A simple geometric algorithm to predict optimal starting gantry angles using equiangular-spaced beams for intensity modulated radiation therapy of prostate cancer

Peter S. Potrebko
Department of Physics and Astronomy, University of Manitoba, Winnipeg, Manitoba, R3T 2N2, Canada

Boyd M. C. McCurdy
Division of Medical Physics, CancerCare Manitoba, 675 McDermot Avenue, Winnipeg, Manitoba R3E 0V9, Canada

James B. Butler and Adel S. El-Gubtan
Department of Radiation Oncology, CancerCare Manitoba, 675 McDermot Avenue, Winnipeg, Manitoba R3E 0V9, Canada

Zoann Nugent
Department of Epidemiology, CancerCare Manitoba, 675 McDermot Avenue, Winnipeg, Manitoba R3E 0V9, Canada

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A fast, geometric beam angle optimization (BAO) algorithm for clinical intensity-modulated radiation therapy (IMRT) was implemented on ten localized prostate cancer patients on the Radiation Therapy Oncology Group (RTOG) 0126 protocol. The BAO algorithm computed the beam intersection volume (BIV) within the rectum and bladder using five and seven equiangular-spaced beams as a function of starting gantry angle for comparison to the $V_{75}$ Gy and $V_{70}$ Gy. A mathematical theory was presented to explain the correlation of BIV with dose and dose-volume metrics. The class solution ‘W’ pattern in the rectal $V_{75}$ Gy and $V_{70}$ Gy as a function of starting gantry angle using five equiangular-spaced beams (with two separate minima centered near $20^\circ$ and $50^\circ$) was reproduced by the 5 BIV within the rectum. A strong correlation was found between the rectal 5 BIV and the rectal $V_{75}$ Gy and $V_{70}$ Gy as a function of starting gantry angle. The BAO algorithm predicted the location of the two dosimetric minima in rectal $V_{75}$ Gy and $V_{70}$ Gy (optimal starting gantry angles) to within $5^\circ$. It was demonstrated that the BIV geometric variations for seven equiangular-spaced beams were too small to translate into a strong dosimetric effect in the rectal $V_{75}$ Gy and $V_{70}$ Gy. The relatively flat distribution with starting gantry angle of the bladder $V_{75}$ Gy and $V_{70}$ Gy was reproduced by the bladder five and seven BIV for each patient.

A geometric BAO method based on BIV has the advantage over dosimetric BAO methods of simplicity and rapid computation time. This algorithm can be used as a standalone optimization method or act as a rapid calculation filter to reduce the search space for a dosimetric BAO method. Given the clinically infeasible computation times of many dosimetric beam orientation optimization algorithms, this robust geometric BIV algorithm has the potential to facilitate beam angle selection for prostate IMRT in clinical practice. © 2007 American Association of Physicists in Medicine.

Key words: intensity-modulated radiotherapy, prostate cancer, Gantry angle optimization

I. INTRODUCTION

Intensity-modulated radiation therapy (IMRT) for the treatment of prostate, head-and-neck, and other cancers has been shown to provide dosimetric improvements over three-dimensional conformal radiotherapy (3D-CRT) and, therefore, has gained widespread clinical acceptance.\textsuperscript{1-4} In current IMRT treatment planning, the selection of a set of suitable beam orientations is based on the experience and intuition of the planner or by a trial-and-error approach. However, manually selected beam orientations may be far from optimal especially considering the counterintuitive effects of intensity modulation.\textsuperscript{5} A common approach has been to avoid the sub-optimal placement of beams altogether by employing a relatively large number of beams, such as nine coplanar equiangular-spaced beams, so that the IMRT plan may not be sensitive to the choice of individual beam angles. The evidence for such a strategy has come from a number of investigations employing various target geometries.\textsuperscript{6-7} Bortfeld and Schlegel\textsuperscript{6} demonstrated that, theoretically, the optimal beam configuration with more than three beams tends to be...
an even distribution over an angular range of 0° to 360° in gantry angle. The rationale for this was that a more even distribution results in a smaller burden on the normal tissue surrounding the target. These authors also demonstrated that as the number of beams increases, the dose distribution becomes less dependent on beam orientation. Söderström and Brahme concluded that if a large number of beam angles (≥5) are used, particularly when the tumor is deep seated, it is often sufficient to select equiangular-spaced beam angles to produce a good treatment plan. However, a larger number of beams may have the undesirable consequence of spreading low doses to larger volumes of normal tissues. It may also increase treatment delivery time, quality assurance efforts, and the probability of patient movement during delivery.

