MORTALITY DEPENDENCE AND LONGEVITY BOND PRICING: A DYNAMIC FACTOR COPULA MORTALITY MODEL WITH THE GAS STRUCTURE

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ABSTRACT
Modeling mortality dependence for multiple populations has significant implications for mortality/longevity risk management. A natural way to assess multivariate dependence is to use copula models. The application of copula models in the multipopulation mortality analysis, however, is still in its infancy. In this article, we present a dynamic multipopulation mortality model based on a two-factor copula and capture the time-varying dependence using the generalized autoregressive score (GAS) framework. Our model is simple and flexible in terms of model specification and is widely applicable to high dimension data. Using the Swiss Re Kortis longevity trend bond as an example, we use our model to estimate the probability distribution of principal reduction and some risk measures such as probability of first loss, conditional expected loss, and expected loss. Due to the similarity in the structure and design of CAT bonds and mortality/longevity bonds, we borrow CAT bond pricing techniques for mortality/longevity bond pricing. We find that our pricing model generates par spreads that are close to the actual spreads of previously issued mortality/longevity bonds.

INTRODUCTION
Mortality improvements around the world are putting more and more pressure on social security systems, pension funds, life insurance companies, and individuals, and thus call for more efficient management of longevity risk. Coping with this trend, longevity risk-related capital market solutions have grown in recent years. Many innovative products, such as mortality/longevity bonds, longevity swaps, buy-ins, and buy-outs, have been adopted (Tan, Blake, and MacMinn, 2015). A better understanding of the correlation among mortality improvements for multiple...
populations appears to be critical for issuers, investors, insurers, pension plans, and governments for several reasons (Cairns et al., 2011; Chen, MacMinn, and Sun, 2015). First, almost all mortality/longevity bonds are written on a weighted index based on the mortality experience of multiple populations. Issuers (or sponsors) need to understand mortality correlations in order to better design the mortality/longevity derivatives and price the premiums. Investors also need to analyze the mortality correlations to evaluate their risk and payoff. Second, life insurers that write both life insurance policies and annuities can naturally hedge mortality risk from insured lives with longevity risk from annuitants. Such a natural hedge, however, is not perfect. Therefore, insurers need to understand mortality correlations between these two groups and manage the residual risk. Third, for mortality/longevity risk hedgers that use capital market instruments, basis risk exists because the mortality experience of their own population is usually different from that of the population(s) associated with the standardized hedging instruments. A model that captures mortality dependence among multiple populations can help them determine hedge ratios and minimize basis risk.  

There has been a growing literature on mortality models for multiple populations in recent years (see, e.g., Li and Lee, 2005; Cairns et al., 2011; Dowd et al., 2011; Jarner and Kryger, 2011; Li and Hardy, 2011; Yang and Wang, 2013; Zhou, Li, and Tan, 2013; Zhou et al., 2014). These models typically assume that the forecasted mortality experiences of two or more related populations are linked together and do not diverge over the long run. This assumption might be justified by the long-term mortality comovements. There is, however, little evidence to suggest mean reversion to a constant difference in relative mortality rates in the short run (Hunt and Blake, 2015). Mackenbach et al. (2003) and Waldron (2007) also document a divergence in mortality between different socioeconomic groups in recent years. We, therefore, develop our multipopulation mortality model following another stream of dependence modeling, that is, copulas. We introduce a time-varying dependence structure using the generalized autoregressive score (GAS) model. We use the Kortis longevity bond, the first longevity trend bond issued by Swiss Re in 2010, as an example to illustrate how this model can be used for mortality risk modeling and pricing.

Copulas have been studied in both actuarial science and finance to examine dependencies among risks (Frees and Valdez, 1998; Embrechts, Lindskog, and McNeil, 2003). In mortality studies, copulas have been applied to model the bivariate survival function of the lives of couples (see, e.g., Frees, Carriere, and Valdez, 1996; Carriere, 2000; Denuit et al., 2001; Shemyakin and Youn, 2006). Surprisingly, the application of copula models in the multipopulation mortality analysis is still in its  

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<1> Another application of multipopulation mortality models is to forecast mortality for a small country. Sometimes the mortality data may not be available or reliable for a small population due to a small number of deaths, a limited number of calendar years of data, age range, or poor data quality. Jointly estimating the small population and a larger linked population allows the small-population mortality forecasts to be consistent with those of the larger population. Due to the space restriction, we do not discuss this application in this article. We refer interested readers to Dowd et al. (2011).
infancy stage. To the best of our knowledge, only a few articles (e.g., Lin, Liu, and Yu, 2013; Chen, MacMinn, and Sun, 2015; Wang, Yang, and Huang, 2015) fall into this category.

Lin et al. (2013) develop a jump diffusion mortality model for six countries where mortality dependence comes from common jumps and the correlation between idiosyncratic risks. Their approach entails the use of normalized multivariate exponential tilting that implicitly assumes a Gaussian copula. This can be problematic since Gaussian copulas lack tail dependence and are, therefore, inadequate to model the joint mortality events. Chen, MacMinn, and Sun (2015) and Wang, Yang, and Huang (2015) share some common features in the sense that both articles build a multi-population mortality model based on a two-stage procedure. In the first stage, the mortality dynamics of each population is modeled using a time-series analysis. In the second stage, the mortality dependence of the residuals from the first stage is captured using a copula model. The two-stage procedure permits the separate development of the marginal distributions and the copula model. Hence, this method allows the use of any existing model for the univariate analysis with the subsequent focus on the copula model.

In spite of the similarity, their models differ in several ways. First, Chen, MacMinn, and Sun (2015) choose the best ARMA-GARCH model for each population, whereas Wang, Yang, and Huang (2015) consider the conditional mean models only. Second, the former article uses a one-factor copula model that is widely applicable to high-dimension data and very flexible in model specification, while the latter exploits copulas in the Elliptical or Archimedean family which use only one or two parameters to characterize dependence and are thus quite restrictive when the number of variables increases. Third, Chen, MacMinn, and Sun (2015) find that the best one-factor model has a common factor with a skewed $t$ distribution, suggesting that mortality dependence is stronger during mortality deteriorations than during mortality improvements. Wang, Yang, and Huang (2015) find that the dynamic Gumbel copula provides the best fit among five copula models of their choice. Gumbel copula captures the upper tail dependence during mortality deteriorations while ignoring the lower tail dependence during mortality improvements. Finally, Wang, Yang, and Huang (2015) adopt the dynamic conditional correlation (DCC) approach to allow time-varying correlations, but, Chen, MacMinn, and Sun (2015) consider a static approach.

