Design of hollow cone pressure swirl nozzles to atomize Newtonian fluids

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Abstract

We present calculation models to predict the drop size depending on the hollow cone nozzle’s geometry and the volume flow for atomization of Newtonian fluids. The observations are valid for the field of the aerodynamic wave break-up of a swirling liquid sheet. The calculation methods are based on the knowledge of the flow at the nozzle outlet. Assisted by a flow simulation program it was possible to calculate the detailed and locally discrete flow field inside the nozzle taking into account the arising phase interface. The method allows to determine the film thickness, the velocities and the angle of the film which leaves the nozzle outlet. The calculation methods developed, however, allow to describe the formation of the sheet, the alteration of the sheet thickness in the direction of its spreading as well as to determine the Sauter mean diameter after the break-up of the sheet and the ligament under the influence of the gas phase which is in interaction with the liquid. Following the calculation results extensive experimental investigations varying the geometry and the transport properties of the fluids, in particular the viscosity, were undertaken. Comparing the numeric results with the experimental data and with citations in the literature the success of the investigations becomes evident. Thus a calculation basis for the design of hollow cone nozzles is available which allows to predict the Sauter mean diameter as a function of individual parameters of operation and material. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Hollow cone nozzles excel themselves by their simple construction and high reliability in operation. They are used, e.g. for atomization of fuels, for spray-drying and in scrubbing towers for dust and gas. The nozzle’s principle of operation is simple. It consists of a distributor having spirally shaped grooves, of a cylindrical and of an adjacent conical part as well as of the nozzle orifice (Fig. 1).

The distributor generates a rotational flow which gets accelerated as the nozzle diameter decreases towards the outlet. Depending on the operation data, the transport properties and the geometry a rotationally symmetric gas core is formed in the area of the nozzle axis. The resulting liquid film at the nozzle outlet widens in the form of a cone after leaving the nozzle and disintegrates downstream into droplets.

In the literature, one finds according to the variety of application fields a significant number of investigations with the aim of which being able to predict the drop size. There are formulations to calculate the frictional flow inside the nozzle in Horvay (1985), Dumouchel, Bloor, Ingham and Ledoux (1991), Dumouchel, Bloor, Dombrowski, Ingham and Ledoux (1993), Löfler-Mang (1992) and Yule and Chinn (1994). However, these publications are limited to the calculation of the laminar nozzle flow without taking into account the phase interface which occurs in the technical use. Chinn and Yule (1997) deal with nozzles where the swirl chamber has tangential holes. Piesche (1981) presented calculation models to describe the formation of the sheet contour neglecting viscous influences. But these are only conditionally suited for a detailed analysis of the atomization of liquids of higher viscosity. A mathematical–physical model for the break-up of plane liquid sheets was developed by Squire (1953) as well as by Dombrowski and Johns (1963). The resulting calculation equations for the Sauter mean diameter exhibit considerable differences when compared to the experimental data concerning the spraying of oils; cf. Dombrowski and Tahir (1977), Rangel and Sirignano (1991) and Camatte, Cousin,
Ledoux and Versaevel (1994) extended these models in the form of non-linear stability analyses. Apart from these publications many empirical relations to determine the drop size are known in the literature. We cite here among others the publications of Walzel (1982, 1990), Schorradt (1984), Richter and Glaser (1987), Richter and Walzel (1989), Dahl (1992). Result: The break-up of the sheet at the nozzle outlet can be described at least qualitatively by half-empirical models. The drop size can be predicted only to a limited extent. However, the calculation equations fail already at medium viscosities. Neither knowledge of the flow processes and the formation of the phase interface inside the nozzle nor the area of the nozzle outlet is satisfying. Yet these flow processes characterize the extension behavior of the liquid sheet, i.e. the value of the radial, azimuthal and axial velocities as well as the sheet thickness and its alteration up to the break-up (Fig. 2).