The selection of optimal beam directions with the fewest number of beams possible would be advantageous in IMRT. Many investigators have studied computerized beam angle optimization (BAO) methods for the automatic selection of the best beam orientations in both 3D-CRT and IMRT. Different methods, including exhaustive search, genetic algorithms, and integer programming have been used. A common approach has been to optimize an objective function (OF) incorporating dose-volume constraints with respect to the beam orientation. However, such an OF can contain multiple local minima, which may lead to a suboptimal solution if the optimization method becomes trapped in a local minimum.

An exhaustive search can circumvent the local minima problem but is time consuming because of the large search space. Stochastic optimization methods such as simulated annealing and genetic algorithms are capable of escaping from local minima but also have the disadvantage of long computation times. Yang et al. employed a faster deterministic method, which used a mixed integer programming algorithm to search the solution space in a systematic manner (thereby avoiding local minima traps), however, it still required at least 30 min to finish the optimization. In general, the computationally intensive nature of dosimetric-based optimization approaches gives rise to clinically infeasible computation times (0.5 to >10 h) per patient.

Only a few investigators have explored geometry-based methods in order to reduce optimization time, however, these studies were limited to 3D-CRT. In this work, we propose a clinical geometric BAO algorithm for IMRT, based on minimizing beam intersection volume (BIV) within organs-at-risk (OARs). A mathematical theory is presented that explains the correlation of BIV with dose and dose-volume metrics. The algorithm is applied to optimize coplanar, equiangular-spaced beam arrangements since these arrangements (or nearly equiangular ones) are commonly used clinically in prostate IMRT. We show that the BIV in the rectum is strongly correlated to the rectal high dose metrics (from a previous study) and, therefore, facilitates choosing an optimal starting gantry angle for rectal sparing using five equiangular-spaced beams.

II. METHODS AND MATERIALS

II.A. Theory

A mathematical relationship between beam geometry and the dose distribution for both 3D-CRT and IMRT is presented here. Specifically, a clear link between the beam intersection volume and the dose distribution can be derived from the work of Pugachev et al. where filtered backprojection was employed to relate the dose distribution on a two-dimensional (2D) dose plane $D(x,y)$ to the 1D incident intensity profile $I(R)$ [Fig. 1(a)]. For simplicity, Pugachev et al. assumed parallel beam geometry (fully divergent beam calculations are performed in the current work) and only considered primary beams (scattering was neglected). For $N_p$ incident beams, with the $i$th beam denoted by direction $\theta_i$, each incident beam was divided into a series of beamlets. The number of activated beamlets in an incident beam was determined by the BEV projection of the planning target area (PTA). The dose at point $(x,y)$ was then given by

$$D(x,y) = \sum_{i=1}^{N_p} \sum_{j=1}^{N_b} I_i(R) \delta(x \cos \theta_j + y \sin \theta_j - R_j),$$

where $N_b$ is the number of pencil beams in the $i$th beam, $I_i(R)$ was the intensity profile of the $i$th beam, and $R$ was the coordinate of the projection line [Fig. 1(a)]. The function $\delta(p)$ corresponded to the propagation of a single beamlet and was defined by

![Fig. 1](image-url)

