Doppler Properties of Polyphase Coded Pulse Compression Waveforms

F. F. KRETSCHMER, JR.
B. L. LEWIS
Naval Research Laboratory

Doppler properties of the Frank polyphase code and the recently derived P1, P2, P3, and P4 polyphase codes are investigated and compared. An approximate 4 dB cyclic variation of the peak compressed signal is shown to occur as the Doppler frequency increases. The troughs in the peak-signal response occur whenever the total phase shift across the uncompressed pulse, due to Doppler, is an odd multiple of π radians.

It is shown that while the P3 and P4 codes have larger zero-Doppler peak sidelobes than the other codes, the P3 and P4 codes degrade less as the Doppler frequency increases. Also, the effects of amplitude weighting and receiver bandlimiting for both zero and nonzero Doppler are investigated.

I. INTRODUCTION

The Frank polyphase coded waveform, and the P1, P2, P3, and P4 polyphase pulse-compression waveforms previously described by the authors [1-3], provide a class of frequency-derived phase coded waveforms that can be sampled upon reception and processed digitally.

These waveforms are derivable from the chirp and step-chirp analog waveforms and are therefore similar in certain respects. There are some important differences, however, which include differences in sidelobe levels, implementation techniques, and Doppler characteristics.

The compressed pulse of the polyphase coded waveforms has sidelobes which decrease as the pulse compression ratio μ is increased. For μ equal to 100, the peak sidelobes range from 26 to 30 dB below the peak signal response depending on the particular code. In contrast, the compressed chirp or step-chirp pulse has approximately 13 dB sidelobes, independent of the pulse compression ratio, which can mask a relatively weak nearby target. An amplitude weighting is generally applied to reduce the sidelobes and the resulting mismatch reduces the output S/N by 1 to 2 dB.

The polyphase coded waveforms are capable of large pulse compression ratios which may be efficiently implemented using the phase shifts provided by a fast Fourier transform (FFT). Thus the FFT can be used directly as the pulse compressor. These waveforms can also be efficiently compressed with another pulse compression technique where the FFT is used to convert to the frequency domain where the matched filtering and weighting are performed. This processing is followed by an inverse FFT to restore the signal to the time domain.

This paper first reviews the properties of the polyphase coded waveforms, then focuses on the Doppler characteristics of these waveforms. A cyclic loss of approximately 4 dB is discussed which is characteristic of frequency derived polyphase coded waveforms having low sidelobes. This cyclic variation was not recognized in the prior literature dealing with Frank codes [4, 5]. A method of compensating for this loss is described.

The Doppler characteristics of the various polyphase codes are investigated in detail. Also, the effects of weighting on the Doppler performance of the codes is presented. This weighting may be due to an applied amplitude weighting and/or it may be caused by bandlimiting in the receiver.

II. PROPERTIES OF POLYPHASE CODED WAVEFORMS

A. Frank-Polyphase-Coded Waveforms

The Frank-polyphase-coded waveform may be described and generalized by considering a hypothetically sampled step-chirp waveform [1, 2]. The Frank code was not originally described in this manner, but was given in
terms of the elements of a matrix [6]. As an example, consider a four-frequency step-chirp waveform as shown in Fig. 1(b) where the $F_i$ denote frequency tones. In this waveform, the frequency steps are equal to the reciprocal of the tone duration $4\tau_c$, where $\tau_c$ denotes the compressed pulsewidth. Assuming this waveform has been beat to baseband $I$ and $Q$ using a synchronous oscillator having a frequency the same as the first tone frequency, the resultant phase-versus-time characteristic consists of four linear sections, as shown in Fig. 1(a). The corresponding baseband frequencies are the subharmonics of the frequency $1/\tau_c$. If the baseband phases of the step-chirp waveforms are sampled every $\tau_c$ seconds and held for $\tau_c$ seconds, the phase sequence shown in Fig. 1(c) is obtained. This sequence of phases constitutes the phases of a Frank code for $N = 4$, corresponding to the four baseband frequencies of the hypothetical step-chirp waveform. The actual transmitted Frank-coded waveform consists of a carrier whose phase is modulated according to the indicated baseband waveform sequence. For each frequency, or section, of the step-chirp phase characteristic, a phase group consisting of $N$ phase samples is obtained and the total number of code phases is $N^2$ which is equal to the pulse-compression ratio. Note that the phase increments within the four phase groups are $0^\circ$, $90^\circ$, $180^\circ$, and $270^\circ$. However, the phases of the last group are ambiguous ($>180^\circ$) and appear as $-90^\circ$ phase steps or as the conjugate of the $F_1$ group of phases, which corresponds to the lower sideband of $F_1$. The last group of phases appears, because of the ambiguity, to complete one $360^\circ$ counterclockwise rotation rather than the $(N-1)$ counter-clockwise rotations of the end frequency of the step-chirp waveform.

