Study of the bottom-hole rock stress field under water jet impact

G. Li, D. Chang, Z. Shen, Z. Huang, S. Tian, H. Shi & X. Song


To link to this article: http://dx.doi.org/10.1080/15567036.2010.531509

Published online: 28 Jan 2016.
Study of the bottom-hole rock stress field under water jet impact

G. Li, D. Chang, Z. Shen, Z. Huang, S. Tian, H. Shi, and X. Song

State Key Laboratory of Petroleum Resource and Prospecting, China University of Petroleum, Beijing, China

ABSTRACT
On the basis of analysis of the bottom-hole rock stress field under water jet impact, there are three types of coupling, which include the coupling of pore fluid and water jet, the coupling of pore fluid and rock matrix, and the coupling of water jet and rock matrix. The fluid-solid coupling model with the four main factors of three-dimensional in-situ stress, fluid column pressure, pore pressure, and water jet velocity is established and calculated by the finite element method and the finite volume method. The results show that the maximum principal stress of the bottom-hole increases with the increase of bottom-hole differential pressure. The maximum jet impact force is proportional to the square of the jet velocity; the pore pressures on the impact surface and along impact axis decrease in the form of cubic parabola with the increase of the distance. The local effect is obvious under water jet impact; the main affected area of the water jet is 2 times jet radius on the impact surface and 2.5 to 3.5 times jet radius along the impact axis when water jet velocity increases from 50 to 250 m/s, which is consistent with the stress wave attenuation theory. Study of the bottom-hole rock stress field under water jet impact provides a numerical research method for study of the water jet rock breaking mechanism under actual drilling conditions and the theoretical basis for faster and more efficient drilling.

KEYWORDS
Bottom-hole rock; fluid-solid coupling; pore pressure; rock breaking mechanism; stress field; water jet

Introduction
It has been proved during the drilling practice that a high pressure water jet can significantly improve the rate of penetration (Cohen et al., 2005; Kolle et al., 2008). At present, scholars at home and abroad have done a lot of research work on the water jet rock-breaking mechanism. There are two theories, which are more recognized of all the water jet impact crushing theories of the rock (Shen, 1998): tension-water wedge theory and compact core-splitting theory. However, it is quite difficult to study the bottom-hole stress field under water jet impact with the consideration of actual drilling conditions, such as three-dimensional in-situ stress, fluid column pressure, pore pressure, and so on. First, the different properties and the complexities of the bottom-hole rock make the big difference of rock materials itself. Second, the formation process of the bottom hole belongs to the end face excavation effect (Wang et al., 2009) but there is no analytic solution thus far. Third, an experimental study of the bottom-hole rock stress field under water jet impact is extremely difficult with the consideration of actual drilling conditions, and so far, there is no report about that. The purpose of the article is to provide a numerical simulation method for study of the bottom-hole stress field under water jet impact with the consideration of actual drilling conditions and water jet rock-breaking mechanism, which is the theoretical basis for faster and more efficient drilling.
Coupling analysis of water jet impinging on rock

The coupling of water jet impinging on rock includes the coupling of pore fluid and water jet, the coupling of pore fluid and rock matrix, and the coupling of water jet and rock matrix (Ni and Wang, 2004; Li et al., 2009; Inoue and Fontoura, 2009).

The coupling of pore fluid and water jet

When water jet is impinging on rock, pore pressure within the rock will be changed in the impacted area. According to Darcy’s law, it can be simplified to the circumstance of seepage from the center of the spherical region to the outside region. By using polar coordinates, the equation (Dewiest, 1969) is as follows:

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{dP}{dr}) = 0.$$  

(1)

The pressure boundary conditions are \( r = R_j \), \( P = P_j \) and \( r = v_c t \), \( P = P_p \), where \( r \) is the seepage flow radius, mm; \( R_j \) is the radius of jet impact hole, mm; \( P_j \) is the water jet impact pressure at the interface, MPa; \( v_c \) is the stress wave propagation speed, m/s; \( t \) is the time of water jet impact, s; and \( P_p \) is initial pore pressure within the rock, MPa.