Thus, the flow field within the nozzle taking into account individual dimensions of geometry and of transport properties is to be calculated by the aid of the numeric flow simulation. The investigations include the development of calculation methods to determine the alteration of the sheet thickness along the flow direction and to describe the break-up of the sheet and afterwards of the ligament. Finally, the aim is to make possible statements about the drop size suitable for industrial applications.

2. Calculation models

The theoretical investigations are based on partitioning of the fluid dynamic processes within and outside the nozzle in the four following model sections:

- flow inside the nozzle,
- sheet contour,
- sheet break-up and
- ligament break-up.

The individual model sections are coupled with each other by the transmission of characteristic model parameters. The flow field in the nozzle is calculated to determine the film thickness and the velocities at the nozzle outlet depending on the distributor geometry and the volume flow. These calculations supply the data which serve as initial values to describe the sheet contour. The contour and the resulting change in film thickness are of particular importance for the drop size. The calculation methods for the instability behavior of the sheet yield the diameter of the ligament in the form of a torus which results at the moment of the break-up and which disintegrates into drops by interaction with the gas phase.

2.1. Calculation of the flow inside the nozzle

The geometry of the distributor and of the nozzle can be seen in Fig. 3. At the inlet of the distributor there is a volume flow \( V \). It is separated into partial flows by inlet grooves. The grooves are inclined towards the vertical to the rotational axis by an angle \( \beta \) and thus create a swirling flow of which the axial and azimuthal velocity components depend on the surface \( f \approx n(r_a - r_i)a/\sin \beta \) axially covered by the flow, where \( n \) is the groove number, \( r_a \) is the outer limiting radius and \( r_i \) is the inner limiting radius and \( a \) is the groove width. In the swirl chamber having the conus angle \( c \), the length \( h \), the outlet length \( h_o \) and the exit radius \( r_o \), the liquid having the density \( \rho_f \), the surface tension \( \sigma_{f,g} \) and the dynamic viscosity \( \mu_f \) undergoes an alteration of velocity, whereby the pressure changes by the difference \( \Delta p_D = p_{D,a} - p_{D,o} \).

The calculation of the flow inside the nozzle was carried out with the aid of the simulation software FLUENT (Nonnenmacher & Piesche, 1999). The numeric investigations were effected under the condition of laminar time-dependent flow. The calculations are based on rotationally symmetric boundary conditions. The width of the bars between the grooves of the distributor has no
influence on the values of interest at the outlet of the nozzle. This will be shown later in comparison with the experiment. Therefore, the gradients of the flow in circumferential direction can be neglected. This reduces the model description to a two-dimensional axisymmetric problem with respect to geometry and grid. Not dependent on this the circumferential velocity with radial and axial gradients is determined by solving the momentum equation in circumferential direction. In order to prevent the boundary conditions located downstream from influencing the calculated flow inside the nozzle in particular at the nozzle outlet the calculation area includes also the outer area of the nozzle nearby the outlet. The computational area was divided into a structured non-uniform grid of 45524 cells. Thereby grid spacing was reduced in regions where the phase interface could be expected. The phase distribution itself was calculated using the volume of fluid (VOF) model. Since the surface tension $\sigma_{f,a}$ is of minor importance during the flow processes in the nozzle, only the Reynolds number

$$Re = \frac{\rho_f V}{\mu_f}$$  \hspace{1cm} (2.1)

occurs as a characteristic dimensionless parameter.

2.2. Calculation of the sheet contour

In Fig. 4, the rotationally symmetric sheet contour which is submitted to spin and to pressure is observed. We have a three-dimensional movement, $u$ being the radial, $v$ the azimuthal and $w$ the axial velocity component. To describe the outer sheet contour having the width $d$, the length $dL$ and the normal tensions $\tau_{rr}$ and $\tau_{\theta\theta}$ in the $r$, $\theta$, $z$ plane, only the Reynolds number

$$Re = \frac{\rho_f V}{\mu_f}$$  \hspace{1cm} (2.1)

