Determination of gas dispersion in vapor extraction of heavy oil and bitumen

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Abstract

In this work, a mathematical model is developed and simulated to determine gas dispersion along with solubility during the vapor extraction (Vapex) of live oil from a laboratory scale physical model. The physical model is a rectangular block of homogenous porous medium saturated with heavy oil and bitumen. At a given temperature and pressure, the block is initially exposed on its side to a solvent gas, which diffuses into the medium and gets absorbed. The absorption of gas reduces the viscosity of heavy oil and bitumen causing it to drain under gravity. The low-viscosity “live oil” is produced at the bottom of the porous block. The production of live oil with time is accompanied by the shrinkage of oil in the block as well as its increased exposure to gas from top. These phenomena of Vapex are described by the mathematical model, which is used to calculate live oil production with various values of gas solubility and dispersion. Their optimal values are determined for the vapor extraction of Cold Lake bitumen with butane by matching calculated live oil production with its experimental values published earlier.

Keywords: Gas dispersion; Vapor extraction; Heavy oil and bitumen; Modeling Vapex front; Enhanced oil recovery

1. Introduction

Vapor extraction or Vapex is the process of heavy oil and bitumen recovery in a horizontal well configuration inside reservoirs using vaporized solvents. These solvents diffuse and absorb into the highly viscous natural reserves, and reduce their viscosity so that they can be easily pumped out. Since the early demonstrations of its effectiveness (Butler and Mokrys, 1991, 1993), Vapex has undergone substantial advancement from laboratory scale studies to pilot plant investigations, and commercial projects (Turta and Singhal, 2004). Principal reasons are the non-thermal nature of Vapex, its environment friendliness, low capital and operating costs, and its suitability to thick as well as frequently occurring thin heavy oil and bitumen reservoirs relative to other recovery processes (Luhning et al., 2003).

In general, when a gas such as a light hydrocarbon is injected into heavy oil and bitumen reservoirs, it creates a second phase. Depending on the system temperature and pressure and whether it is below or above the minimum miscibility pressure, the first contact miscibility or multi-contact miscibility occurs. Whether solvent vaporization or condensation occurs in the gas phase, it reduces the viscosity of the heavy oil and bitumen.
(or a combination of both) happens, the following mechanisms take place to a greater or smaller degree: (a) swelling of the oil phase, (b) lowering of oil viscosity, (c) reduction of interfacial tension, and (d) miscibility and mixing of gas and oil.

Vapex is an enhanced oil recovery technique developed for the oil sands reservoirs. In its typical operation, ethane, propane, butane, or a mixture of light hydrocarbon and non-condensable gas is used as a solvent gas for its injection into the reservoir just below the corresponding dew point pressure. Such an injection causes strong molecular diffusion at the gas–oil interface where the highest solubility of gas is achieved. Following the multi-contact principle, the solvent gas penetrates at the pore scale of the reservoir. Its porous medium influences the mixing of the gas with the reserves. In fact, the mass transfer increases due to the increased interfacial area in the porous medium, increased solubility, capillary imbibitions, and surface renewal. Depending on the diffusion and dispersion coefficient of the solvent gas in the reserves, the gas concentration in the reservoir varies with time and distance from the interface.

Crucial to the economic viability of Vapex is the mixing of solvent with crude oil, which is governed by the phenomenon of dispersion. The porosity of reservoir media augments this phenomenon, which otherwise would include molecular diffusion only. In a reservoir, a solvent gas mixes with heavy oil and bitumen phase through the combined phenomena of molecular diffusion, viscosity reduction, gravitational flow, capillary action, and surface renewal (Das and Butler, 1998). The net effect is dispersion, which cannot be accurately described by molecular diffusion alone. As a matter of fact, the recovery rates of heavy oil and bitumen with Vapex correspond to effective diffusivities, which are two to five orders of magnitudes higher than the molecular diffusivities.