The Frank-code phase may be stated mathematically as follows. The phase of the $i$th code element in the $j$th phase group, or baseband frequency, is

$$\phi_{i,j} = (2\pi/N) (i-1) (j-1)$$

where the index $i$ ranges from 1 to $N$ for each of the values of $j$ ranging from 1 to $N$. An example of a brute force Frank-code pulse generation for $N = 3$ is shown in Fig. 2. The Frank-code phases are the same as the negative of the steering phases of an $N$ point discrete Fourier transform where the $j$th frequency coefficient is

$$F_j = \sum_{i=1}^{N} a_i \exp \{-j(2\pi/N) (i-1) (j-1)\}$$

where $a_i$ is the $i$th complex input time sample. This means that a considerable savings in hardware can be achieved by using the efficiency of an FFT.

The matched-filter output for an $N = 10$ (100-element) Frank code is shown in Fig. 3. This figure and the following figures showing the compressed pulse were obtained by sampling the input baseband waveform once per code element or per reciprocal bandwidth, unless stated otherwise. Using a discrete-time matched filter, the output signal is also a discrete-time sampled signal. However, for ease of plotting and viewing, the points were connected by straight lines. The four side lobe peaks on each half of the match point (peak response) are of equal magnitude. The first peak side lobe at sample number 5 in Fig. 3 occurs as the last phase group having $-36^\circ$ phase.

---

**Fig. 1.** Step-chirp and Frank polyphase code relationships.

**Fig. 2.** Simplified Frank code generation.

**Fig. 3.** Compressed pulse of 100-element Frank code.
increments indexes halfway into the first phase group of zero phase vectors in the autocorrelation process. In general, at sample number \( N/2 \), there are \( N/2 \) vectors adding to complete a half circle. The end phase group indexing into the first phase group of \( 0^\circ \) vectors makes an approximate circle since the phases of the last phase group make only one rotation as stated previously. The peak sidelobe amplitude may be approximated by the diameter \( D \) of the circle from the relation,

\[
\text{perimeter} = N = \pi D \tag{3}
\]

or

\[
D = N/\pi. \tag{4}
\]

At the match point the amplitude is \( N^2 \) so that the peak response to peak-sidelobe power ratio \( R \) is

\[
R = N^4/(N/\pi)^2 = N^2 \pi^2 = \rho \pi^2. \tag{5}
\]

For a 100-element Frank code, this ratio is approximately 30 dB, as shown in Fig. 3.

Had the phases of the polyphase coded waveform been generated by using the phases of the step-chirp phase characteristic sampled at 1/5 of the interval used for the Frank code, the compressed code would appear as shown in Fig. 4. In this figure, five samples are equal in time to one sample in Fig. 3. Note in Fig. 4 that the near-in sidelobes are approximately 13 dB and that the envelope of the sidelobe peaks is approximately that of a sin \( x/x \) pulse. The 13 dB sidelobes also appear for an oversampling of 2:1. These comments also apply to the other polyphase codes described in this paper. Also note that the compressed pulsewidth in Fig. 4 has not decreased since it is determined by the underlying bandwidth of the step-chirp waveform.