2.2. Calculation of the sheet contour

In Fig. 4, the rotationally symmetric sheet contour which is submitted to spin and to pressure is observed. We have a three-dimensional movement, $u$ being the radial, $v$ the azimuthal and $w$ the axial velocity component. To describe the outer sheet contour having the radius $r_L$, cylindrical coordinates with the nomenclature $r$, $\theta$, $z$ are taken as a basis. Outside the liquid sheet there is the pressure $p_{a,i}$ and inside there is the pressure $p_{f,i}$ which is in our case constant and not necessarily equal to $p_{D,i}$. The force balance at an annular element having the width $dr$ and the length $dz$ provides under the presumptions and preconditions

- steady, laminar, elongational sheet flow,
- neglect of viscous forces in the gas phase,
- constant axial hollow jet velocity $w$, and
- small curvature radius of the contour in the $r$, $z$ plane opposed to that in the $r$, $\theta$ plane,
- the equation of motion in radial direction

$$\rho_f \left( u \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial r} + \tau_{rr} + \frac{1}{r} (\tau_{rr} - \tau_{\theta\theta})$$  \hspace{1cm} (2.2)

with the normal tensions

$$\tau_{rr} = 2\mu_f \frac{\partial u}{\partial r} \quad \text{and} \quad \tau_{\theta\theta} = 2\mu_f \frac{u}{r}.$$  \hspace{1cm} (2.3)

In addition in azimuthal direction the torque sentence applies

$$\nu r = r_o \nu_{o,m}.$$  \hspace{1cm} (2.4)

Here the index $m$ means the averaged value of the velocity components at the nozzle outlet (index $o$).

The radial velocity change in the liquid film results from the continuity equation

$$u(r, z) = \frac{r_L}{r} \frac{dr_L}{dt} = \frac{w}{r} \frac{dr_L}{dz}.$$  \hspace{1cm} (2.5)

The relationship between contour radius $r_L$ and the film width $b$ results from the integral mass conservation

$$r_L^2 - (r_L - b)^2 = \text{constant.}$$  \hspace{1cm} (2.6)

Refs. (2.4) and (2.5) are inserted into the Eq. (2.2). Afterwards the integration concerning the film width takes place. Introducing the boundary conditions

$$p(r_L) - \tau_{rr}(r_L) = p_{\infty,a} + \frac{\sigma_{f,a}}{r_L}$$  \hspace{1cm} (2.7)

and

$$p(r_L - b) - \tau_{rr}(r_L - b) = p_{\infty,i} - \frac{\sigma_{f,a}}{r_L - b}$$  \hspace{1cm} (2.8)

as well as the approximation

$$\ln \left( \frac{r_L}{r_L - b} \right) = \frac{b}{r_L} + \frac{1}{2} \left( \frac{b}{r_L} \right)^2 + \ldots.$$  \hspace{1cm} (2.9)
the equations to describe the contour and the film width are

\[
\left( \frac{B}{R_L} \right)^2 \left( 1 + 2 \frac{r_o}{b_o} \frac{R_L}{B} \right) \left( \frac{r_o}{b_o} \frac{R_L}{B} - 1 \right)^2 \times \left[ R_L \frac{d^2 R_L}{dZ^2} + \left( \frac{dR_L}{dZ} \right)^2 \right] \frac{r_o}{b_o}^{-2} \\
+ \left( 1 - 2 \frac{r_o}{b_o} \frac{R_L}{B} \right) \left( \frac{dR_L}{dZ} \right)^2 + \left( 1 - 2 \frac{r_o}{b_o} \frac{R_L}{B} \right) \frac{D^2}{R_L^2} \\
- 2 \left( \frac{r_o}{b_o} \frac{R_L}{B} - 1 \right)^2 E_u + \left( 2 \frac{r_o}{b_o} \frac{R_L}{B} - 1 \right) \frac{r_o}{b_o} \frac{R_L}{B} - 1 \right) \times \frac{2}{WcR_L} \left( 1 - 2 \frac{r_o}{b_o} \frac{R_L}{B} \right) \frac{4}{RcR_L} \frac{dR_L}{dZ} = 0,
\]