(a) Backprojection geometry (2D) with an arbitrary planning target area (PTA). (b) Case I geometry. The $i$th beam is at $\theta=0^\circ$, has width $2w$ (encompassing PTA), and traverses a distance $2L$ within the phantom. Assume the intensity profile $I(R)$ is given by a “top-hat” function. (c) Case II geometry. An organ-at-risk (OAR) of width $2a$ lies within the beam. Assume the intensity profile $I(R)$ is given by a “well” function. (d) Case III geometry. An OAR of width $2a$, which is offset to the side of the beam. This scenario emulates what would be seen if the beam were incident at an oblique angle relative to the Case II example. Assume the intensity profile $I(R)$ is given by an asymmetric “step” function.
\( \delta(p) = \begin{cases} 1, & \text{if} |p| < \Delta R/2 \\ 0, & \text{otherwise}, \end{cases} \) (2)

where \( \Delta R \) was the width of a beamlet.

**Case I (3D-CRT)**

For mathematical simplicity suppose the \( i \)th beam is at \( \theta_i = 0° \), has width \( 2w \) to encompass the PTA, and traverses a distance \( 2L \) within the phantom [Fig. 1(b)]. Assume the idealized intensity profile is given by

\( I_i(R) = \begin{cases} 1, & \text{if} -w \leq R \leq w \\ 0, & \text{otherwise}. \end{cases} \) (3)

According to Eq. (1), the dose distribution from the \( i \)th beam is

\( D_i(x, y) = I_i(R). \) (4)

In this 2D example, if we wish to consider the total dose over an area of the beam rather than simply a point dose, we define a more useful quantity called the areametric dose (or area integrated dose) of the \( i \)th beam given by

\[
\tilde{D}_i = \int_{-L}^{+L} \int_{-w}^{+w} D_i(x, y) \, dx \, dy = 1 \cdot 4Lw = 1 \cdot \text{Area}_i.
\] (5)

Thus, we see that the areametric dose is directly proportional to the beam area weighted by the intensity profile. Note that if the integration limits were taken over an OAR area inside the beam one would conclude that the areametric dose in the OAR is proportional to the area of the OAR encompassed by the beam. The intersection area of all \( N_b \) beams of equal intensity will correspond to the area of maximum dose of the total dose distribution, which is given by

\[
\tilde{D}_{\text{max}} = \bigcap_{i=1}^{N_b} \tilde{D}_i \approx \bigcap_{i=1}^{N_b} \text{Area}_i.
\] (6)

Generalizing to three dimensions, one can in a similar manner define a volumetric dose such that the volume of maximum dose will be given by the intersection volume of all beams:

\[
\tilde{D}_{\text{max}} = \bigcap_{i=1}^{N_b} \text{Volume}_i.
\] (7)

**Case II (IMRT-OAR directly in beam path)**

Suppose we have an OAR of width \( 2a \) in the beam [Fig. 1(c)]. Let \( \Delta_i \) be a parameter representing an arbitrary level of intensity modulation in the beam. Assume the intensity profile is now given by

\[
I_i(R) = \begin{cases} 1, & -w \leq R < -a \\ (1 - \Delta_i), & -a \leq R \leq a \\ 1, & a < R \leq w \\ 0, & \text{otherwise}. \end{cases} \) (8)

According to Eq. (1), the dose distribution from the \( i \)th beam is

\[
D_i(x, y) = I_i(R).
\] (9)

The areametric dose of the \( i \)th beam is

\[
\tilde{D}_i = \int_{-L}^{+L} \int_{-w}^{+w} D_i(x, y) \, dx \, dy
= \int_{-L}^{-a} \int_{-w}^{+w} 1 \cdot dx \, dy + \int_{-w}^{+w} \int_{-a}^{+a} (1 - \Delta_i) \, dx \, dy
+ \int_{-L}^{a} \int_{-2a}^{-w} 1 \cdot dx \, dy
\]

\[
= 1 \cdot 4Lw - \Delta_i \cdot 4La = 1 \cdot \text{Area}_i - \Delta_i \cdot 4La.
\] (10)

It is evident that the areametric dose has been reduced by a strip of width \( 2a \) and length \( 2L \) weighted by the reduction in intensity \( \Delta_i \) due to the presence of the OAR, as compared to the simple “top-hat” intensity function assumed in the previous example.