(1) Effects of Bandlimiting Prior to Pulse Compression: A Frank-coded waveform is depicted in Fig. 5(a) where the \( G_K \) denote the phase groups corresponding to the sampled phases of a step-chirp waveform as previously discussed. Each group consists of \( N \) vectors beginning with a vector at a phase angle of \( 0^\circ \). The phase increments within the \( K \)th group are

\[
\Delta \phi_K = K \frac{360^\circ}{N}. \tag{6}
\]

Thus \( G_0 \) consists of \( N \) vectors at \( 0^\circ \). \( G_1 \) has vectors separated by \( 360^\circ/N \) until at the center of the coded waveform the phase increments approach or become \( 180^\circ \) depending on whether \( N \) is odd or even. For phase increments greater than \( 180^\circ \), the phases are ambiguous with the result that the phasors of phase group \( G_{N-K} \) are the conjugates of the phasors of phase group \( G_K \) so that the vectors have the same increments but rotate in opposite directions. The result is that the phase increments are small at the ends of the code and become progressively larger toward the center of the code where the increments approach \( 180^\circ \) from opposite directions.

If a receiver is designed so that it has an approximately rectangular bandwidth corresponding to the 3 dB bandwidth of the received waveform, the received waveform becomes bandlimited and a mismatch occurs with the compressor. This bandlimiting would normally occur prior to sampling in the A/D conversion process in order to prevent noise foldover and aliasing. The result of any bandlimiting is to average (or smooth) the vectors constituting the coded waveform, and for the Frank code, a weighting \( W(t) \) such as illustrated in Fig. 5(a) takes place due to the larger phase increments toward the middle of the code. This weighting causes an unfavorable mismatch with the compressor which results in a degradation of the sidelobes relative to the peak response.

The new symmetrical codes found by the authors have the common property that the phase groups with the small phase increments are at the center of the code and the larger increment groups progress symmetrically toward the ends of the code. This is illustrated in Fig. 5(b) where a favorable amplitude weighting resulting from bandlimiting prior to pulse compression is shown.

B. \( P_1, P_2, P_3, \) and \( P_4 \) Polyphase Codes

The new polyphase codes which tolerate bandlimiting are referred to as the \( P_1, P_2, \) and \( P_4 \) codes. The \( P_1 \) code was derived from use of the previously described rela-
rionship between the Frank-code phases and those of a sampled step-chirp waveform. The desired symmetry, having the dc or small incremental phase group at the center of the code, can be achieved by determining the phases which result from placing the hypothetical synchronous oscillator at the band center of the step-chirp waveform. For an odd number of frequencies, the synchronous oscillator frequency corresponds to one of the waveform frequencies and the resulting phases are the same as the Frank code except the phase groups are rearranged as indicated in Fig. 5(b). If there is an even number of frequencies, the synchronous oscillator frequency placed at the band center does not correspond to one of the frequencies in the step-chirp signal. The phase of the $i$th element of the $j$th group is given in degrees by

$$\phi_{i,j} = -(180/N) [N - (2j - 1)] [(j - 1)N + (i - 1)]$$  

(7)

where $i$ and $j$ are integers ranging from 1 to $N$.

The $P2$ code, which also has the desired features, is similar to the Butler matrix steering phases used in antennas to form orthogonal beams. The $P2$ code is valid for $N$ even, and each group of the code is symmetric about zero phase. The usual Butler matrix phase groups are not symmetric about zero phase and result in higher side-lobes. For $N$ even, the $P1$ code has the same phase increments, within each phase group, as the $P2$ code except that the starting phases are different. The $i$th element of the $j$th group of the $P2$ code is given in degrees by

$$\phi_{i,j} = (90/N) [N + 1 - 2i] [N + 2 - 2j]$$  

(8)

where $i$ and $j$ are integers ranging from 1 to $N$ as before. The requirement for $N$ to be even in this code stems from the desire for low autocorrelation sidelobes. An odd value for $N$ results in high autocorrelation sidelobes. This code has the frequency symmetry of the $P1$ code and also has the property of being a palindromic code which is defined as a code having symmetry about the center.

The $P4$ code is similar to the $P1$ code except that the phase samples are those of a sampled chirp waveform rather than step-chirp waveform. In each case, the synchronous oscillator is placed at the band center with the result that the codes are symmetrical. The $P3$ code is also derived from a chirp waveform and is the counterpart of the Frank code where the synchronous oscillator is put at the lowest frequency to determine the phases. The $P3$ code is therefore intolerant of bandlimiting like the Frank code.