In this article, we adopt the factor copula to model mortality dependence because of its simplicity and flexibility. The factor copula is based on a simple linear, additive structure and is particularly attractive for high-dimension applications; it also permits more flexibility for the number of variables and available data. We extend the one-factor copula model used in Chen, MacMinn, and Sun (2015) to a two-factor copula model, with one common factor representing the market force that drives mortality dynamics in all countries and one country-specific factor capturing mortality dependence across age groups in each country. We also incorporate a time-varying structure into the factor copula model by using the GAS framework. The GAS model was developed by Creal, Koopman, and Lucas (2013). They argue that the score function is an effective choice for introducing a driving mechanism for time-varying parameters. We find that two-factor models can better capture the
dependence structure than one-factor models, which is very important to multipopulation mortality modeling and pricing. We also find that dynamic models are better than their static counterparts in terms of overall goodness of fit, though the improvement in model fit is limited based on our estimation.

When it comes to mortality/longevity derivative pricing, we face the difficulty in selecting an appropriate risk-neutral measure with sparse market data. This difficulty drives us to consider alternative pricing approaches in CAT bond markets. CAT bond risk premiums can be proportional to expected loss itself, or proportional to the variance or standard deviation of the loss, or determined by a risk cubic model (Lane and Movchan, 1999; Lane, 2000). Multiple linear (Lane and Beckwith, 2008; Lane and Mahul, 2008; Berge, 2005) or log-linear (Major and Kreps, 2003; Dieckmann, 2010) models have been suggested by adding other macroeconomic factors and/or bond-specific characteristics. Mortality/longevity bonds are similar to CAT bonds in design, so CAT bond pricing methodologies can be readily applied to mortality/longevity bonds. Lane (2000) and Chen and Cummins (2010) use the risk cubic model to price mortality bonds. We exploit the multiple linear model in this article. After experimenting several model specifications, we find that a simple linear model with expected loss as the pricing factor generates premiums that are very close to actual ones for mortality/longevity bonds.

Our article contributes to the existing literature in several ways. First, we are among the first to develop a copula-based mortality model to capture mortality correlations among multiple populations. Copulas are powerful tools to model the dependence of tail events. Therefore, our model has significant applications to pricing and hedging correlated mortality risks. Second, we extend the one-factor copula model to a two-factor copula model, so our model can capture the complex structure of intracountry and intercountry mortality correlations. Third, we incorporate the time-varying dependence structure, leading to a flexible yet parsimonious dynamic model for high-dimension application. Fourth, we propose a simple model to price mortality/longevity bonds. Our model can replicate announced mortality/longevity premiums to a large extent.

The remainder of this article is organized as follows. In the “A Dynamic Factor Copula Model for MultiPopulations” section, we propose our multipopulation mortality model based on a two-factor copula and the GAS structure. We introduce the Kortis bond in “The Kortis Longevity Bond” section, calibrate model parameters in the “Model Estimation” section, and price the bond in the “Longevity Bond Pricing” section. We conclude the article and discuss other applications of our multipopulation mortality model in the “Conclusions” section.

**A Dynamic Factor Copula Model for MultiPopulations**

Copula-based multivariate models allow researchers to specify the marginal distributions separately from the dependence structure. This enriches the class of multivariate distributions and permits a much greater degree of flexibility in model specification. If the parameters of the marginal distributions are separable from those of the copula, we may estimate those parameters separately in two stages, which
greatly simplifies the estimation procedure and facilitates the study of high-
dimension multivariate problems.2

Suppose we need to model the joint dynamics of time-series data, $y_{it}$ ($i = 1, 2, \ldots, N, t = 1, 2, \ldots, T$). The first step is to model the conditional marginal distribution of each
time series. Based on the Box–Jenkins (1976) approach, we can fit $y_{it}$ with a conditional
mean model ARMA ($r, m$),

$$y_{it} = c_i + \sum_{k=1}^{r} \varphi_{ik} y_{i,t-k} + \sum_{k=1}^{m} \rho_{ik} e_{i,t-k} + \epsilon_{it},$$

and a conditional variance model GARCH ($p, q$)

$$\sigma_{it}^2 = \omega_i + \sum_{k=1}^{p} \alpha_{ik} \sigma_{i,t-k}^2 + \sum_{k=1}^{q} \beta_{ik} \epsilon_{i,t-k}^2,$$

where $\epsilon_{it} = \sigma_{it} \eta_{it}$ and $\eta_{it}$ are independently and identically distributed sequence of
standardized innovations.

In the next step, we use a copula model to describe the joint distribution of
innovations,

$$\eta_i \equiv [\eta_{i1}, \ldots, \eta_{iN}]' \sim C_t(F_{I1}, \ldots, F_{Ni}; \gamma),$$

where $C_t$ denotes the time-varying copula function with a vector of parameters $\gamma$.

To estimate $\gamma$, we consider a set of latent variables $X_t \equiv [X_{i1}, \ldots, X_{Ni}]'$ whose
dependence structure follows the same copula model as the standardized residuals $\eta_t$. We use a two-factor copula model proposed by Oh and Patton (2017) as follows,

$$X_{it} = \lambda_{it}^0(\gamma_{\alpha}) Z_{0t} + \lambda_{it}^c(\gamma_{\alpha}) Z_{ct} + \epsilon_{it}, i = 1, \ldots, N$$

$Z_{0t} \sim \text{iid} F_{\gamma_{\alpha}}(\gamma_{\alpha})$;

$Z_{ct} \sim \text{iid} F_{\gamma_{\alpha}}(\gamma_{\alpha}), c = \text{US or UK}; Z_{ct} \perp Z_{0t}$ and $Z_{US,t} \perp Z_{UK,t}$

$\epsilon_{it} \sim \text{iid} F_{\gamma_{\alpha}}(\gamma_{\alpha}), i = 1, \ldots, N; \epsilon_{it} \perp Z_{0t}$ and $\epsilon_{it} \perp Z_{ct}, \forall i$;

$X_t \equiv [X_{i1}, \ldots, X_{Ni}]' \sim C_t(G_{I1}(\gamma), \ldots, G_{Ni}(\gamma); \gamma).$