Substituting boundary conditions into Eq. (1), the pore pressure within the rock is:

$$P_r = P_p - (P_p - P_j) \left( \frac{1}{r} - \frac{1}{v_c t} \right) / \left( \frac{1}{R_j} - \frac{1}{v_c t} \right).$$  

(2)

The coupling of pore fluid and rock matrix

Rock is a typical porous medium based on the effective stress principle of the porous medium. The coupling equation of pore fluid and rock matrix (Jaeger et al., 2007) is:

$$\sigma_{ij}^e + \delta_{ij} \alpha P_p - \lambda e \delta_{ij} - 2 \mu \varepsilon_{ij} = 0,$$  

(3)

where \( e \) is volumetric strain; \( \delta_{ij} \) is Kronecker delta function; \( \lambda \) is lame constant; \( \lambda = E\mu / [(1 + \mu)(1 - 2\mu)] \); \( \mu \) is Poisson’s ratio; \( P_p \) is the pore pressure, MPa; \( \varepsilon_{ij} \) is elastic strain tensor; \( \sigma_{ij}^e \) is the effective stress of rock matrix; \( \alpha \) is effective stress coefficient and is different to different types of rocks. It is assumed to be 1 during the computational process.

The coupling of water jet and rock matrix

Coupling of the water jet and rock matrix first occurs at the fluid-solid interface, and the displacement and stress compatible conditions (Li et al., 2009) should satisfy at the interface:

$$d_f = d_s; \quad n \cdot \tau_f = n \cdot \tau_s,$$  

(4)

where \( d_f \) is fluid displacement, mm; \( d_s \) is solid displacement, mm; \( \tau_f \) is fluid stress, MPa; and \( \tau_s \) is solid stress, MPa.

Model of the bottom hole under water jet impact

There are two main types of the bottom-hole geometry, one is the plan form and the other one is the smooth form, which is tangent to the outside of the drill bit. The figure of the bottom hole will change during drilling. Due to the actual shape of the borehole wall and because the bottom-hole surface is very complex, the shape of the bottom hole is assumed to be the plane shape in order to carry on quantitative theoretical study of the bottom-hole rock stress field. At present, there is no
kind of model that simultaneously includes the factors of three-dimensional in-situ stress, fluid column pressure, pore pressure, and water jet impact pressure (Warren and Smith, 1985; Ni and Wang, 2004; Sun et al., 2005; Caicedo et al., 2005; Peng et al., 2006; Wang et al., 2008). In this article, the schematic diagram of the bottom-hole model under water jet impact is shown in Figure 1.

**Fundamental assumptions**

The bottom-hole rock stress field under water jet impact with consideration of actual drilling conditions is so complicated that the following basic assumptions are made for the fluid-solid coupling model: (1) Rock is isotropic and full of pore fluid; (2) pore fluid flow is steady flow and conforms to Darcy’s law; (3) without considering the boundary layer and liquid compressibility; (4) the stand-off distance is within the potential core of the jet; and (5) without considering the effect of well deviation, fractures, and temperature.

**Computational parameters**

The computational parameters mainly include borehole geometrical parameters, material characteristic parameters, fluid pressures, three-dimensional in-situ stresses, and water jet parameters. The main parameters in detail are shown in Table 1.

**Boundary conditions**

In the solid model, the vertical total stress, the maximum and minimum total horizontal stresses are 75, 60, and 50 MPa, respectively; normal constraint is applied around and at the bottom of the model. In the fluid model, no-slip boundary condition is applied inside the nozzle, initial pore pressure inside the rock is 30 MPa, water jet velocity at the nozzle inlet is 50, 100, 150, 200, and 250 m/s, respectively, and ambient pressure, which is caused by the fluid column pressure, is 26, 28, 30, 32, and 34 MPa,
respectively. The bottom-hole differential pressure (Arash et al., 2009) is used to describe the three different drilling states: underbalanced drilling, balanced drilling, and overbalanced drilling.