The phases of a modified $P4$ code\textsuperscript{1} are given by

$$\phi_i = (45/\rho) (2i - 1)^2 - 45(2i - 1), \quad 1 \leq i \leq \rho$$  

(9)

and the phases of the $P3$ code are

$$\phi_i = (180/\rho) (i - 1)^2, \quad 1 \leq i \leq \rho.$$  

(10)

\textsuperscript{1}This code varies slightly from the one given in [3]. In effect, the first sample of the Nyquist sampling of a chirp signal has been shifted by $1/2$ of a sample period to produce a palindromic code.

The compressed pulse for a zero-Doppler, 100-element $P3$ or $P4$ code is shown in Fig. 6. It is similar to the Frank, $P1$, and $P2$ 100-element codes except that the peak sidelobes are approximately 4 dB higher.

![Fig. 6. Compressed pulse of 100-element $P3$ or $P4$ code.](image)

III. DOPPLER PROPERTIES OF POLYPHASE CODES

A. Ambiguity Functions and Cyclic Losses

A partial ambiguity function for a 100-element Frank code is shown in Fig. 7 which shows the amplitude in dB of a matched-filter output for given Doppler shifts of the input. The Doppler is normalized to the signal bandwidth and the delay axis is normalized to the uncompressed pulse length. The cut through this diagram at zero Doppler shows the output of a perfectly matched receiver. In this case, the output pulse is the same as the autocorrelation function of the input waveform. A cut along any other Doppler axis shows the output of the receiver for an input waveform having a Doppler which is mismatched to the receiver by the stated amount. The vertical scale ranges from 0 dB to $-60$ dB, and the $-30$ dB sidelobes for zero Doppler are evident. The normalized Doppler

![Fig. 7. Partial ambiguity function for 100-element Frank code.](image)
shift of $-0.05$ shown in this figure corresponds to a mach-50 target for an $L$-band radar having a signal bandwidth of 2 MHz. The first Doppler cut shown in the literature [4] is taken at this normalized Doppler and the resultant high-peak sidelobes shown in Fig. 8 have per-

haps discouraged usage of the Frank code. The region shown between zero and mach-5 Doppler, and a delay interval of $\pm 0.3$ is of interest, and it is shown on an expanded scale in Fig. 9. In this region, the Doppler response is good in terms of the sidelobe levels. (The blank spot on the plot was caused by a computer plotting glitch.) The corresponding ambiguity function for the 100-element $P4$ code is shown in Fig. 10 where the peak response is seen to have the same cyclic variation as the Frank code. The differences between these ambiguity functions are discussed in later sections and it will be shown that amplitude weighting or bandlimiting will reduce the image lobes at the ends of the compressed pulse. At the Doppler shift of $-0.005$, or more generally $\pm 1/ (2p)$, the total phase shift across the uncompressed pulse is $\pi$ and the peak response drops approximately 4 dB. At this Doppler, there is a range-Doppler coupling of $1/2$ of a range cell with the result that the signal splits between two range cells. This is illustrated in Fig. 11 for the Frank code. At a normalized Doppler shift of $-0.01$, or

Fig. 8. Compressed pulse of 100-element Frank code, Doppler = $-0.05$.

Fig. 9. Magnified ambiguity function of 100-element Frank code.

Fig. 10. Partial ambiguity function for 100-element $P4$ code.

Fig. 11. Compressed pulse of 100-element Frank code, Doppler = $-0.005$.

in general $\pm 1/p$, there is a range-Doppler coupling of one range cell resulting from a total phase shift of $2\pi$ radians across the uncompressed pulse, and the main peak response is restored to nearly full amplitude as shown in Figs. 7 and 10. This effect is cyclic and a loss of approximately 4 dB is encountered when the total phase shift due to Doppler is an odd multiple of 180°.