In Equation (4), $Z_{0t}$ is the common factor that affects both U.S. and U.K. mortality
rates, $Z_{ct}$ is the country-specific factor that affects U.S. or U.K. mortality rates only,
and $\epsilon_{it}$ are idiosyncratic factors. $F_{Z_{\alpha}}(\gamma_{\alpha})$, $F_{Z_{\alpha}}(\gamma_{\alpha})$, and $F_{\gamma_{\alpha}}(\gamma_{\alpha})$ are univariate
distributions for the common, country, and idiosyncratic factors. $\lambda_{it}^0(\gamma_{\alpha})$ and $\lambda_{it}^c(\gamma_{\alpha})$

2Clearly, two- or multiple-stage estimation is asymptotically less efficient than one-stage
estimation. However, simulation studies in Joe (2005) and Patton (2006) indicate that this loss is
not great in many cases.
are time-varying factor loadings. \( G_t \) is the marginal distribution function for the latent variable \( X_t \), and \( \gamma \equiv [\gamma_{x0}, \gamma_{z0}, \gamma_{e0}, \gamma_{k0}] \) is a collection of all copula parameters.\(^3\)

An important question is how to specify the evolution process for the copula parameters over time. We adopt the GAS model in Creal, Koopman, and Lucas (2013). These authors propose using the lagged score of the density model as the forcing variable. Generally speaking, for a copula \( C(\delta_t | \gamma) \) with time-varying parameters \( \delta_t \) governed by fixed parameters \( \gamma \), we define the following process

\[
\delta_t = w + B\delta_{t-1} + A s_{t-1}, \tag{5}
\]

where \( s_{t-1} = S_{t-1}\nabla_{t-1}, \) \( S_{t-1} \) is a scaling matrix and \( \nabla_{t-1} = \frac{\partial \log(\lambda_{i,t-1})}{\partial \delta_{i-1}} \).

This specification has several advantages. First, compared to modeling the time-varying parameters as a latent time series, such as stochastic volatility models (see, e.g., Shephard, 2005) and related stochastic copula models (Manner and Segers, 2011; Hafner and Manner, 2012), modeling the time-varying parameters as some functions of lagged observables avoids the need to “integrate out” the innovation terms that drive the dynamics of the latent variable.\(^4\) This feature is very important for the tractability of the model in high dimensions. Second, the GAS model exploits the complete density structure to update the time-varying parameters based on the density score rather than means and higher moments only. Third, the recursion uses the density score as the forcing variable. This is similar to numerical optimization such as the Gaussian–Newton algorithm in the sense that it uses the steepest ascent direction for improving the model fit in terms of the likelihood, given the current value of model parameter \( \delta_t \) (Oh and Patton, 2016).

Applying the general GAS framework in Equation (5) to our specific case in Equation (4), the \( 2N \) time-varying factor loadings would require the estimation of a \( 2N \times 1 \) vector \( w \) and two \( 2N \times 2N \) matrices \( B \) and \( A \), totally \( 2N \times (1 + 4N) \) parameters. The computation cost becomes extremely large for even moderate values of \( N \). To keep the model simple, we suppose that the coefficient matrices \( B \) and \( A \) are diagonal with a common scalar parameter on the diagonal (\( \beta \) and \( \alpha \), respectively), as in the DCC model of Engle (2002). We also set \( S_t = I \) to avoid the estimation of the scaling matrix. This simplifies the evolution model to be (in logs),

\[
\log \lambda_{i,t} = \alpha_{i,t} + \beta \log \lambda_{i,t-1} + \alpha s_{i,t-1}, j \in \{0, c\} \tag{6}
\]

\(^3\)Although we impose the same dependence structure, that is, the copula model, for the latent variables \( X \) and the standardized innovations \( \eta \), their marginal distributions might be different.

\(^4\)Stochastic volatility models allow volatility to depend on some unobserved components or latent structure. The major challenge of stochastic volatility models is the intractability of the likelihood function. Because the variance is an unobserved component and the model is non-Gaussian, the likelihood function is only available in the form of a multiple integral.
where $s^i_t = \partial \log c(u_t; \lambda_t; \gamma_z, \gamma_z, \gamma_z) / \partial \lambda^i_t$ and $\lambda_t = [\lambda^0_{1,t}, \ldots, \lambda^0_{N,t}, \lambda^\zeta_{1,t}, \ldots, \lambda^\zeta_{N,t}]'$.

To further simplify our task, we adopt the “variance targeting” method proposed in Oh and Patton (2016), the details of which can be found in the Supplementary Internet Appendix (Chen, MacMinn and Sun, 2017). Simply speaking, using the result from Creal, Koopman, and Lucas (2013) that $E_t[1/s^i_t] = 0$, we have $E[\log \lambda^i_t] = \omega^i/(1 - \beta)$.

Then, Equation (6) can be rewritten as

$$\log \lambda^i_t = E[\log \lambda^i_t] (1 - \beta) + \beta \log \lambda^i_{t-1} + \sigma^i_{t-1},$$

where $E[\log \lambda^i_t]$ can be estimated using sample rank correlations. After the above procedure, we only need to estimate two parameters for the GAS dynamics (i.e., $\beta$ and $\sigma$) and the shape parameters for the common, country, and idiosyncratic factors. This can be done through maximum likelihood estimation. Factor copulas do not have a closed-form likelihood, so we approximate the likelihood using standard numerical integration methods, as shown in the Supplementary Internet Appendix (Chen, MacMinn and Sun, 2017).

**THE KORTIS LONGEVITY BOND**

In this article, we use the Kortis longevity bond as one example to illustrate how our proposed multipopulation mortality model can be used for mortality risk modeling and pricing. Other applications of our model will be discussed in the “Conclusions” section.

The Kortis bond was issued by Swiss Re via a Cayman Island domiciled, special purpose vehicle called Kortis Capital Ltd. in December 2010. It is the first issued “longevity trend” bond to capital market investors and designed to transfer the residual risk remained in Swiss Re’s business in life insurance and annuities. The issue size was $50 million. The bond pays coupon payments at a rate of LIBOR plus 507 basis points (adjusted annualized premium) and matures in January 2017 (extendable until July 2019). The principal repayment is indexed on a Longevity Divergence Index Value (LDIV), defined as the divergence in the mortality improvements between males aged 75 and 85 in England & Wales, representing Swiss Re’s pension exposures, and males aged 55 and 65 in the United States, representing Swiss Re’s life insurance exposure (Swiss Re, 2012).