**Case III (IMRT-OAR offset to side of beam path)**

Suppose we have an OAR of width \( 2a \), which is offset to the side of the beam [Fig. 1(d)]. This scenario emulates what would be observed if the beam in Case II was incident at an oblique angle. Assume the intensity profile is given by

\[
I_i(R) = \begin{cases} 1, & -w \leq R < -2a \\ (1 - \Delta_i), & -2a \leq R \leq w \\ 0, & \text{otherwise}. \end{cases}
\] (11)

According to Eq. (1), the dose distribution from the \( i \)th beam is

\[
D_i(x, y) = I_i(R).
\] (12)

The areametric dose of the \( i \)th beam is

\[
\tilde{D}_i = \int_{-L}^{+L} \int_{-w}^{+w} D_i(x, y) \, dx \, dy
= \int_{-L}^{-w-2a} \int_{-w}^{+w} 1 \cdot dx \, dy + \int_{-w}^{+w} \int_{-a}^{+a} (1 - \Delta_i) \, dx \, dy
+ \int_{-L}^{w+2a} \int_{-w}^{+w} 1 \cdot dx \, dy
\]

\[
= 1 \cdot 4Lw - \Delta_i \cdot 4La
\] (13)

As was seen in Case II, the areametric dose consists of two terms. The first term represents the unmodulated component with unit intensity (from Case I), and the second term represents the intensity modulated component producing a reduction in areametric dose by a strip of width \( 2a \) and length \( 2L \) weighted by the reduction in intensity \( \Delta_i \) due to the presence of the OAR.

In summary, the mathematical theory presented establishes a relationship between geometry (BIV) and dose. For \( N_b \) incident beams, the theory postulates that the volume of maximum dose will occur in the \( N_b \) beam intersection volume. An ideal treatment would consist of the \( N_b \) BIV exactly corresponding to the planning target volume (PTV) in order to minimize high dose regions in the surrounding healthy
to scale the magnitude of the BIV distributions toward an absolute dose variation with beam angle. However, it will be demonstrated that the relative variation of the BIV distributions can accurately predict the relative variation of the dose distribution with beam angle and thus a more time-consuming calculation of dose is unnecessary.

II.B. IMRT treatment plans

The generation of the IMRT treatment plans using a cohort of prostate patients has been discussed in detail in a previous publication. In brief, ten patients with localized prostate cancer treated in the supine position from October 2004 to January 2006 at CancerCare Manitoba according to the Radiation Therapy Oncology Group (RTOG) protocol were selected for this retrospective study. The Arm 2 prescription (79.2 Gy in 44 fractions) of the protocol using IMRT with five and seven equiangular-spaced beam arrangements was applied. The starting gantry angles of 0°, 72°, 144°, 216°, 288° for five beams and 0°, 51°, 102°, 153°, 204°, 255°, 306° for seven beams were incremented by 5° until the starting beam reached the initial angular position of the second beam, resulting in 15 five beams and 11 seven beams plans per patient. All plans were generated in Pinnacle3 using direct machine parameter optimization with 6 MV photon beams and static multileaf collimator delivery.

The PTV coverage satisfied the RTOG 0126 protocol requirements (V 79.2 Gy at least 98%) for all plans. Therefore, the target coverage remained constant for all plans, and the variation in several bladder and rectum dose metrics based on the RTOG 0126 protocol as a function of starting gantry angle was investigated. In the current work, minimization of \( N_b \) BIV in the rectum is employed to find optimal starting gantry angles that minimize the high dose rectal metrics such as \( V_{75\,Gy} < 15\% \) and \( V_{70\,Gy} < 25\% \) and compare to results established in previous work.