The loss may also be shown by considering the misalignments of the vectors at the pulse compressor output due to a Doppler shift which results in phase shifts across the uncompressed pulse duration $T$. The loss occurs because the matched filter does not de steer the phases due to Doppler. For a Doppler frequency $f_d$, the phase increments from subpulse to subpulse are

$$\Delta \theta = 2\pi f_d T/p$$

and the resultant unit-normalized signal is

$$S = (1/p) \sum_{n=0}^{p-1} \exp(-jn\Delta \theta)$$

$$= (1/p) \frac{\sin \rho (\Delta \theta/2)}{\sin (\Delta \theta/2)}$$

($\rho$ = 1/p)
For $\Delta \theta = 0$, the maximum normalized output of one is obtained. When the total phase shift across the uncompressed pulse is $\pi$, one finds that

$$S = 2/\pi.$$  (13)

This is equivalent to an approximate 4 dB loss and corresponds to a range-Doppler coupling of $1/2$ of a range cell. As mentioned previously, for a range Doppler coupling of 1 range cell, the total phase shift across the uncompressed pulse is $2\pi$ and the peak amplitude is nearly restored. The trough following each peak is down approximately 4 dB from the peak.

It should be noted that this 4 dB loss also occurs in a pseudorandom binary shift register code when the Doppler phase shift across the uncompressed pulse is $\pi$. However, the response is not cyclic and monotonically decreases as the Doppler frequency is increased. For this reason, a Doppler filter bank is usually instrumented to cover the Doppler band of interest.

The cyclic loss indicated for the polyphase code is a consequence of deriving the phases of the polyphase code from those of a step-chirp or chirp waveform which is sampled at the Nyquist rate. Had the phases been sampled faster, the cyclic loss would decrease and the peak sidelobes of the compressed pulse would increase to approximately 13 dB as previously described. This is a general property of the polyphase codes described in this paper.

B. FFT Implementations and Doppler Compensation

Another property of the polyphase codes described in this paper is that they can be implemented using a modified FFT phase structure. An example is shown in Fig. 12 for a $P1$ code. Each code can be generated or compressed using the same standard FFT phase filter shown in Fig. 13. The phase shifts used before and after the FFT phase filter depend on the particular code.

One way to reduce the 4 dB cyclic variation of the polyphase codes is to provide an additional output port for the compressed pulse which provides an approximate phase compensation of $\pi$. This could be achieved by using additional phase shifters and delay lines in the $F_i$ output ports of the FFT phase filter shown in Fig. 12.

![Diagram of FFT phase filter](image)

**Fig. 13.** FFT phase filter.

C. Comparison of Doppler Responses and Effects of Weighting

In this section we compare the Doppler responses of the various 100-element codes by examining the Doppler variation of the peak compressed signal, the peak sidelobes or secondary maxima, and the image signal. The image signal is due to the polyphase codes being derived from a Nyquist rate sampling of the step-chirp or linear-chirp phase characteristics. These are illustrated in Fig. 14 for a Frank-coded receive waveform having a normalized Doppler frequency of $-0.05$.

![Graph of Doppler responses](image)

**Fig. 14.** Compressed pulse of 100-element Frank code, Doppler $= -0.05$.

The Doppler behavior of the Frank, $P1$, and $P2$ codes is the same and is shown in Fig. 15 where the cyclic variation of the peak amplitude is evident. There is also a cyclic behavior of the secondary maxima and the envelope of the peak signal response every 0.1 in normalized Doppler. This Doppler corresponds to a range-Doppler coupling of 10 range cells for the 100-element code, which is the equivalent duration of one phase or frequency group. These secondary maxima are nearly the same as those which occur for an analog step-chirp compressed pulse having the same Doppler shifted input waveform [4]. Fig. 15, except for the rapid cyclic behav-
ior, is similar to [4, fig. 8.28]. However, the peak response in Fig. 15 does not fall off as fast with Doppler. This is attributed to our sampling the received Frank code once per code element. Also, our secondary maxima and image signal include the zero Doppler values.

The P3 and P4 codes also have the same Doppler behavior which is shown in Fig. 16, where it is evident that

Fig. 15. Doppler properties of 100-element Frank, P1, and P2 codes.

the secondary maxima sidelobes are generally much less than for the Frank, P1, and P2 codes shown in Fig. 15. At zero Doppler, the P3 and P4 codes have sidelobes that are approximately 4 dB higher than the Frank, P1, and P2 codes. The compressed pulse for the P3 or P4 code having a Doppler shift of \(-0.05\) is shown in Fig. 17, which should be compared with the Frank, P1, and P2 compressed pulse in Fig. 14.