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5Swiss Re has a strong track record of developing the capital markets for insurance perils, initially with natural catastrophe bonds and more recently by periodically securitizing its life risks. To hedge that risk, it has sold about $2.35 billion in mortality bonds to date. In the meanwhile, Swiss Re has been renewing its efforts to provide capacity for longevity risk. As an example, in 2009, Swiss Re announced that it would insure around £1 billion of pension liabilities for the Royal County of Berkshire Pension Fund. Insuring longevity risk, then, helps Swiss Re naturally hedge its mortality risk. Such a natural hedge is imperfect because of the mismatch of the reference populations in its pension business and its life insurance business. To manage the residual longevity risk, Swiss Re issued the first longevity trend bond in 2010.
The LDIV is constructed in several steps. First, the mortality improvement for each population of men at different ages is averaged over $n$ years, that is,

\[ \Delta m^j(t - n, t) = 1 - \left( \frac{m^j_{x,t}}{m^j_{x,t-n}} \right)^{1/n}, \]  

(8)

where $m^j_{x,t}$ is the mortality rate observed at age $x$ in year $t$ for population $j$. For the Kortis bond, an averaging period of 8 years is used, that is, from January 1, 2009 to December 31, 2016.

Second, the mortality improvement is averaged across age groups (ages $x_1$ to $x_2$) for each year and country, that is,

\[ \Delta m^j_t(x_1, x_2) = \frac{1}{1 + x_2 - x_1} \sum_{x=x_1}^{x_2} \Delta m^j_t(t - 8, t). \]  

(9)

$\Delta m_t^{EW}(75, 85)$ is calculated using ages 75–85 for England & Wales, while $\Delta m_t^{US}(55, 65)$ is calculated using ages 55–65 in the United States.

Finally, the LDIV is calculated for year $t$ as

\[ \text{LDIV}_t = \Delta m_t^{EW}(75, 85) - \Delta m_t^{US}(55, 65). \]  

(10)

The principal reduction factor (PRF) is indexed on the LDIV and structured as a call option spread, that is,

\[ \text{PRF} = \frac{[\text{LDIV}_{2016} - 3.4\%]_+ - [\text{LDIV}_{2016} - 3.9\%]_+}{3.9\% - 3.4\%}, \]  

(11)

where 3.4 percent is the attachment point and 3.9 percent is the detachment point. In other words, if the mortality improvement rate in England & Wales between 2008 and 2016 is significantly higher than that observed in the United States (i.e., the LDIV is greater than 3.4 percent), the principal of the bond is reduced linearly until full exhaustion when the LDIV is greater than 3.9 percent.

When the Kortis bond was issued, Standard & Poor’s rated the single tranche Series 2010-I Class E Notes as BB+. This was based in part on the modeling work of Risk Management Solutions (RMS). The distribution of the PRF estimated by RMS and given in Standard & Poor’s (2010) is reported in Table 3 in the “Model Estimation” section.

**Model Estimation**

We collect mortality data for U.S. males aged 55 to 65 and U.K. (England & Wales) males aged 75 to 85 from Human Mortality Database over the period.
1933–2010. The crude mortality rates for each age group are plotted in a figure in the Supplementary Internet Appendix (Chen, MacMinn, and Sun, 2017). We observe a clear trend of mortality improvements and a very strong intracountry mortality dependence. These crude mortality rates are not stationary, so we compute the mortality improvement rates, denoted by $y_{j,t} = \ln m_{j,t} - \ln m_{j,t-1}$. Phillips–Perron test results (available in the Supplementary Internet Appendix (Chen, MacMinn, and Sun, 2017)) reject the null hypothesis of a unit root for mortality improvement rates at the 1 percent level for all groups.

Conditional Marginal Distribution

In the previous literature, ARMA-GARCH models have been used extensively to model mortality dynamics. We run the LBQ test and the ARCH test on mortality improvement rates (results available in the Supplementary Internet Appendix (Chen, MacMinn, and Sun, 2017)). We find evidence of strong autocorrelation for each group and conditional heteroskedasticity for some groups. These properties imply that an ARMA model or an ARMA-GARCH model is proper to use, so we follow this vein of literature to model the mortality improvement rates for each group separately. We consider three types of distributions for error terms: (1) the standard normal distribution, (2) the Student’s $t$-distribution with the degree of freedom $v$, and (3) the variance gamma distribution (see Wang, Huang, and Liu, 2013). The Bayesian information criterion (BIC) is used to determine the appropriate lags for lag orders up to five. We find that assuming variance gamma innovations generates much higher BICs than Gaussian and Student’s $t$-distributions. Assuming Gaussian is slightly better than Student’s $t$ for all groups except for United Kingdom age 84 where the BICs are about the same. We hence choose the best ARMA-GARCH model with Gaussian innovations, the results of which are shown in Table 1. Overall, the conditional mean of mortality improvement can be explained by the MA(1) process consistently across groups; only four age groups in the United States need AR(1) or AR(2). As to the conditional variance model, most of the U.S. groups exhibit constant variance over time while

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8A positive $y_{j,t}$ indicates mortality deterioration and a negative $y_{j,t}$ suggests mortality improvement.
9Lee and Carter (1992) use an ARIMA model to fit the time-varying mortality factor. They assume that the error terms are white noise with zero mean and a small constant volatility; that assumption of homoskedasticity, however, is not realistic. Lee and Miller (2001) argue that the observed logarithm of central death rates is quite variable and the volatility is time varying. Recently, GARCH-related models have been used on modeling mortality trends in the Lee-Carter model (e.g., Gao and Hu, 2009; Giacometti, Ornobelli, and Bertocchi, 2009; Wang, Huang, and Liu, 2013; Wang, Yang, and Huang, 2015) or the central mortality rates (e.g., Giacometti et al., 2012; Chen, MacMinn, and Sun, 2015).
10We also estimate the ARMA-GJR-GARCH model. However, the GJR coefficient is not significant for all age groups at the 1 percent level. In addition, ARMA-GARCH models without the “leverage” effect are preferred for all (but one) age groups based on the model BICs.
most of the U.K. groups illustrate conditional heteroskedasticity. In total, there are 10 out of 22 groups requiring GARCH models to remove the ARCH effect.11

After the ARMA-GARCH fitting, we perform the LBQ test and the ARCH test again on the standardized residuals (results available in the Supplementary Internet Appendix (Chen, MacMinn, and Sun, 2017)). Almost all test statistics are insignificant up to lag 10, which indicates that our selected model for each group sufficiently accounts for autocorrelation and conditional heteroskedasticity. The mean of standardized residuals is close to zero and the variance is about one for each group. In addition, we observe very small skewness and reasonable kurtosis (around 3) except for United States age 63.12 All of these observations provide evidence that Gaussian innovations is a valid assumption.