II.C. Algorithm

A starting gantry angle optimization algorithm was developed, which interfaces to Pinnacle3 version 7 (Philips Radiation Oncology Systems, Milpitas, CA) in order to extract the Cartesian coordinates of the physician-delineated contours of the PTV, rectum, and bladder from patient treatment plans. The following represent the main steps of the starting gantry angle optimization algorithm: (1) For each beam source position (gantry angle) in the equiangular-spaced beam arrangement, a beams-eye-view (BEV) image of the PTV is produced. (2) Each BEV is divided into a grid of incident ray lines (beamlets) and ray tracing is performed through the BEV to generate a 3D matrix, which models the geometrically diverging primary beam in 3D space. (3) The coincidence volume of all 3D beams in the equiangular-spaced arrangement and all individual OARs is then calculated. (4) Each gantry angle in the equiangular-spaced arrangement is

![Fig. 2. The normalized (Z transform) rectal (a) V 75 Gy, (b) V 70 Gy, and (c) five BIV, as a function of starting gantry angle for all ten patients using five equiangular-spaced beams. The characteristic “W” pattern (with two separate minima centered near 20° and 50°) observed in the rectal V 75 Gy and V 70 Gy was reproduced by the rectal five BIV.](image-url)
then incremented by 5° and steps 1–4 are repeated until beam 1 surpasses the original position of beam 2. In essence, the algorithm implements a very simple geometric case of the backprojection method as discussed in the theory and exhaustively searches the coplanar, equiangular solution space. The algorithm calculated the \( N_b \) BIV in the rectum and bladder using \( N_b = 5 \) and \( N_b = 7 \) equiangular-spaced beams for a ten patient cohort.

The Cartesian coordinates of the 75, 70, 65, and 60 Gy isodose contours were imported into the algorithm from the treatment plans. The algorithm calculated the \( N_b \), \( N_b - 1 \), \( N_b - 2 \), \( N_b - 3 \), and \( N_b - 4 \) BIV components in the rectum within each isodose volume in order to demonstrate that the BIV within an isodose volume can reproduce the dose-volume metric. In fact, it will be demonstrated that the rectal dose-volume metrics are the superposition of the BIV components in the rectum within each isodose volume.

### II.D. Statistical analysis

The coefficient of variation (\( C_{var} \)), defined as the ratio of the standard deviation to the mean, and the range of variation (\( R_{var} \)), defined as the difference between the maximum value and the minimum value, were used to quantify the magnitude of variation with starting gantry angle for the dose metrics from the planning studies as well as the \( N_b \) BIV distributions. The similarity in the pattern of variation with starting gantry angle over the ten patient cohort for the dose metrics and \( N_b \) BIV distributions was assessed by performing a normalization of the data via the Z transform \( \frac{(x_i - \bar{x})}{\sigma} \), defined as the quotient of the difference between the data value and the mean (\( \bar{x} \)) with the standard deviation (\( \sigma \)). Pearson’s correlation coefficient (\( r_{corr} \)), based on Z transformed data, was used to correlate the variation with starting gantry angle of the metrics to the \( N_b \) BIV. For five equiangular-spaced beams, an \( r_{corr} \) value greater than 0.514 (corresponding to a \( p \) value of less than 0.05) was considered to be a significant correlation. For seven equiangular-spaced beams, an \( r_{corr} \) value greater than 0.602 (corresponding to a \( p \) value of less than 0.05) was considered to be a significant correlation.