(2.10)

\[ B^2 - 2 \frac{r_o}{b_o} (R_L B - 1) - 1 = 0. \]

(2.11)

Here the time variable is already substituted by the space variable (Piesche, 1981) and the equations are brought into a dimensionless form. The dimensionless, geometric values are

\[ R_L = \frac{r_L}{r_o}, \quad Z = \frac{z}{r_o} \quad \text{and} \quad B = \frac{b}{b_o}. \]

(2.12)

By these normalizations we obtain the following characteristic dimensionless numbers:

- **Length ratio**: \( \frac{r_o}{b_o} \),
- **Euler number**: \( E_u = \frac{\rho_{\infty, i} - \rho_{\infty, a}}{\rho_f w_{o,m}^2} \),
- **Weber number**: \( W_e = \frac{\rho_f w_{o,m}^2 r_o}{\sigma_{f,a}} \),
- **Reynolds number**: \( R_e = \frac{\rho_f w_{o,m} r_o}{\mu_f} \),
- **Swirl parameter**: \( D = \frac{w_{o,m}}{w_{o,m}} \).

(2.13) to (2.17)

The boundary conditions needed for the integration are

\[ R_L(Z = 0) = 1 \quad \text{and} \quad \frac{dR_L}{dZ}(Z = 0) = \frac{u_{o,m}}{w_{o,m}}. \]

(2.18)

With the values known from the simulation of the flow inside the nozzle for the film width \( b_o \), the averaged axial velocity \( w_{o,m} \), the azimuthal velocity \( v_{o,m} \) and the radial velocity \( u_{o,m} \), the numeric solution of the differential Eq. (2.10) is effected.

### 2.3. Calculation of the break-up of the sheet

In the following, the break-up process of a rotationally symmetric liquid film is discussed. The calculation model refers to the configuration in Fig. 5.

The break-up is due to disturbances induced by flow forces which lead to the formation of an oscillation with increasing amplitude. The break-up into liquid ligaments is the result. With the aid of a force balance (Dombrowski & Johns, 1963) at an annular element having the longitudinal extension \( dx \) and width \( b \cos \gamma \) the amplitude growth can be determined. In order to make the problem posed accessible for a theoretical treatment the arithmetic effort is limited as follows:

- The frictional forces at the phase interface are not taken into account.
- The shear flow is laminar and unsteady.
- The radius of the sheet contour \( r_L \) is much greater than the sheet width \( b \) and the amplitude of the oscillation \( y \).
- The influence of the centrifugal force is neglected.
- The surface tension force in circumferential direction is not taken into account.

With these limitations the following forces in \( y \)-direction occur:

- **Pressure force**: \( dF_p = 2 \pi \rho g (w^2 + v^2 + u^2) y 2 \pi r_L \) \( dx \),
- **Surface tension force**: \( dF_s = \frac{\partial}{\partial x} \left( 2 \pi r_L \frac{\partial y}{\partial t} \right) dx \),
- **Inertia force**: \( dF_i = - \frac{\partial}{\partial t} \left( 2 \pi r_L \left[ 1 + \left( \frac{dr_L}{dz} \right)^2 \right]^{-0.5} \frac{\partial y}{\partial t} \right) dx \),
- **Friction force**: \( dF_u = \frac{\partial}{\partial x} \left( \mu_f 2 \pi r_L \left[ 1 + \left( \frac{dr_L}{dz} \right)^2 \right]^{-0.5} \frac{\partial y}{\partial t} \right) dx \).