A considerable amount of experimental and theoretical research exists on the determination of dispersion in different solvent-medium systems (Perkins and Johnston, 1963). However, the phenomenon of dispersion is poorly understood in systems like solvent–crude oil in porous media, which undergo significant viscosity changes (Oballa and Butler, 1989). Laboratory experiments show that the oil production rates in physical model tests using reservoir sand and falling film geometry can be several folds higher than the rates predicted through molecular diffusion. Possible reasons for this enhancement could be the increase in molecular diffusivity with solvent concentration leading to decreased oil viscosity, an increase in gas/oil interfacial area, and interfacial instabilities. To match Vapex oil production rates, researchers (Dunn et al., 1989; Lim et al., 1996; Das and Butler, 1998; Oduntan et al., 2001) had to use different values of dispersion, all of which are quite higher than molecular diffusion. These values indicate a strong concentration dependence of dispersion (Boustani and Maini, 2001) as observed in the case of molecular diffusion (Upeti and Mehrotra, 2002).

For the accurate prediction of heavy oil and bitumen recovery with Vapex leading to effective designs and optimal operations of commercial Vapex applications, it is essential to experimentally determine the dispersion of various solvents used in Vapex. Literature survey indicates a paucity of the much-needed dispersion data. In this work, we develop and simulate a detailed mathematical model to determine solvent dispersion in Vapex. A laboratory scale physical model of heavy oil and bitumen saturated porous medium is utilized for this purpose. The physical model was invented, and has been used by the researchers at University of Waterloo in various studies on Vapex (Oduntan et al., 2001; Ramakrishnan, 2003). Their experimental data of live oil production from Vapex are used in the simulation of the developed model to determine the dispersion of butane in Cold Lake bitumen.

2. Mathematical model

Fig. 1 shows a lateral cross-section of the physical model (Oduntan et al., 2001), which is a rectangular block of porous medium saturated with heavy oil and bitumen throughout at the onset of Vapex. Under specified temperature and pressure conditions, the block is initially exposed to a solvent gas on the vertical face on right hand side. The gas diffuses into the block, and gets absorbed in heavy oil and bitumen. The absorption of gas reduces the viscosity of heavy oil and bitumen causing it to flow under gravity through porous medium. The low-viscosity oil or “live oil” is produced at the bottom of the block. The flow of live oil at the exposed surface renews it continuously, thereby enhancing the transfer of gas into heavy oil and bitumen. The production of live oil with time causes the block to shrink in size. The height of block reduces with time, and varies along x-axis, i.e., along the width of the block. The reduction of height exposes the block to the gas from top, thereby augmenting the
exposed surface area. This effect further enhances the transfer of gas. In the mathematical model developed below, the main assumptions are as follows:

1. Vapex is carried out at constant temperature and pressure.
2. The porous medium has uniform porosity and permeability.
3. The mass fraction of gas at the exposed surface of porous medium is the saturation mass fraction under equilibrium.
4. The dispersion of gas takes place along \( x \)-direction. The transfer of gas along \( y \)-direction is governed by the \( y \)-component of Darcy velocity in porous medium.
5. The dispersion of gas incorporates molecular diffusion, the effects of surface renewal and augmentation, and any convective component along \( x \)-direction.
6. There is no mass transfer across the vertical face of the block on left hand side, which is a wall.
7. There is no variation of state variables along the thickness of the block.

The unsteady state mass balance for solvent gas in the constant density block results in the following partial differential equation:

\[
\phi \frac{\partial \omega}{\partial t} = \frac{\partial}{\partial x} \left( D \frac{\partial \omega}{\partial x} \right) - v \frac{\partial \omega}{\partial y}
\]  

(1)

In Eq. (1), \( \phi \) is the porosity of medium, \( \omega \) is the mass fraction of gas in the block, \( D \) is the dispersion of gas along \( x \)-direction, and \( v \) is the Darcy velocity along \( y \)-direction given by

\[
v = \frac{K_r K \rho g \cos \theta}{\mu}
\]  

(2)