### The Two-Factor Copula Model

In this subsection, we investigate the dependence structure among standardized residuals for all age groups. We first look into Spearman’s rho and quantile dependence measures.13 The left panel in Figure 1 depicts the heat map of the Spearman’s rho rank correlation matrix. A lighter color in the heat map indicates a higher Spearman’s rho. As we can see, most of the Spearman’s rho are positive for both intra- and intercountry correlations. The result also suggests that the U.K. groups

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11 As a separate experiment, we also use the Lee–Carter model to estimate the mortality trend, $k_t$, and apply the ARMA-GARCH model on the mortality trend improvement, $\Delta k_t$. We find that we need to model the U.S. mortality trend and the U.K. mortality trend separately, that is, MA(1) with Student’s t error terms for $\Delta k^{US}_t$ and MA(1)-GARCH (1,1) with Gaussian error terms for $\Delta k^{UK}_t$. This result on the mortality trend improvement ($\Delta k_t$) is largely consistent with our main results on the mortality rate improvement ($\Delta (m_{x,t})$).

12 We report the summary statistics for the standardized residuals in each group in the Supplementary Internet Appendix.

13 Those measures are “pure” measures of dependence, meaning that they are solely affected by the changes in the copula, not by the changes in the marginal distribution. For a detailed discussion of such “pure” dependence measures, see Joe (1997) and Nelsen (2006).
have stronger intracountry dependence than the U.S. groups; the inter-country
dependence between U.K. and U.S. groups is the weakest.

The right panel in Figure 1 shows the heat map for the pairwise quantile dependence. The upper (lower) triangle matrix reports the 90th (10th) percentile dependence. We observe an asymmetric dependence structure, that is, the upper-tail dependence is larger than the lower-tail dependence, especially for the intercountry tail dependence. This observation is consistent with our conjecture that mortality dependence is stronger when the market faces extreme mortality deterioration than it does when the market sees mortality improvement.

We use the factor copula model with varying loadings on the common factor and the country factor to capture the heterogeneity of pairwise dependence. We assume the idiosyncratic factor ε follows the standard normal distribution. Based on distribution assumptions for the common and country factors, we consider four model specifications: normal—normal, Student t—normal, Student t—Student t, and skewed t—Student t. For example, in the normal—normal model, we assume the common factor \( Z_0 \sim N(0, \sigma_0^2) \) and the country factor \( Z_c \sim N(0, \sigma_c^2) \); in the skewed t—Student t model, \( Z_0 \) follows a skewed t-distribution with a skewness parameter, \( \psi_0 \in (-1, 1) \), and a degree of freedom parameter, \( v_0 \in (2, \infty) \), and \( Z_c \) follows a Student’s t-distribution with a degree of freedom, \( v_c \in (2, \infty) \). We further assume that all factors (common, country, and idiosyncratic) are independent to each other.

We first use the “variance targeting” method to estimate \( E[\log \lambda_{ij}^\ell] \). We then estimate the GAS parameters and the distribution parameters for the common and country factors. Parameter estimates for dynamic two-factor copula models are reported in Panel A of Table 2. For comparison purposes, we report parameter estimates for their static counterparts in Panel B.

Our results show that dynamic models are in general better than their static counterparts regarding overall goodness of fit, according to the log-likelihood, AIC
Among all two-factor copula models, the dynamic skewed $t$–Student $t$ model provides the best goodness of fit. Under this model, the common factor follows a skewed $t$-distribution, which generates stronger upper-tail dependence (during mortality deteriorations) than lower-tail dependence (during mortality improvements). However, the log-likelihood (AIC and BIC) is only improved by a small percentage (about 1.5 percent) from the static model to its dynamic version.14

**Table 2**

Parameter Estimates for Two-Factor Copula Models

### Panel A: Dynamic Two-Factor Copula Models

<table>
<thead>
<tr>
<th>Normal–Normal</th>
<th>Student $t$–Normal</th>
<th>Student $t$–Student $t$</th>
<th>Skewed $t$–Student $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$</td>
<td>$0.9979^{***}$</td>
<td>$0.1211^{***}$</td>
<td>$0.1004^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0350)</td>
<td>(0.0040)</td>
<td>(0.0034)</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>$1.0060^{***}$</td>
<td>$0.5013^{***}$</td>
<td>$0.1002^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0418)</td>
<td>(0.0236)</td>
<td>(0.0154)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$0.1996^{***}$</td>
<td>$0.1998^{***}$</td>
<td>$0.0980^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0046)</td>
<td>(0.0037)</td>
<td>(0.0028)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0.9501^{***}$</td>
<td>$0.9470^{***}$</td>
<td>$0.9007^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0011)</td>
<td>(0.0009)</td>
<td>(0.0024)</td>
</tr>
</tbody>
</table>

Log $L$ $-2,807.55$  Log $L$ $-2,779.98$  Log $L$ $-2,874.78$  Log $L$ $-2,668.67$

AIC 5,711.1  AIC 5,655.96  AIC 5,845.56  AIC 5,435.34

BIC 5,705.38  BIC 5,650.24  BIC 5,839.84  BIC 5,429.50

### Panel B: Static Two-Factor Copula Models

<table>
<thead>
<tr>
<th>Normal–Normal</th>
<th>Student $t$–Normal</th>
<th>Student $t$–Student $t$</th>
<th>Skewed $t$–Student $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$</td>
<td>$1.001^{***}$</td>
<td>$0.9991^{***}$</td>
<td>$0.9995^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0189)</td>
<td>(0.0038)</td>
<td>(0.0029)</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>$1.001^{***}$</td>
<td>$1.0001^{***}$</td>
<td>$0.9994^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0352)</td>
<td>(0.0308)</td>
<td>(0.0280)</td>
</tr>
</tbody>
</table>

Log $L$ $-2,818.41$  Log $L$ $-2,843.24$  Log $L$ $-2,882.29$  Log $L$ $-2,710.28$

AIC 5,728.82  AIC 5,778.48  AIC 5,856.58  AIC 5,514.56

BIC 5,723.34  BIC 5,773.00  BIC 5,851.10  BIC 5,508.96

Note: Standard errors are reported in parentheses. *** (**) or (*) indicates significance at the 1% (5% or 10%) level.