(2.19) to (2.22)

If one neglects minor terms, with the reduced coordinates \( X = x/r_o, Y = y/r_o \) and the time \( T = t w_{o,m}/r_o \) as well as...
the velocities \( W = w/w_{o,m} \), \( U = u/w_{o,m} \) and \( V = v/w_{o,m} \) the balance of forces in dimensionless form is
\[
\frac{\partial^2 Y}{\partial T^2} = \frac{2}{B} \frac{\partial \rho_f}{\partial Z} \left[ 1 + \left( \frac{\partial R_L}{\partial Z} \right)^2 \right]^{0.5} (W^2 + V^2 + U^2) Y \\
+ \frac{2}{B \omega_e b_o} \left[ 1 + \left( \frac{\partial R_L}{\partial Z} \right)^2 \right]^{0.5} \frac{\partial Y}{\partial X^2} + \frac{1}{Re} \frac{\partial^3 Y}{\partial T} 
\]
(2.23)
with the wave number \( \tilde{a} = \pi r_a \).

The stability investigation is now an eigenvalue problem. With the solution
\[
Y = Y_e \exp (\tilde{a} X),
\]
we obtain the equation to determine the growth rate \( F(T) \) for
\[
\frac{d^2 F}{dT^2} + \left( \frac{dF}{dT} \right)^2 + \frac{2}{Re} \frac{\partial F}{\partial T} - \frac{2}{B} \frac{\partial \rho_f}{\partial Z} \left[ 1 + \left( \frac{\partial R_L}{\partial Z} \right)^2 \right]^{0.5} (W^2 + V^2 + U^2) + \frac{2}{B \omega_e b_o} \left[ 1 + \left( \frac{\partial R_L}{\partial Z} \right)^2 \right]^{0.5} = 0.
\]
(2.25)

The initial conditions are
\[
F(T = 0) = 0 \quad \text{and} \quad \frac{dF}{dT} (T = 0) = 0.
\]
(2.26)

Experimental investigations (Dombrowski & Hooper, 1962) for atomization of liquids of low viscosity show that for a critical wave length \( \lambda_{krit} = 2\pi/\lambda_{krit} \) the growth rate aims at the limiting value \( F = 12 \). When atomizing liquids of higher viscosity the maximal growth rate has to be determined via the measurement of the break-up time in an extension of the experiments. From the break-up of the liquid film a ligament results having the radius
\[
R_{Lg} = \frac{r_{Lg}}{r_o} = \left( \frac{B}{\lambda_{krit}} \frac{r_o}{b_o} \right)^{0.5}.
\]
(2.27)

The solution of the Eq. (2.25) to determine the critical wave number is done simultaneously with Eqs. (2.10) and (2.11) so that at each moment the film thickness, the contour radius \( R_L \) and the velocities \( W, U \) and \( V \) are known.

### 2.4. Calculation of the break-up of the ligament

In the following, the approximation method to determine the drop size at the break-up of the ligament induced by transversely oncoming flow is deduced. The calculation model is outlined in Fig. 6. The surface is subject to an unsteady deformation having the deviation \( \delta \). A surface pressure is the result which depends functionally on the surface tension, the radii of curvature and on the outer pressure \( p_{Lg} \) at the phase interface. The outer pressure in turn is coupled with the flow conditions in the vicinity of the oscillating contour. The acting forces now attempt to increase the deviation \( \delta \) of the initial cylinder contour having the radius \( r_{Lg} \). For the mathematical determination some presumptions and preconditions are set:

- The ligament flow is laminar and unsteady.
- The annular ligament moves at the velocity \( (u^2 + v^2 + w^2)^{0.5} \) like it is present at the moment of the film break-up.
- The influence of the friction in the gas phase is neglected.\textcolor{red}{\text{1}}
- In the vicinity of the phase interface the gas phase moves having the velocity components \( u_{g,t}, v_{g,t} \) and \( w_{g,t} \), whereby a variable pressure \( p_{Lg} \) at the surface of the phase interface results.
- The axial velocity of the ligament \( w_c \) has a constant distribution in the direction of the radial coordinate \( \eta \).
- The local acceleration (\( \hat{\nabla}w_c/\hat{\nabla}t \)) is much greater than the convective component (\( \nabla w_c/\nabla \zeta \)).