The effects of weighting, using a cosine on a pedestal of 0.2, are depicted in Fig. 18 for the Frank, P1, and P2 codes (Case 1) and in Fig. 19 for the P3, P4 codes (Case 2). Figs. 20 and 21 show the compressed pulses for the weighted Case 1 and Case 2 codes. Figs. 20(a) and 21(a) and 20(b) and 21(b) show normalized Doppler frequencies of 0.0 and \(-0.05\), respectively. A comparison of these figures with the corresponding unweighted responses shown in Figs. 3, 14, 6, and 17, respectively, allows several observations. First, weighting reduces the percentage cyclic variation of the compressed pulse peak with Doppler. Second, weighting increases the ratios of the peak signal to the image lobe, the mean-squared sidelobes, and to a lesser extent, the secondary maxima. An-
Fig. 20. Compressed pulse of weighted 100-element Frank, P1, and P2 codes. (a) Doppler = 0.0. (b) Doppler = −0.05.

other aspect of weighting is that it can be shown to reduce the fluctuation of the compressed pulsewidth as the Doppler varies.

D. Bandlimitation Effects on Doppler Responses

In this section we describe the effects of bandlimitation prior to pulse compression. It was previously described how bandlimitation acts as an adverse amplitude weighting on the Frank and P3 codes but improves the P1, P2, and P4 symmetrical codes.

The behavior of the polyphase codes was compared in the presence of bandlimiting. The bandlimiting effect was simulated by oversampling the received coded waveform by a factor of 5 to approximate the analog received waveform and then filtering this waveform with a 5-sample sliding window average. The inputs to the pulse compressor were then taken using every fifth sample. The particular set of inputs is therefore dependent on the starting time. The outputs of the compression filter were computed for the five sets of input data, corresponding to different sampling times within the subpulsewidth of the coded element. This data was averaged and is shown in Figs. 22, 23, and 24 for the Frank, P2, and P4 codes, respectively, for normalized Doppler frequencies of 0 and −0.05. The results for the P3 and P1 codes were similar to the Frank and P2 codes, respectively, and are not shown. In Figs. 22(a), 23(a), and 24(a) showing the zero Doppler cases, the peak responses are each reduced approximately 2.4 dB. However, for the Frank code, the secondary maxima and the image sidelobes are not reduced. Thus, it is seen that in the presence of bandlimiting, the Frank code compressed pulse degrades. Referring to Figs. 23(a) and 24(a), it is seen that the secondary maxima and image sidelobes are approximately 5 dB lower with the result that the peak signal to sidelobe ratio is improved. In Figs. 22(b), 23(b), and 24(b), we show the same codes having a normalized Doppler shift of −0.05. Comparing 22(b) with Fig. 14 for the Frank code, it may be seen that the sidelobe levels are approximately the same although the peak signal is reduced by nearly 4 dB. This again shows that the Frank code is degraded in the presence of bandlimiting. For the P2 code in Fig. 23(b), the ratio of the peak signal to secondary maxima is the same as for no bandlimiting (this ratio is the same as for the Frank code shown in Fig. 14); how-
ever, the ratio of the peak signal to the image lobe has improved by approximately 3 dB. In Fig. 24(b), showing
the bandlimited $P_4$ coded waveform having a Doppler
shift of $-0.05$, the ratio of the peak signal to image lobe
is also improved by approximately 3 dB compared with
the case shown in Fig. 17 for no bandlimiting.

From these results and prior comments, it is seen that
for zero and nonzero Doppler shifts, the Frank and $P_3$
codes degrade in the presence of bandlimiting. On the
other hand, the symmetrical $P_1$, $P_2$, and $P_4$ codes
improve mainly in terms of the ratios of the peak signal to
the image lobe and the mean-squared sidelobes. The peak
signal to secondary maxima ratio is improved by several
dB for zero Doppler, and for higher Dopples the ratio
is approximately the same. The large secondary maxima of
the Frank, $P_1$, and $P_2$ codes which occur at the higher
Doppler frequencies are not present with the $P_3$ or $P_4$
codes.