The magnitude of improvement in the model fit is largely in line with the findings in Oh and Patton (2016). They examine the systemic risk of S&P 100 firms using their daily credit default swap (CDS) spreads. They find that the dynamic factor copula with the GAS structure (heterogeneous dependence) can improve the model fit (the log-likelihood, AIC and BIC) by about 3 percent.

14The magnitude of improvement in the model fit is largely in line with the findings in Oh and Patton (2016). They examine the systemic risk of S&P 100 firms using their daily credit default swap (CDS) spreads. They find that the dynamic factor copula with the GAS structure (heterogeneous dependence) can improve the model fit (the log-likelihood, AIC and BIC) by about 3 percent.
We also compare the model fit in terms of the dependence structure. Particularly, we calculate the sample Pearson correlation matrix of the standardized residuals and the fitted Pearson correlation matrix using the dynamic (or static) skewed $t$–Student $t$ model. We then calculate the sum of squared errors (SSE) between the sample and the fitted Pearson correlation matrices. A smaller SSE indicates a better fit in the dependence structure. The dynamic skewed $t$–Student $t$ model is associated with an SSE of 5.54, which is slightly smaller than the SSE calculated by the static model (6.43).\footnote{The small improvement in the model fit is due to two main reasons. First, dynamic models are rather complicated compared to static ones. We have to make compromise between the simplicity of the model and the goodness of fit. On the one hand, extending static models to dynamic ones allows us to better capture the time-varying dependence structure of the data, which in turn improves the goodness of fit. On the other hand, we have to make simplifying assumptions to keep the model tractable and reduce the computational burden. For example, in the GAS model (Equation (5)) we assume that the coefficient matrices ($\mathbf{B}$ and $\mathbf{A}$) are diagonal with a common scalar ($\beta$ and $\alpha$, respectively). We also assume $S_1 = I$ to avoid the estimation of the scaling matrix. Such simplifying assumptions restrict the improvement of the model fit to some extent. Second, in the GAS model if $\beta = 1$ and $\alpha = 0$, then a dynamic model degenerates to a static one. As can be seen from Table 2, the dynamic skewed $t$–Student $t$ model has a $\beta$ that is close to 1 ($\beta = 0.9847$) and $\alpha$ close to 0 ($\alpha = 0.1$); therefore, the dynamic model is largely the same as the static one. We see similar patterns when comparing other pairs of dynamic and static models.}

Next, we focus on comparing the performance of two-factor copula models with corresponding one-factor models.\footnote{We also compare the fitting and forecasting results for the LDIV index based on the Lee–Carter model (as specified in footnote 11) with those obtained by our dynamic two-factor copula model. We find that the former is slightly better than the latter in terms of in-sample fitting—the mean squared error for the LDIV index fitting based on the Lee–Carter model is 2.4897e-8 (vs 6.1805e-05 based on our dynamic two-factor copula model). However, these two models are very close in the power of out-of-sample validation if we use mortality data 1933–2003 to forecast the LDIV index in 2004–2010 (mean squared error is about 0.0001 for both models). In addition, the latter generates LDIV forecasts with a larger variance so more simulation paths can trigger the Kortis bond. As a result, our two-factor copula model generates the closest loss distribution and the premium estimate for the Kortis bond, compared to the Lee–Carter model.} To save space, the results are reported in the Supplementary Internet Appendix (Chen, MacMinn, and Sun, 2017). We find that one-factor copula models in general have better goodness of fit than two-factor copula models, based on the log-likelihood, AIC and BIC. However, two-factor models capture the dependence structure much better than one-factor models. We illustrate this using the dynamic skewed $t$–Student $t$ model and the dynamic skewed $t$ model. The SSE (between the sample and fitted Pearson correlation matrices) generated from the dynamic skewed $t$ model is 21.10, much larger than that from the dynamic skewed $t$–Student $t$ model (5.54). We depict the heat maps for those correlation matrices in the Supplementary Internet Appendix (Chen, MacMinn, and Sun, 2017). It is clear that the dynamic skewed $t$–Student $t$ model produces the fitted correlation matrix that is closer to the sample correlation matrix than the dynamic skewed $t$ model does. We also compare these two models in terms of the LDIV forecasts. Figure 2 shows the fan charts...
of the forecasted LDIV index with its 90, 95, and 99.5 percent confidence intervals. We can see that the LDIV predicted by the skewed \( t \)-Student \( t \) model (the left panel) has a much wider confidence interval (or a larger variance) than the one predicted by the dynamic skewed \( t \) model (the right panel). At the 99.5th percentile, the former generates some simulated paths that can trigger the Kortis bond, while the latter does not.

Based on our simulation results, we estimate the probabilities corresponding to each LDIV threshold and several risk measures of the Kortis bond, that is, the probability of first loss (PFL), conditional expected loss (CEL), and expected loss (EL). These results are reported in Table 3. We can see that the dynamic skewed \( t \)-Student \( t \) model generates a loss distribution that is close to the one estimated by RMS, while the probability of loss at each loss level calculated based on the dynamic skewed \( t \) model is far less than the RMS estimate.\(^{17}\) Though the CEL predicated by both models is about the same, the PFL is 0.96 percent (0.14 percent) based on the dynamic skewed \( t \)-Student \( t \) model (the dynamic skewed \( t \) model). As a result, the expected loss for the Kortis bond is 0.57 versus 0.08 percent using these two models, respectively.

\(^{17}\)The Kortis bond is the first longevity trend bond issued in history. Other than the RMS estimates, there is no other benchmark for us to compare our estimates with the probability distribution of losses. The fact that our estimates are close to those reported by RMS validates our model to some extent. The closeness also guarantees that our estimated par spread for the Kortis bond based on our economic pricing model is close to the announced spread. Therefore, it provides evidence that our mortality model and pricing approach can be used as a reference for future mortality/longevity securities.
LONGEVITY BOND PRICING

Pricing Methodology

Longevity/mortality bonds pay out periodical coupon payments at a rate of LIBOR plus a risk premium. In an incomplete market with sparse market price data (e.g., longevity/mortality markets), some prevalent pricing methodologies, such as the arbitrage-free pricing method (Cairns, Blake, and Dowd, 2006; Bauer, Boerger, and Russ, 2010), the Wang transform (Dowd et al., 2006; Denuit, Devolder, and Goderniaux, 2007; Lin and Cox, 2008; Chen and Cox, 2009), or the Esscher transform (Chen, Cox, and Wang, 2010; Li, Hardy, and Tan, 2010), often require the user to make one or more subjective assumptions to derive the market price of risk. The pricing process becomes even more difficult when multiple risks are involved.