With these data in the model description the following equations result:

**Equation of motion:**
\[
p \rho Lg^2 \frac{\hat{\nabla}w_c}{\hat{\nabla}t} = \frac{\hat{\nabla} \left[ \eta Lg d \left( p_{\infty} - p \right) \right]}{\hat{\nabla} \zeta} + 2 \sigma_{Lg} \frac{\hat{\nabla} \left( \frac{\nabla \eta Lg \left( \hat{\nabla} \eta Lg \right)^2 \right)^{-0.5}}{\hat{\nabla} \zeta} + 3 \eta Lg \frac{\hat{\nabla} \left( \frac{\nabla \eta Lg \left( \hat{\nabla} \eta Lg \right)^2 \right)^{-1.5}}{\hat{\nabla} \zeta}. \tag{2.28}
\]

**Laplace equation:**
\[
p = p_{Lg} + \sigma_{Lg} \left\{ \frac{1}{\eta Lg} \left[ 1 + \left( \frac{\nabla \eta Lg}{\hat{\nabla} \zeta} \right)^2 \right]^{-0.5} - \frac{\nabla^2 \eta Lg}{\hat{\nabla} \zeta} \left[ 1 + \left( \frac{\nabla \eta Lg}{\hat{\nabla} \zeta} \right)^2 \right]^{-1.5} \right\}. \tag{2.29}
\]

**Continuity equation:**
\[
u_{g,t} = \frac{\hat{\nabla} \delta}{\hat{\nabla} t} = -\frac{1}{2} \eta Lg \frac{\hat{\nabla} w_c}{\hat{\nabla} \zeta}. \tag{2.30}
\]
The unknown pressure \( p_L \) at the phase interface is derived from the equation of motion of a potential flow around a moved cylinder having a surface deviation

\[
\delta = \delta_0 \exp(qt + ik(\xi + \zeta)),
\]  
(2.31)

(Kitamura & Takahashi, 1976); \( q \) is the growth rate of an initial disturbance having the amplitude \( \delta_0 \) and the wave number \( k \). The velocity potential of the gas phase

\[
\Phi = (u^2 + v^2 + w^2)^{0.5} \left( \eta + \frac{r_L}{\eta} \right) \cos \xi
\]

\[
+ \Phi_1(\eta) \exp(qt + ik(\xi + \zeta))
\]

(2.32)

with the disturbance potential \( \Phi_1 \) fulfills the potential equation

\[
\eta^2 \frac{d^2 \Phi_1}{d\eta^2} + \eta \frac{d\Phi_1}{d\eta} - (1 + k^2 \eta^2) \Phi_1 = 0.
\]

(2.33)

Its solution is

\[
\Phi_1 = \text{Const.} M_1(\eta k).
\]

(2.34)

The constant \( \text{Const.} \) is determined as a result of the boundary conditions of the oscillating phase interface

\[
\text{Const.} = - \frac{\delta_0}{kM_0 + \frac{M_1}{r_L}}
\]

\[
\times \left[ q - (u^2 + v^2 + w^2)^{0.5} \frac{i}{r_L} \sin \xi \right].
\]

(2.35)

With the Bernoulli equation

\[
\rho_0 \frac{d\Phi}{dt} + \frac{1}{2} \rho_0 (v_\eta^2 + w_\eta^2 + u_\eta^2) \eta = \eta_\alpha + p_L - \text{constant}
\]

(2.36)

the relations outlined provide the maximal pressure difference \( p_L - p_\infty \) at the phase interface

\[
p_L = p_\infty + \rho_0 \left[ q^2 - \frac{4}{r_L^2} (u^2 + w^2 + v^2) \right]
\]

\[
\times \left\{ \frac{M_1 \delta}{kM_0 + \frac{M_1}{r_L}} - \frac{1}{2} \left[ 1 - \left( \frac{M_1}{M_0 + \frac{M_1}{r_L}} \right) \right] \right\}^2
\]

\[
- \left( \frac{M_1}{M_0 + \frac{M_1}{r_L}} \right)^2 \delta^2 \right\},
\]

(2.37)

where \( M_0 \) and \( M_1 \) are the Mac-Donald functions (Bronstein & Semendjajew, 1989).