In Eq. (2), \( K_r \) is relative permeability, and \( K \) is permeability of porous medium. \( \rho \) is the density of live oil, \( g \) is gravity, and \( \theta \) is the angle between the base of physical model and the horizontal. \( \mu \) is the concentration-dependent viscosity of live oil given by the following correlation (Oduntan, 2001):

\[
\mu = \mu_o \omega^{-2}
\]  

(3)
For the dispersion of gas in heavy oil and bitumen, we use the following dependence

$$D \propto \mu^{-0.5}$$

(4)

based on an empirical model correlated by Das and Butler (1996) for the diffusion of butane in Peace River bitumen. Eqs. (3) and (4) when combined (Oduntan, 2001) yield the following linear, concentration-dependent model for dispersion

$$D = D_o \omega$$

(5)

where $D_o$ is the dispersion coefficient, or gas dispersion for $\omega=1$.

The change in the height of block ($Y$) with time at any location on $x$-axis is the negative of Darcy velocity at the bottom of block, i.e.,

$$\frac{\partial Y}{\partial t} = -v$$

(6)

Initially, there is no gas in the block except at its exposed vertical face on right hand side, where gas concentration is its equilibrium saturation value under prevailing temperature and pressure. Furthermore, there is no production of live oil at the beginning so that the initial height of block is its specified height, $Y_o$. Thus, the initial conditions at $t=0$ are

$$\omega = \begin{cases} 0, & 0 \leq y \leq Y_o, \quad 0 \leq x < X, \\ \omega_{sat}, & 0 \leq y \leq Y_o, \quad x = X \end{cases}$$

(7)

$$Y = Y_o$$

(8)

At all times, any exposed surface of block is saturated with gas. This condition applies to the top face of block when it gets exposed to gas. Furthermore, the concentration gradient is zero at the wall on the vertical face on left hand side across which there is no mass transfer. Thus, the boundary conditions for $t \geq 0$ are

$$\omega = \omega_{sat} \begin{cases} \text{for } 0 \leq y \leq Y, \quad x = X \\ \text{for } 0 \leq x < X, \quad y = Y; \quad \text{if } Y < Y_o \end{cases}$$

(9)

$$\frac{\partial \omega}{\partial x} = 0; \quad \text{for } 0 \leq y \leq Y \text{ and } x = 0$$

(10)

At any time, the cumulative volume freed up within the initial boundaries of porous block due to oil production is given by

$$V = \phi^{2/3} Z \int_0^X (Y_o - Y) \, dx$$

(11)

and is the cumulative volume of oil produced. However, this volume corresponds to the volume of “dead oil” (i.e., oil sans gas) initially present in the porous block. Hence, the cumulative volume of live oil produced at any time is given by

$$V_{cal} = \alpha V = \alpha \phi^{2/3} Z \int_0^X (Y_o - Y) \, dx$$

(12)

where $\alpha$ is the coefficient for volume correction.

2.1. Solution

The above mathematical model cannot be solved analytically. In this work, it was numerically solved using finite differences. The application of second order finite differences along $x$ and $y$ directions results in the set of simultaneous
ordinary differential equations with time as an independent variable. With $N_i$ and $N_j$ grid points, respectively, along $x$ and $y$ directions as shown in Fig. 1, the finite-differenced ordinary differential equations are as follows:

For intermediate grid points in the block:

$$\frac{d\omega_{i,j}}{dt} = \frac{D_o}{\phi} \left[ \omega_{i,j} \left( \frac{(\omega_{i+1,j} - 2\omega_{i,j} + \omega_{i-1,j})}{(\Delta x)^2} \right) + \left( \frac{(\omega_{i+1,j} - \omega_{i-1,j})}{2\Delta x} \right)^2 \right] - \frac{K_i \rho g \cos \theta}{\mu_o \phi} \omega_{i,j}^2 \left( \frac{(\omega_{i+1,j} - \omega_{i-1,j})}{2\Delta y_i} \right) ; \quad 0<i<N_i-1,$$

and $0<j<N_j-1$  \hfill (13)