### III. RESULTS

#### Five equiangular-spaced beams

Figure 2 illustrates the normalized (Z transform) rectal V 75 Gy, V 70 Gy, and 5 BIV with the rectum, as a function of starting gantry angle for all ten patients using five equiangular-spaced beams. The similarity in “W” pattern (with two separate minima centered near 20° and 50°) of the normalized average, minimum, and maximum values for the ten patient cohort indicated a class solution for both the rectal V 75 Gy and V 70 Gy. This distinctive pattern was reproduced by the rectal 5 BIV. The range of variation (\( R_{var} \)) and the coefficient of variation (\( C_{var} \)) in the rectal 5 BIV were comparable to those in the rectal V 75 Gy and V 70 Gy for each patient (Table I). A high correlation coefficient (\( r_{corr} \)) was found between the rectal 5 BIV and the rectal V 75 Gy and V 70 Gy indicating a strong correlation between the geometric BIV and high dose metrics (Table I). The algorithm predicted the location of the two minima in rectal V 75 Gy and V 70 Gy (optimal starting gantry angles) to within 5° (Tables II and III). The predicted minima in rectal V 75 Gy and V 70 Gy differed at most by only 0.9% and 1.2%, respectively, from the observed minima for the ten patient cohort. Figure 3 illustrates the exact reproduction (\( r_{corr} = 0.99–1.00 \)) of the rectal V 75 Gy, V 65 Gy, and V 60 Gy variation with starting gantry angle using the superposition of the BIV components (5 BIV+4 BIV+3 BIV+2 BIV +1 BIV) in the rectum and within each isodose volume (total BIV 75 Gy, total BIV 65 Gy, and total BIV 60 Gy) for a typical prostate patient.

It was interesting to observe that the bladder V 75 Gy, V 70 Gy, and 5 BIV did not exhibit any “W” pattern as was seen for the rectum. In fact, the small coefficient of variation (\( C_{var} \)) observed in the relatively flat distribution with starting gantry angle of the bladder V 75 Gy and V 70 Gy was reproduced by the bladder 5 BIV for each patient (Table IV).

#### Seven equiangular-spaced beams

Figure 4 illustrates the normalized (Z transform) rectal V 75 Gy, V 70 Gy, and 7 BIV, as a function of starting gantry angle for all ten patients using seven equiangular-spaced beams.
beams. The 7 BIV demonstrated a characteristic “W” pattern, however, unlike with five beams, there was no such dosimetric pattern in the rectal V 75 Gy and V 70 Gy. The range of variation (R_{var}) and the coefficient of variation (C_{var}) in the rectal V 75 Gy, V 70 Gy, and 7 BIV were reduced compared to those with five beams (Table V). Only one patient (7) demonstrated a significant correlation (r_{var}) between the rectal 7 BIV and the rectal V 75 Gy and V 70 Gy.

As was observed with five beams, the bladder V 75 Gy, V 70 Gy, and 7 BIV did not exhibit any “W” pattern. The small coefficient of variation (C_{var}) observed in the relatively flat distribution with starting gantry angle of the bladder V 75 Gy and V 70 Gy was reproduced by the bladder 7 BIV for each patient (Table IV).

IV. DISCUSSION

It has been demonstrated that, even with intensity modulation, there was a strong correlation between the characteristic “W” pattern observed in the rectal V 75 Gy and V 70 Gy with the 5 BIV as a function of starting gantry angle. This was a verification of the theory presented in that it confirmed that the volume of maximum dose within a critical structure (as assessed by the rectal V 75 Gy and V 70 Gy) was proportional to the intersection volume of all beams within that structure despite the ability to highly modulate the radiation beams. It is important to note that even with the assumption of uniform incident intensity (equal beamlet weights) for all beams in the purely geometric BIV algorithm, there was a strong correlation between the geometric 5 BIV and high dose rectal metrics. This can be understood from the theory presented in Sec. II in that, for each beam, the intensity acts as a weighting in the proportionality between the maximum dose in a critical structure and the volume of beam intersection within that structure (herein referred to as the “proportionality relationship”). In current practice, an IMRT beam will never be so highly modulated to have its intensity completely diminished within a critical structure since a minimum non-zero level of intensity is required for target coverage. Therefore, a proportionality relationship will always exist. In prostate IMRT, the results of this work demonstrate that the low level of intensity modulation with five beams produces a very strong proportionality relationship.