With the Eqs. (2.28)–(2.31) and (2.37) one obtains an equation to determine the growth rate \( Q = q r_n/w_{m} \) with the predetermined wave number \( K = k r_n \). When neglecting minor terms this is in dimensionless form

\[
\left( 1 + \frac{\rho_g}{2 \rho_f} R_{Li} K \right) \left( \frac{M_1}{M_0 + \frac{M_1}{K R_{Li}}} \right) Q^2 + \frac{3}{Re} K^2 Q
\]

\[
- \frac{K^2}{2 We R_{Li}} (1 - R_{Li}^2 K^2)
\]

\[
- \frac{2 \rho_g}{\rho_f} \frac{K}{R_{Li}} \left( \frac{M_1}{M_0 + \frac{M_1}{K R_{Li}}} \right) \left( U^2 + W^2 + V^2 \right) = 0.
\]

(2.38)

For a critical wave number \( K_{krit} \) a maximal growth rate \( Q \) results. Thus the Sauter mean diameter is calculated

\[
\frac{d_{32}}{r_n} = \left( \frac{12 \pi}{K_{krit} R_{Li}} \right)^{1/3}
\]

(2.39)

3. Comparison of calculation results and experiments

A comparison of numeric results with measurements was effected on the basis of the data taken from the literature (Dahl, 1992; Klein, 1990) as well as with own experimental data. The main emphasis concentrated on the atomization of liquids in the viscosity range of \( 10^{-1} \) Pas > \( \mu_f \geq 10^{-3} \) Pas. Water and glycerin — water mixtures served as experimental liquids. The gas phase was air at room temperature and atmospheric pressure. Table 1 shows the dimensions of the distributors and nozzles used which are the basis for the comparison.

The groove number \( n = 6 \) and conus angle \( c = 45^\circ \) were constant. The basis to determine the drop size was the laser diffraction spectroscopy.

Figs. 7 and 8 show the results of a simulation calculation of the flow inside the nozzle. Next to the calculated phase interface and the gas core, respectively, the velocity curves and pressure curve in the liquid phase and in the gas phase are qualitatively shown as a function of the radius at scaled distances from the nozzle outlet. The
Fig. 7. Phase interface, pressure and axial velocity inside the nozzle.

Fig. 8. Phase interface, radial and azimuthal velocity inside the nozzle.
The fluid for atomization in this example is a mixture of glycerin and water having a viscosity \( \mu_f \) of 10 mPas and a density \( \rho_f \) of 1150 kg/m\(^3\). The volume flow \( \dot{V} \) is 0.8 m\(^3\)/h. The surrounding gas phase is air at room temperature and atmospheric pressure. The radius of the nozzle outlet \( r_o \) is 4.0 mm and the angle of the distributor is 30°. The Reynolds number of the nozzle \( Re_D \) is 2000, approximately. Like the phase interface which is experimentally observed with a transparent nozzle, the calculated phase interface extends up to the distributor. The film thickness at the orifice area which decreases continuously and the detaching of the sheet are also correctly represented in the simulation.

In the following some typical results from a great number of equivalent measurements are chosen to show the comparison with numeric results and this concerning the pressure loss in the nozzle, the thickness of the liquid film, the velocities at the nozzle outlet, the sheet contour and the Sauter mean diameter.

In Fig. 9 the pressure loss in the form of the Euler number \( Eu_D = \frac{(p_{D,a} - p_{D,o})}{\frac{\mu_f}{2}(\dot{V}/f)^2} \) and in Fig. 10 the dimensionless thickness of the liquid film \( b_o/r_o \) are shown as a function of the Reynolds number \( Re_D \). It shows that the Reynolds number with geometric similarity is the relevant dimensionless number to predict the film thickness and the pressure loss inside the nozzle. An approximation solution for the case of inviscid flow is also mentioned (Piesche & Nonnenmacher, 1998) which allows a first estimation of the thickness of the liquid film and the change in pressure when predetermining the surface \( A = f/\pi r_o^2 \), the swirl angle \( \beta \) and the outer radius \( R_o = r_o/r_a \) of the swirl chamber. The example shows that it is sufficient at least in the range \( Re_D > 10^3 \) to take as a basis the results of the approximation method to predict the film thickness and the change in pressure.

