency of the optimization iteration will be our future work. One possible solution is to use parallel algorithms.

Besides the minimum compliance problem with multiple materials, the proposed MM-LS topology description model can also be applied to other topology and shape optimization problems to achieve specific purpose, such as the design of thermoelastic metamaterials with multiple materials.

Acknowledgments

This research is partially supported by the Chancellor’s Research Fellowship, University of Technology Sydney (UTS) (2032063), the Key Project of Chinese National Programs for Fundamental Research and Development (2010CB832703) and the Natural Science Foundation of China (91130025). The authors are also grateful to the anonymous reviewers for their valuable suggestions for improving the manuscript.

References

学霸图书馆

www.xuebalib.com

本文献由“学霸图书馆-文献云下载”收集自网络，仅供学习交流使用。

学霸图书馆（www.xuebalib.com）是一个“整合众多图书馆数据库资源，提供一站式文献检索和下载服务”的24小时在线不限IP图书馆。

图书馆致力于便利、促进学习与科研，提供最强文献下载服务。

图书馆导航：

图书馆首页 文献云下载 图书馆入口 外文数据库大全 疑难文献辅助工具