For all grid points (except corners) on left hand side:

$$\frac{d\omega_{0,j}}{dt} = - \frac{K_i \rho g \cos \theta}{\mu_o \phi} \omega_{0,j}^2 \left( \frac{(\omega_{0,j+1} - \omega_{0,j-1})}{2\Delta y_0} \right) ; \quad 0<j<N_j-1 \hfill (14)$$

For the corner grid point on left hand side at and bottom:

$$\frac{d\omega_{0,0}}{dt} = - \frac{K_i \rho g \cos \theta}{\mu_o \phi} \omega_{0,0}^2 \left( -\omega_{0,2} + 4\omega_{0,1} - 3\omega_{0,0} \right) \hfill (15)$$

For all grid points (excluding corners) at the bottom:

$$\frac{d\omega_{N_i-1,j}}{dt} = - \frac{K_i \rho g \cos \theta}{\mu_o \phi} \omega_{N_i-1,j}^2 \left( \frac{3\omega_{N_i,j-1} - 4\omega_{N_i,j-2} + \omega_{N_i,j-3}}{2\Delta y_0} \right) \hfill (16)$$

For grid points on right hand side exposed to gas:

$$\frac{d\omega_{N_i-1,0}}{dt} = 0 ; \quad 0<j<N_j \hfill (17)$$

For grid points (excluding corners) at the top:

$$\frac{d\omega_{i,0}}{dt} = \frac{D_o}{\phi} \left[ \omega_{i,0} \left( \frac{(\omega_{i+1,0} - 2\omega_{i,0} + \omega_{i-1,0})}{(\Delta x)^2} \right) + \left( \frac{(\omega_{i+1,0} - \omega_{i-1,0})}{2\Delta x} \right)^2 \right] - \frac{K_i \rho g \cos \theta}{\mu_o \phi} \omega_{i,0}^2 \left( -\omega_{i,2} + 4\omega_{i,1} - 3\omega_{i,0} \right) ; \quad 0<i<N_i-1 \hfill (18)$$

For grid points (excluding corners) at the top:

$$\frac{d\omega_{N_i-1,j}}{dt} = \left\{ \begin{array}{ll}
\frac{D_o}{\phi} \left[ \omega_{N_i-1,j} \left( \frac{2\omega_{N_i-1,j} - 2\omega_{N_i-1,j-1} + \omega_{N_i-1,j-2}}{(\Delta y)^2} \right) + \left( \frac{2\omega_{N_i-1,j} - \omega_{N_i-1,j-1}}{2\Delta y} \right)^2 \right] - \frac{K_i \rho g \cos \theta}{\mu_o \phi} \omega_{N_i-1,j}^2 \left( \frac{3\omega_{N_i,j-1} - 4\omega_{N_i,j-2} + \omega_{N_i,j-3}}{2\Delta y_j} \right) ; & \text{if } Y_i < Y_o \\
0 ; & \text{if } Y_i \geq Y_o
\end{array} \right. \hfill (19)$$

In the above equations, $\omega_{i,j}$ is the mass fraction of gas at the node, $(i,j)$ corresponding to the coordinate, $(x,y)$. $\Delta x$ and $\Delta y$ are the distances between equispaced grid points, respectively, along $x$ and $y$ directions. Note that $Y_i$ which is the height of block, changes with time and $x$, and is calculable though the simultaneous integration of the following equation:

$$\frac{dY_i}{dt} = -\omega_{i,0} \frac{K_i \rho g \cos \theta}{\mu_o} \omega_{i,0} ; \quad 0<i<N_i-1 \hfill (20)$$

While $\Delta x$ is constant, and is given by

$$\Delta x = \frac{X}{N_i} \hfill (21)$$
Δyi varies along x-direction with time, and is given by

$$\Delta y_i = \frac{Y_i}{N_j}; \quad 0 \leq j \leq N_j$$

(22)

Eqs. (13)–(20) are coupled, highly nonlinear and stiff to solve. In this work, they were numerically integrated using semi-implicit Bader–Deu hard algorithm, and adaptive step-size control (Press et al., 2002). Analytical Jacobians of Eqs. (13)–(20) were employed for integration. To fix the number of grid points, $N_i$ and $N_j$, the equations were integrated with the increasing number of grid points until the changes in solution became negligible. Table 1 lists the various parameters used in the simulation of the mathematical model.