IV. SUMMARY

The basic properties of the Frank and the $P_1$, $P_2$, $P_3$, and $P_4$ polyphase codes were presented. It was shown
how these codes may be obtained from considering the
sampled phases of the step-chirp and chirp baseband
waveforms. These codes can be digitally compressed by
using FFTs directly or by a fast convolution technique.

The Doppler properties of these waveforms were in-
avestigated in detail. It was shown that these waveforms
have a cyclic loss of approximately 4 dB which is attributed to the $\rho$ discrete phases that are used. As the
number of phase samples increases, this loss diminishes and
the sidelobe levels approach the 13 dB level of the chirp
waveform. It was shown that this loss occurs when the
total phase shift across the uncompressed pulse is an odd
multiple of $\pi$ radians. This loss can therefore be reduced
by providing a phase compensated channel which has ap-
proximately a $\pi$ phase shift across the uncompressed
pulse. Then, for example, the channel having the largest
signal is selected. Also, it was shown or stated that
weighting reduced the cyclic variation of the peak re-
response and the variation of the pulsewidth with Doppler.

The Doppler responses of the $P_3$ and $P_4$ codes were
shown to have much lower secondary maxima than the
Frank, $P_1$, and $P_2$ codes and to have comparable image
lobes. However, the effects of amplitude weighting were
shown to primarily increase the ratios of the peak signal to the image lobe, the mean-squared sidelobes, and, to a smaller extent, the ratio of the peak signal to the secondary maxima.

The effects of bandlimiting were investigated for zero and nonzero Doppler-shifted waveforms; for the symmetrical \( P1, P2, \) and \( P4 \) codes, the results were found to be similar to amplitude weighting. For these codes, it was found that the ratios of the peak signal to the image lobe, the mean-squared sidelobes and, to a lesser extent, the secondary maxima are improved. However, for the unsymmetrical Frank and \( P3 \) codes, these ratios are degraded.

The \( P4 \) code, in addition to being tolerant of bandlimiting, was shown to provide better Doppler tolerance than the other codes in the presence of relatively large Doppler shifts. However, for small normalized Doppler shifts of less than approximately \( 1/(2\pi) \), the \( P1 \) or \( P2 \) codes have lower peak sidelobes. The preferred code therefore depends on the expected Doppler shifts.

ACKNOWLEDGMENTS

The assistance of Mrs. F.C. Lin in obtaining the computer plots is gratefully acknowledged.
REFERENCES


IEEE Transactions on Information Theory, IT-9 (June 1963), 43–45.

Frank F. Kretschmer, Jr. (M’64—SM’77) was born in Philadelphia, Pa., on July 31, 1930. He received the B.S. degree in 1957 from the Pennsylvania State University, University Park, Pa., the M.S. degree in 1961 from Drexel Institute of Technology, Philadelphia, Pa., and the Ph.D. degree in 1970 from Johns Hopkins University, Baltimore, Md., all in electrical engineering.

He was employed by the Burroughs Corporation, Paoli, Pa., from 1957 to 1958. From 1958 to 1964 he was employed by Bendix Radio in Towson, Md., where he was engaged in radar systems work. From 1964 to 1970 he was a Research Associate at Johns Hopkins University. Since 1970 he has been with the Naval Research Laboratory, Washington, D.C., where his research interests include adaptive signal processing, moving target indicators (MTI), and waveform design.

Dr. Kretschmer is a member of Sigma Xi.

Bernard L. Lewis (M’53—SM’57) was born in Storm Lake, Iowa, in 1923. He received the B.S. and M.S. degrees in physics from Tulane University, New Orleans, La., in 1946 and 1948, respectively.

From 1948 to 1957 he was employed by the Radar Division of the Naval Research Laboratory, Washington, D.C., where he aided in the development of monopulse radar and studied target induced radar errors. From 1957 to 1972 he worked in radar and optics in industry; in 1972 he returned to the Naval Research Laboratory, where he is currently a Research Physicist working in adaptive signal processing, antenna design, and radar sea scatter studies.

Mr. Lewis is a member of Sigma Xi.