### Numerical simulation solution of the fluid-solid coupling model

#### Solid model

The solid element (Zienkiewicz and Taylor, 2000; Zeng, 2004) is a three-dimensional, 20-node hexahedron element, and the solid physical model is divided into 22,400 units. The shape functions of the element node are:

\[
\begin{align*}
N_i &= \frac{1}{8}(1 + \xi_0)(1 + \eta_0)(1 + \zeta_0)(\xi_0 + \eta_0 + \zeta_0 - 2) & (i = 1, 3, 5, 7, 13, 15, 17, 19) \\
N_i &= \frac{1}{4}(1 - \xi^2)(1 + \eta_0)(1 + \zeta_0) & (i = 2, 6, 14, 18) \\
N_i &= \frac{1}{4}(1 - \eta^2)(1 + \xi_0)(1 + \zeta_0) & (i = 4, 8, 16, 20) \\
N_i &= \frac{1}{4}(1 - \zeta^2)(1 + \xi_0)(1 + \eta_0) & (i = 9, 10, 11, 12)
\end{align*}
\]

For node \(i \), \(\xi_0 = \xi_1 \xi, \eta_0 = \eta_1 \eta, \) and \(\zeta_0 = \zeta_1 \zeta.\)

The solid model is solved by the finite element method and the equation is:

\[
\int_\Omega B^T D B^e d\Omega - \int_\Omega N^T b d\Omega + \int_s N^T \bar{p} dA = 0,
\]

where \(B\) is the geometric stiffness matrix; \(B = |\partial|N, |\partial|\) is operator matrix of geometric equation; \(N\) is shape function matrix; \(D\) is elastic coefficient matrix; \(b\) is body force matrix; \(\bar{p}\) is boundary force matrix; and \(q^e\) is node displacement matrix.

#### Fluid model

The fluid element (Anderson, 1995) is a three-dimensional 8-node hexahedron element and the fluid model is divided into 60,000 units. Annulus fluid and pore fluid physical models are solved by the finite volume method, and the discrete equation of annulus fluid is:

\[
\int_V (h' \phi' + Q' \cdot \nabla h') dV - \oint h' Q' \cdot dS = 0,
\]

where \(f\) is \(p\) (pressure), \(\mathbf{v}\) (velocity vector), and \(\theta\) (temperature) for the continuity equation, momentum equation, and energy equation, respectively; \(h'\) are step functions and...
\[ G^p = \frac{\partial p}{\partial t} + \nabla \cdot (\rho v), \quad G^v = \frac{\partial p v}{\partial t} + \nabla \cdot (\rho v v - f^B), \]
\[ G^\theta = \frac{\partial p E}{\partial t} + \nabla \cdot (\rho v E - \tau \cdot v) - f^B \cdot v - q^B, \]
\[ Q^p = 0, \quad Q^v = \tau, \quad Q^\theta = -q. \] (7)

The discrete equation of continuity equation and energy equation of pore fluid is the same as the discrete equation of annulus fluid; the discrete equation of momentum equation of pore fluid is:
\[ \int_V (\mu k^{-1} \cdot v - f^B) dV + \int_S p dS = 0. \] (8)

The influence of temperature is not taken into consideration, hence, \( q^B = 0, \quad q = 0. \)

Besides Eqs. (1)–(8) above, the computational parameters and boundary conditions are also used during the numerical simulation solution.

**Results**

In this article, the influences of the water jet and the bottom-hole differential pressure on the bottom-hole rock stress field are primarily studied.

**Distribution of the pore pressure on the impact surface**

In order to study the influence of water jet on the pore pressure within the rock, overbalanced drilling is taken as an example to be studied. The bottom-hole differential pressure is 4 MPa and the water jet velocity is 50, 100, 150, 200, and 250 m/s. Figure 2 indicates that the pore pressure on the impact surface rapidly decreases in the form of cubic parabola with the increase of the radial distance within the radial distance from 0 to 20 mm (two times nozzle radius). When the radial distance is more than 20 mm, it tends to be stable and the value is about 34.0 MPa. With the water jet velocity increasing from 50 to 250 m/s, the maximum pore pressure on the impact surface increases from 35.25 to 65.25 MPa.

By regression analysis of the computational data of pore pressure on the impact surface, the regression equation in polynomial form is:
\[ P_x = A^* x^3 + B^* x^2 + C^* x + D, \]

Figure 2. Distribution of the pore pressure on the impact surface.
where \( x \) is radial distance, mm; \( A, B, C, \) and \( D \) are the constant coefficients; and \( P_x \) is the pore pressure at the radial distance of \( x \) on the impact surface, MPa. The regression coefficients are shown in Table 2.