We, therefore, turn to the alternative CAT bond pricing approach as mortality/longevity bonds are very similar to CAT bonds in terms of the bond design and payoff structure. In CAT bond markets, the risk premium can be proportional to the expected loss, or to the variance (or standard deviation) of the loss, or determined by a risk-cubic model (Lane, 2000). Extending the previous work, Lane and Beckwith (2008) and Lane and Mahul (2008) suggest a multiple linear model using the expected loss and a factor covering cycle effects to decide risk premiums. Other multiple linear models include those established by Berge (2005) and Dieckmann (2010). Both studies identify CAT-bond-specific factors, such as peril, trigger, size, and rating, to capture risk loadings. Log-linear models have also been used in Major and Kreps (2003) and Dieckmann (2010). Comparing different CAT bond pricing models, Galeotti, Guertler, and Winkelvos (2013) document that a linear model is better than a log-linear model for in-sample fitting, but CAT-bond-specific information does not improve out-of-sample forecasts. Braun (2016) finds that expected loss, transactions covering U.S. or multiple geographic areas, transactions sponsored by Swiss Re, the reinsurance cycle, and the BB corporate bond spreads are major drivers of CAT bond spreads.

### Table 3

Comparison Between RMS Estimation Results and Our Estimation Results

<table>
<thead>
<tr>
<th>LDIV</th>
<th>PRF</th>
<th>Probability (RMS Estimates)</th>
<th>Probability (Dynamic Skewed t–Student t)</th>
<th>Probability (Dynamic Skewed t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4%</td>
<td>0</td>
<td>0.88%</td>
<td>0.96%</td>
<td>0.14%</td>
</tr>
<tr>
<td>3.5%</td>
<td>20%</td>
<td>0.72%</td>
<td>0.82%</td>
<td>0.12%</td>
</tr>
<tr>
<td>3.6%</td>
<td>40%</td>
<td>0.58%</td>
<td>0.58%</td>
<td>0.10%</td>
</tr>
<tr>
<td>3.7%</td>
<td>60%</td>
<td>0.47%</td>
<td>0.52%</td>
<td>0.08%</td>
</tr>
<tr>
<td>3.8%</td>
<td>80%</td>
<td>0.38%</td>
<td>0.34%</td>
<td>0.04%</td>
</tr>
<tr>
<td>3.9%</td>
<td>100%</td>
<td>0.30%</td>
<td>0.30%</td>
<td>0.04%</td>
</tr>
<tr>
<td>PFL</td>
<td></td>
<td>0.88%</td>
<td>0.96%</td>
<td>0.14%</td>
</tr>
<tr>
<td>CEL</td>
<td>62.50%</td>
<td>59.88%</td>
<td>59.82%</td>
<td></td>
</tr>
<tr>
<td>EL</td>
<td>0.55%</td>
<td>0.57%</td>
<td>0.08%</td>
<td></td>
</tr>
<tr>
<td>Premium</td>
<td>507 bps</td>
<td>502 bps</td>
<td>36 bps</td>
<td></td>
</tr>
</tbody>
</table>
Therefore, we propose to use a multiple linear model in this article as follows:

\[
\text{Premium} = \alpha + \beta \times EL + \lambda \times Z + \epsilon,
\]

(12)

where \(Z\) is a set of pricing factors, including issue size (in million USD), maturity term (in months), a dummy variable \(\text{SwissRe}\) equal to one if Swiss Re is the sponsor and zero otherwise, a dummy variable \(\text{US}\) equal to one in case U.S. soil is part of the stipulated region and zero otherwise, and two variables capturing the cyclical spread drivers (\(\Delta \text{ROL}\), the change in the rate on line index, and \(\text{BBspread}\), the BB corporate bond spread, both in basis points).

Implementation

We consider mortality bonds issued from 2003 to 2010. We compile the data from two sources. The main sources are publications and trade notes by Lane Financial L.L.C. that report adjusted premium spread, EL, size, and the maturity of the CAT bonds. Other information, such as sponsor and covered territory, comes from Aon’s quarterly review of the insurance-linked security market. Risk measures such as EL are not available from Lane Financial L.L.C. for some mortality bonds (e.g., Tartan and Nathan), which leaves us 19 tranches of mortality bond for analysis. To identify outliers, that is, cases with an abnormally large impact on the least squares coefficients, we perform the Cook’s \(D\) test and all these tranches pass the test.

We try various model specifications and report the regression results in Table 4. The adjusted \(R^2\) ranges from 76 to 81 percent, indicating good model fit. The EL alone explains about 76 percent of the variation of mortality bond spreads and its impact is significantly positive, which is consistent with the previous literature. We also find that, except for the change in the rate on line index, other factors do not affect pricing at all.\(^{18}\)

For all 19 tranches, we plot the announced spreads (the curve with dots) and the fitted spreads using the simplest model, Model 1 (the curve with boxes) and the best-fitting model, Model 7 (the curve with triangles) in Figure 3. Both models generate par spreads that are close to the actual ones for previously issued mortality bonds, although Model 7 is slightly better. We then use these two models and the EL estimated by the dynamic skewed \(t\)–Student \(t\) model to price the Kortis bond. The calculated premium based on Model 1 (Model 7) is 503 bps (502 bps), again very close to the announced premium (507 bps).

We also run the regression models using all CAT bond transactions between 2000 and 2010 and the results are reported in the Supplementary Internet Appendix (Chen, MacMinn, and Sun, 2017)\(^{19}\). We find that most of the explanatory variables (i.e., expected loss, maturity, transactions covering U.S. territories, transactions sponsored by Swiss

\(^{18}\)Since our sample only includes 19 tranches of mortality bonds, the small sample size leads to the large standard errors for some variables and the insignificance of some variables.