The “W” pattern observed in the rectal V 75 Gy, V 70 Gy and 5 BIV as a function of starting gantry angle was produced by the beam geometry. Figure 5 illustrates this effect in two dimensions on a representative computerized tomography (CT) slice for a typical prostate patient (patient 9).
Figures 5(a) and 5(c) illustrate the “W” pattern maxima in rectal 5 BIV at starting gantry angles of 0° and 35°. These two equiangular-spaced beam configurations are nearly mirror reflections of each other about the patient midline. From a geometrical point of view, a perfectly symmetric “W” pattern with equal magnitude maxima at starting gantry angles of 0° and 35° should be produced if the rectum and PTV are perfectly symmetric about their midline on each CT slice. This is approximately the case on the CT slice shown in Figs. 5(a) and 5(c) where the rectal 5 BIV at starting gantry angles of 0° and 35° are approximately equal. However, the observed “W” patterns in rectal V 75 Gy, V 70 Gy, and 5 BIV are not symmetric, demonstrating a slightly lower maximum at 35° compared to 0°. This likely results from the fact that the rectum and PTV are not perfectly symmetric about their midline in 3D. Figures 5(b) and 5(d) also demonstrates the “W” pattern minima in rectal 5 BIV at starting gantry angles of 20° and 50°. Again, these two equiangular-spaced beam configurations are nearly mirror reflections of each other about the patient midline. The optimal configurations for rectal sparing occur with a nearly lateral beam gantry 92° or 266°, which minimizes the 5 BIV in the rectum, and furthermore we suggest helps to form a sharp dose gradient between the PTV and rectum.

Schreibmann et al. investigated beam orientation class solutions in prostate IMRT. The authors used a genetic algorithm to optimize the beam orientations and a gradient-based method to optimize the intensity profiles of the beams. They concluded that the optimized five beam configurations in all 15 patient cases they examined had a similar beam setup, with nearly equiangular-spaced beams starting at the beam position of 20°–45°. They proposed a class solution with five incident nearly equiangular-spaced beams (gantry =35°, 110°, 180°, 250°, 325°) by averaging the optimal gantry angles of all 15 patients even though optimal starting gantry angle solutions for individual patients were either near 15° or 50°. However, the results from the current work and previous work indicate that there is degeneracy in the optimal starting gantry angle solution with two possible solutions not attributable to prostate patient variability. We have demonstrated that a class solution with starting gantry angle of 35° is not optimal since it coincides with a local maximum in the “W” pattern of the rectal V 75, V 70 Gy, and 5 BIV. Schreibmann et al. suggested that this class solution seems to be physically sensible since setting the angles of beams 1, 2, and 4 to 35°, 110°, and 250°, respectively, balances the dose to the femoral heads and the locations of the third (180°) and fifth (325°) beams are chosen to balance the dose requirements of the PTV and rectum. However, this work has reproduced the optimal starting gantry angles of Schreibmann et al. based solely on variations in 5 BIV with the rectum as a function of starting gantry angle. Therefore, the optimal starting gantry angles seem to be primarily the...

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result of rectal sparing required by the objective function of the IMRT optimization and are highly correlated with the geometric effect of rectal 5 BIV. These results are a validation of the optimal five equiangular-spaced beam configurations for individual patients presented by Schreibmann et al. and provide further refinement by recognizing that two optimal class solutions exist for prostate IMRT with equiangular-spaced beams. Furthermore, we have been able to reproduce the Schreibmann et al. dosimetrically optimized beam configurations based on simple geometrical 5 BIV with the rectum without the need to invoke complicated, time-consuming and computationally-intensive dosimetric optimization methods.

The exact reproduction of the rectal dose-volume metric variation with starting gantry angle through the superposition of the BIV components within the isodose volume using five equiangular-spaced beams (Fig. 3) demonstrates, as expected, that a dose-volume histogram is the superposition of volume-weighted BIV components. The small discrepancy (1%–3%) between the normalized volume of the rectal dose-volume metric and the total BIV for each starting gantry angle is due to the algorithm performing several image processing operations (region filling, dilation, erosion) on the rectal and PTV contours, which discretize the smooth contours into coarser pixels. Even though the superposition of the BIV components within an isodose volume was calculated with a priori knowledge of the dose distribution, this demonstrates that beam geometry (BIV) can reproduce effects in the dose distribution and is the subject of future investigation. This is the first work to reproduce dose-volume metrics based on geometry alone and has exciting implications for radiotherapy optimization.