**Distribution of the pore pressure along the impact axis**

Figure 3 indicates that the pore pressure along the impact axis rapidly decreases in the form of cubic parabola with the axial distance increasing from 0 to 40 mm (4 times nozzle radius). The higher the water jet velocity is, the more quickly it decreases. When the axial distance is more than 40 mm, it will gradually decrease and tend to be stable, and its value is about 30.0 MPa.

Also, by regression analysis of the computational data of pore pressure along the impact axis, the regression equation in polynomial form is:

\[
P_z = a \cdot z^3 + b \cdot z^2 + c \cdot z + d,
\]

where \( z \) is the axial distance, mm; \( a, b, c, \) and \( d \) are the constant coefficients; and \( P_z \) is the pore pressure along the impact axis, MPa. The regression coefficients are shown in Table 3.

Distribution of the bottom hole pore pressure in this article is well in agreement with the theoretical and experimental results (Hu et al., 2001; Zuo et al., 2007).

### Table 2. Regression coefficients of the pore pressure on the impact surface.

<table>
<thead>
<tr>
<th>Jet Velocity, m/s</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( D )</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.0007</td>
<td>-0.0202</td>
<td>0.0361</td>
<td>35.25</td>
<td>-3E-05</td>
<td>0.0025</td>
<td>-0.0755</td>
<td>34.83</td>
</tr>
<tr>
<td>100</td>
<td>0.0036</td>
<td>-0.0914</td>
<td>0.1452</td>
<td>39.00</td>
<td>-0.0001</td>
<td>0.0090</td>
<td>-0.2742</td>
<td>36.91</td>
</tr>
<tr>
<td>150</td>
<td>0.0066</td>
<td>-0.1849</td>
<td>0.2795</td>
<td>45.25</td>
<td>-0.0002</td>
<td>0.0222</td>
<td>-0.6724</td>
<td>41.07</td>
</tr>
<tr>
<td>200</td>
<td>0.0108</td>
<td>-0.3197</td>
<td>0.4717</td>
<td>54.00</td>
<td>-0.0004</td>
<td>0.0383</td>
<td>-1.1441</td>
<td>45.87</td>
</tr>
<tr>
<td>250</td>
<td>0.0170</td>
<td>-0.5050</td>
<td>0.7073</td>
<td>65.25</td>
<td>-0.0006</td>
<td>0.0552</td>
<td>-1.6065</td>
<td>50.44</td>
</tr>
</tbody>
</table>

### Table 3. Regression coefficients of the pore pressure along the impact axis.

<table>
<thead>
<tr>
<th>Jet Velocity, m/s</th>
<th>Cubic Coefficient, ( a )</th>
<th>Quadratic Coefficient, ( b )</th>
<th>Linear Coefficient, ( c )</th>
<th>Constant Coefficient, ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>-2E-6</td>
<td>0.0005</td>
<td>-0.0520</td>
<td>35.25</td>
</tr>
<tr>
<td>100</td>
<td>-1E-5</td>
<td>0.0028</td>
<td>-0.2118</td>
<td>39.00</td>
</tr>
<tr>
<td>150</td>
<td>-3E-5</td>
<td>0.0071</td>
<td>-0.5009</td>
<td>45.25</td>
</tr>
<tr>
<td>200</td>
<td>-7E-5</td>
<td>0.0137</td>
<td>-0.9248</td>
<td>54.00</td>
</tr>
<tr>
<td>250</td>
<td>-0.0001</td>
<td>0.0230</td>
<td>-1.5024</td>
<td>65.25</td>
</tr>
</tbody>
</table>

*Figure 3. Distribution of the pore pressure along the impact axis.*
Distribution of the bottom-hole stress field under different differential pressures

With the water jet velocity of 100 m/s and the bottom-hole differential pressure of –4, –2, 0, 2, and 4 MPa, respectively, distributions of the maximum principal stress of the bottom-hole surface in the radial and axial direction are mainly studied. Figure 4 indicates that the maximum principal stresses on the bottom-hole surface under different differential pressures in the radial direction are similar. They rapidly decrease with the radial distance increasing from 0 to 20 mm (two times nozzle radius) and slowly decrease with the radial distance increasing from 20 to 150 mm. When the bottom-hole differential pressure decreases from 4 to –4 MPa, the corresponding maximum principal stress at the same radial distance shows a tendency of decrease with the same gradient.