\(^{19}\)We find 318 CAT bond transactions during this period. We perform the outlier detection by using the Cook’s \(D\) test and end up with 299 CAT bond transactions.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>EL</td>
<td>7.8079***</td>
<td>7.8080***</td>
<td>7.6000***</td>
<td>7.5592***</td>
<td>7.4407***</td>
<td>7.6385***</td>
<td>11.1222***</td>
</tr>
<tr>
<td></td>
<td>(1.3626)</td>
<td>(1.3647)</td>
<td>(1.2063)</td>
<td>(1.1060)</td>
<td>(1.2480)</td>
<td>(1.2544)</td>
<td>(3.4171)</td>
</tr>
<tr>
<td>Size</td>
<td>0.4758*</td>
<td>0.4172</td>
<td>0.3808</td>
<td>0.3543</td>
<td>0.6142**</td>
<td>0.6576</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2717)</td>
<td>(0.2590)</td>
<td>(0.2824)</td>
<td>(0.2882)</td>
<td>(0.2384)</td>
<td>(0.3879)</td>
<td></td>
</tr>
<tr>
<td>Maturity</td>
<td>-2.0430</td>
<td>0.0376</td>
<td>0.4959</td>
<td>1.4910</td>
<td>2.3126</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.2817)</td>
<td>(2.5042)</td>
<td>(2.7318)</td>
<td>(1.6041)</td>
<td>(1.9940)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SwissRe</td>
<td>-53.2714</td>
<td>-44.5698</td>
<td>23.1891</td>
<td>70.6117</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(70.4076)</td>
<td>(76.4054)</td>
<td>(77.9947)</td>
<td>(82.5167)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>-19.4858</td>
<td>-34.0692</td>
<td>-54.8725</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(51.9489)</td>
<td>(50.2296)</td>
<td>(32.6467)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔROL</td>
<td>0.0467***</td>
<td>0.0455**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0107)</td>
<td>(0.0164)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BBspread</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.8270</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.6637)</td>
</tr>
<tr>
<td>Constant</td>
<td>54.3767***</td>
<td>8.2862</td>
<td>118.3417</td>
<td>60.6000</td>
<td>48.1507</td>
<td>-85.0508</td>
<td>-0.9911</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.7716</td>
<td>0.7904</td>
<td>0.8032</td>
<td>0.8128</td>
<td>0.8141</td>
<td>0.8485</td>
<td>0.8835</td>
</tr>
<tr>
<td>Adj R-squared</td>
<td>0.758</td>
<td>0.764</td>
<td>0.764</td>
<td>0.759</td>
<td>0.743</td>
<td>0.773</td>
<td>0.809</td>
</tr>
</tbody>
</table>

Note: The dependent variable is adjusted premium spread (annualized). Robust standard errors are reported in parentheses. *** (** or *) denotes significance at the 1% (5% or 10%) level.
Re, the change in the rate on line, and the BB corporate bond spreads) are statistically significant, consistent with the findings by Braun (2016). We use this regression model to fit the premium spreads for the 19 mortality bonds (the curve with diamonds in Figure 3) and to predict the Kortis bond premium. Surprisingly, the fitted mortality bond premiums deviate from the actual ones to a large extent and the predicted Kortis bond premium is only 285 bps.

These results provide us with evidence that CAT bond pricing methodologies can be applied to mortality/longevity bonds. More importantly, a simple linear model using expected loss as the pricing factor may suffice to price mortality-linked securities. However, our results need to be interpreted with caution because our pricing methodology works with mortality bond data only, not all CAT bond data. Parameter uncertainty remains a major issue due to the limited number of mortality/longevity bond transactions. With the development of the longevity market and more issues of mortality/longevity securities, our model can provide higher accuracy and reliability in premium determination.

**CONCLUSIONS**

Although there is a rich literature for mortality modeling in a single population, only a few articles (see, e.g., the references cited in the “Introduction”) examine two-population mortality models. These models are based on a critical assumption that mortality rates of two populations do not diverge in the long run, which seems too strong for short-term mortality forecasts. In spite of the extensive use of copula models in finance and economics, only two studies (i.e., Chen, MacMinn, and Sun, 2015; Wang, Yang, and Huang, 2015) have explored their application to multi-population mortality analysis.
In this article, we propose a dynamic multivariate mortality model based on a factor copula and the GAS structure. The factor copula model that we use has a simple (additive) form and is flexible in model specification according to data availability and the number of variables that need to be specified. This characteristic is important when the number of populations that we need to model is large. We extend the one-factor model in Chen, MacMinn, and Sun (2015) to a two-factor copula model, with one common factor affecting all populations and the other factor having an impact on groups in the same country. We also introduce dynamic dependence among multiple populations using the GAS structure. We find that two-factor models capture the dependence structure much better than one-factor models, so this feature is important for multi-population mortality modeling and pricing; dynamic models are in general better than their static counterparts but the improvement in model fit is not significant based on our data and estimation.

We also contribute to the literature by providing a parsimonious, economic pricing model for mortality/longevity bonds. Our regression model can explain about 81 percent of the premium variations for previously issued mortality bonds and can be used to price future issues. We find that the fitted par spreads are very close to the premiums observed on the primary mortality/longevity bond market.

In addition to pricing longevity-linked derivatives, our multipopulation mortality model can have other applications. For example, it can be used to evaluate longevity basis risk for a longevity hedger. Many life insurers and reinsurers operate internationally and pool policies across countries. It is, therefore, increasingly important for them to understand the joint mortality dynamics for multiple populations and assess mortality/longevity risk in their own books of business. To hedge the risk, they actively issue or trade mortality-linked securities. The reference populations associated with these hedging instruments usually exhibit mortality improvement rates that are different from those of the hedger’s population, and this introduces basis risk that the hedger must manage. Guy et al. (2011) propose a framework for assessing longevity basis risk and hedge effectiveness. A proper multipopulation mortality model can help us identify the correlation (both short run and long run) between these two populations and evaluate the longevity basis risk.

A multipopulation mortality model can also be applied to risk management of an insurance company, for example, to calculate the Solvency Capital Requirement (SCR) (see, e.g., Borger, 2010; Zhou et al., 2014). Under the Solvency II, the total asset required is the sum of the best estimated liability, the SCR and a risk margin. The SCR can either be calculated via a standard formula or an internal model. In contrast to the standard formula, internal models are generally stochastic and thus technically more complex. For a life insurance company, mortality/longevity risk is clearly a major concern. The amount of SCR for longevity risk is calculated as the variation of net asset value (NAV). We can approximate the change in NAV as the difference between the value of liabilities under best estimate assumptions and that in the scenario corresponding to the 99.5th percentile stress (Zhou et al., 2014). A good multipopulation mortality model is needed to capture the mortality dependence among different age groups in the insurer’s life insurance and
annuity portfolios. In addition, it needs to provide sufficient insight into the extreme risks that drives the SCR. In this sense, the two-factor copula model seems better than the one-factor copula model, not only because the former can capture a rather complex dependence structure but also because it can generate a larger volatility of mortality than the latter does. We will leave these applications for future research.

**References**


**Supporting Information**

Additional supporting information may be found in the online version of this article at the publisher’s website.