Figs. 9 and 10 show additionally that in the range \( Re_D < 10^3 \) the atomization process, i.e. the drop formation, is very much influenced by increases in viscosity and decreases in volume flow.

Fig. 11 shows the comparison of numerically calculated radial, azimuthal and axial velocities at the outlet \( u_o,m, v_o,m, w_o,m \) with the experimental data for a glycerin–water mixture having a dynamic viscosity \( \mu_f = 0.075 \) Pas, a density \( \rho_f = 1215 \) kg/m\(^3\) and a surface tension \( \sigma_f,g = 0.066 \) N/m. The outlet radius is \( r_o = 2.65 \) mm and the distributor angle \( \beta = 30^\circ \). The experimental determination of the film thickness and the outlet velocities \( w_o,m \) and \( v_o,m \) is effected indirectly by measuring in a predetermined distance to the nozzle the axial and azimuthal momentum forces of the hollow jet leaving the nozzle and by correlating them with the...
conditions at the orifice postulating that the momentum and the mass are maintained.

Fig. 12 shows sheet contours for the operation point $\dot{V} = 0.6 \text{ m}^3/\text{h}$. The data concerning the contours which are experimentally investigated based on photographic pictures. The oscillating liquid film exhibits a longer break-up distance when the viscosity increases. The spray angles near the nozzle orifice are reproduced correctly, i.e. within the frame of measurement exactitude the consistency of calculation and experiment is quite satisfactory.

Analogous statements can also be made for the Sauter mean diameter. Fig. 13 shows a comparison of experiment and calculation for a determined nozzle geometry when water is atomized. Here, the calculations for the drop size are based on the frictionless approximation solution for the flow inside the nozzle. Certain deviations result at larger volume flows. This is because the break-up of the liquid film no longer occurs only due to aerodynamic disintegration but at large Reynolds numbers also due to turbulence of the liquid.

With media of higher viscosity and thus with smaller Reynolds numbers, the parameters at the nozzle outlet have to be determined with the aid of the numeric simulation (cf. Figs. 9 and 10). Regarding a nozzle which is somewhat larger, Fig. 14 shows the comparison of the calculated Sauter mean diameter with the measured Sauter mean diameter for the borderline case — seen from the technical viewpoint — when a fluid having a viscosity of 25 mPa s is atomized. Experiment and calculation correspond well in view of the complexity of the atomization process.

A comparison of theoretical with experimental results show that with the aid of numeric simulation and with the calculation model for drop formation under the precondition of Newtonian fluid properties the prediction of the Sauter mean diameter is possible and the reflections made are suited for the practical need to construct a hollow cone nozzle.

**Notation**

- $a$: groove width
- $b$: film thickness
- $d$: drop diameter
- $f$: surface
- $F$: growth rate
- $h$: length
- $k$: wave number
- $n$: groove number
- $p$: pressure
- $q$: growth rate
\( r \) radius
\( r, \varphi, z \) coordinates
\( u, v, w \) radial, azimuthal, axial velocity
\( t \) time
\( \dot{V} \) volume flow
\( x, y \) coordinates

**Greek letters**

\( x \) wave number
\( \beta \) swirl angle
\( \gamma \) opening angle of the sheet
\( \delta \) deviation
\( \psi \) conus angle of the nozzle
\( \eta \) radius
\( \mu \) dynamic viscosity
\( \rho \) density
\( \sigma \) surface tension
\( \zeta, \eta, \xi \) coordinates

**Subscripts**

\( a \) outside
\( D \) nozzle
\( f \) liquid
\( g \) gas
\( i \) inside
\( L \) sheet
\( o \) nozzle outlet
\( L_g \) ligament
32 Sauter mean diameter
krit critical
\( m \) averaged

**References**