The use of seven equiangular-spaced beams resulted in the dose distribution becoming less dependent on rectal BIV. This was demonstrated by the disappearance of the dosimetric “W” pattern in rectal V 75 Gy and V 70 Gy although the 7 BIV still demonstrated such a pattern. However, the range of variation \(R_{var}\) and the coefficient of variation \(C_{var}\) in the rectal 7 BIV were reduced by approximately half compared with those with 5 BIV. Apparently, the 7 BIV geometric variations were too small to translate into a strong dosimetric effect. Also, the IMRT optimization had two more degrees of freedom (beams) to compensate for beam directions delivering high rectal dose. These results support the findings of other investigators also demonstrating that as the number of IMRT beams is increased, the dose dependence on beam orientation diminishes.\(^5,7\)

It was observed that both the bladder 5 BIV and 7 BIV did not exhibit any “W” pattern as was seen for the rectum. Instead, a relatively flat distribution (small coefficient of variation, \(C_{var}\)) as a function of starting gantry angle in the 5 BIV and 7 BIV reproduced the flat distribution in bladder V 75 Gy and V 70 Gy. The different responses with starting gantry angle observed between the rectum and bladder are due to the volume of each organ in the primary beam paths. Specifically, a much smaller bladder volume is exposed to the primary beams compared to the rectum, therefore, dose-volume variations are magnified over the larger rectal volume. For example, for a typical prostate patient using five equiangular-spaced beams, the maximum in 5 BIV as a function of starting gantry angle was 35% for the rectum and only 8% for the bladder.

The presented BIV algorithm has been applied only to coplanar beam geometry, however, it may be easily used to compute BIV using non-coplanar beams. A generalized BIV algorithm that selects optimal deliverable beam orientations (that minimize the BIV within OARs) from the complete space of gantry and couch angles is being developed. The

![Fig. 4. The normalized (Z transform) rectal (a) V 75 Gy, (b) V 70 Gy, and (c) seven BIV, as a function of starting gantry angle for all ten patients using seven equiangular-spaced beams. The seven BIV demonstrated a characteristic “W” pattern, however, unlike with five beams, there was no such dosimetric pattern in the rectal V 75 Gy and V 70 Gy.](image-url)
BIV optimization approach has potential to produce improved OAR sparing in other treatment sites such as the head and neck, lung, and abdomen, and is the subject of future investigation. The current algorithm is written using the Interactive Data Language RSI, Boulder, CO. With the use of this high-level programming language for developmental purposes, typical computation times for the current BIV algorithm are of the order of a few minutes but can be reduced by a factor of 100 or more by using a lower level programming language such as C.

V. CONCLUSIONS

It was demonstrated that the rectal 5 BIV is strongly correlated to the rectal high-dose metrics of IMRT plans. The geometric minima in rectal 5 BIV corresponded to the dosimetric minima in rectal V 75 Gy and V 70 Gy. The implication of this is that a geometric quantity such as BIV can be used to predict the optimal dose distribution for rectal sparing in prostate IMRT using five equiangular-spaced beams. It was shown that a dose-volume metric is the superposition of volume-weighted BIV components within the isodose volume. A mathematical theory was presented that explains the correlation of BIV with dose and dose-volume metrics. A geometric optimization method based on BIV has the advantage over dosimetric methods of simplicity and rapid computation time. In this work, the geometric algorithm was able to predict the dosimetric minima to within 5° (the angular step size used here). This algorithm can be used as a stand-alone optimization method or act as a rapid calculation filter to reduce the search space for a dosimetric beam orientation optimization method. Given the clinically infeasible computation times of many dosimetric beam orientation optimization algorithms, this robust BIV algorithm has the potential to facilitate beam angle selection for prostate IMRT in clinical practice.

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Author to whom correspondence should be addressed. Electronic mail: peter.potrebko@cancercare.mb.ca Telephone: (204) 787-8023; Fax: (204) 775-1684


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