Figure 5 indicates that the maximum principal stresses along the impact axis under different differential pressures vary from the tensile stress to the compressive stress with the increase of the axial distance. When the axial distance is increasing from 0 to 25 mm, they linearly decrease to 0 MPa with the increase of the axial distance and decrease with the decrease of differential pressure at the same axial distance. When the axial distance is more than 25 mm, they approximately linearly decrease and do not change with the differential pressure.

Figure 4. Distribution of the maximum principal stresses on the bottom-hole surface under different bottom-hole differential pressure.

Figure 5. Distribution of the maximum principal stresses along the borehole axis under different bottom-hole differential pressure.
**Distribution of the bottom-hole stress field under different water jet velocity**

With the bottom-hole differential pressure of 4 MPa and the water jet velocity of 0, 50, 100, 150, 200, and 250 m/s, respectively, distributions of the maximum principal stress of the bottom-hole surface in the radial and axial direction are mainly studied. Figure 6 indicates that the maximum principal stresses on the bottom-hole surface under different water jet velocity are the tensile stress in the radial direction. They rapidly decrease with the radial distance increasing from 0 to 20 mm and the higher the water jet velocity is, the more quickly it decreases. When the radial distance is more than 20 mm, the maximum principal stress at the same radial distance slowly decreases and does not change with the bottom-hole differential pressure.

Figure 7 indicates that the maximum principal stresses along the impact axis under different water jet velocities change from the tensile stress to the compressive stress. When the axial distance is from 0 to 25 mm, they are all the tensile stress and increases with the increase of the water jet velocity. The depth of the tensile stress along the impact axis increases from 25 to 35 mm with the water jet velocity increasing from 50 to 250 m/s. When the axial distance is more than 35 mm, they decrease with the increase of the axial distance and keep steady under the different water jet velocities.

![Figure 6](image1)

**Figure 6.** Distribution of the maximum principal stresses on the bottom-hole surface under different jet velocity.

![Figure 7](image2)

**Figure 7.** Distribution of the maximum principal stresses of the borehole axis under different jet velocity.
Conclusion

(1) The local effect of a water jet is obvious; the main affected area is about 2 times jet radius on the impact surface and 2.5 to 3.5 times jet radius along the impact axis when water jet velocity increases from 50 to 250 m/s. The maximum jet impact force is proportional to the square of the jet velocity, and distributions of the pore pressure on the impact surface and along the impact axis has been obtained by regression analysis of the result on the basis of calculated parameters in the article.

(2) When the water jet velocity is 100 m/s, the maximum principal stresses on the impact surface increase with the differential pressure increasing from –4 to 4 MPa on the whole bottom-hole surface. The maximum principal stresses along the impact axis increase with the differential pressure increasing from –4 to 4 MPa within the distance of 2.5 times jet radius and do not change with the differential pressure within the distance more than 2.5 times jet radius.

(3) When the bottom-hole differential pressure is 4 MPa, the maximum principal stresses on the impact surface rapidly increase with the jet velocity increasing from 0 to 250 m/s within the distance of two times jet radius and do not change with the jet velocity within the distance of more than two times jet radius. The maximum principal stresses along the impact axis rapidly increase with the jet velocity increasing from 0 to 250 m/s within the distance of 2.5 to 3.5 times jet radius and do not change with the jet velocity within the distance of more than 3.5 times jet radius.

Funding

The authors wish to express their appreciation for the support provided by the National Basic Research Program of China (973 Program, 2010CB226704).

References

Cohen, J. H., Deskins, G., and Rogers, J. 2005. High-pressure jet kerf drilling shows significant potential to increase ROP. SPE Paper 96557. SPE Annual Technical Conference and Exhibition, Dallas, TX, October 9–12.