The cumulative volume of live oil produced at any time is given by the following equation:

$$V_{cal} = \alpha \phi^{2/3} \Delta x Z \sum_{i=0}^{N-2} \left( Y_o - \frac{Y_i + Y_{i+1}}{2} \right)$$

(23)

The algorithm was programmed to generate $V_{cal}$ at the experimental time instants for its direct comparison with its experimental counterpart, $V_{exp}$. The root mean square fractional error between these volumes was calculated as

$$e = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} \left( 1 - \frac{V_{cal,n}}{V_{exp,n}} \right)^2}$$

(24)

In Eq. (24), $N$ is the number of experimental data points, and $n$ denotes the $n$th experimental time instant.

In this work, the experimental data published earlier by Oduntan (2001) were used for the operation time of about 4 h. These data are for the vapor extraction of Cold Lake bitumen from a laboratory scale physical model using butane at temperature in the range, 19–22 °C with pressure close to dew point, i.e., in the range, 0.21–0.23 MPa.

### 3. Results and discussion

Root mean square errors were obtained by solving Eqs. (13)–(24) with various values of $\omega_{sat}$ and $D_o$ in the range 0.7–1.0 and 0.194–1.39 cm²/s, respectively. Table 2 shows the errors that are less than 8%, and surround the minimum error. It is 2.2198% with the optimal values of $\omega_{sat}=0.87$, and $D_o=0.556$ cm²/s. This optimal value of $D_o$ is four orders of magnitude higher than the corresponding coefficient reported for the molecular diffusion of butane in Peace River bitumen. The high value of $D_o$ underlines the role of convection and surface renewal encapsulated through dispersion in Vapex.
Further calculations with $\omega_{\text{sat}} > 0.88$, and $D_o > 1.39$ cm$^2$/s did not result in the root mean square error lower than its optimal value of 2.2198%. At very high values of $D_o$, the integration of the set of differential equations, i.e., Eqs. (13)–(20) became very time consuming as the step size of integration became prohibitively small. The reason is that $D_o$ appears only in Eqs. (13), (18) and (19) of the set of ordinary differential equations. A higher value of $D_o$ as such increases the stiffness of the whole set of equations requiring smaller step sizes for integration.

Predicted live oil production versus time for the optimal values of $\omega_{\text{sat}}$ and $D_o$ is compared with the experimental data of Oduntan (2001) in Fig. 2. It is observed that the predicted production follows experimental production very closely during the operation time of about 4 h. The rate of production during this time stays practically constant as has been experimentally reported earlier (Oduntan, 2001).

Table 2
Percentage root mean square error (in gray cells) between predicted and experimental live oil production

<table>
<thead>
<tr>
<th>$D_o \times 10^3$ (cm$^2$/s)</th>
<th>Saturation mass fraction of butane</th>
<th>0.85</th>
<th>0.86</th>
<th>*0.87</th>
<th>0.88</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.94</td>
<td>7.9925</td>
<td>5.9630</td>
<td>4.2130</td>
<td>3.1795</td>
<td></td>
</tr>
<tr>
<td>2.50</td>
<td>6.8543</td>
<td>4.4852</td>
<td>3.2361</td>
<td>2.7811</td>
<td></td>
</tr>
<tr>
<td>2.78</td>
<td>6.4580</td>
<td>4.4752</td>
<td>2.9202</td>
<td>2.7616</td>
<td></td>
</tr>
<tr>
<td>5.56</td>
<td>4.7998</td>
<td>2.9416</td>
<td>*2.2198</td>
<td>3.4657</td>
<td></td>
</tr>
<tr>
<td>8.33</td>
<td>4.2823</td>
<td>2.5685</td>
<td>2.3366</td>
<td>3.8916</td>
<td></td>
</tr>
<tr>
<td>11.1</td>
<td>4.0360</td>
<td>2.4226</td>
<td>2.4497</td>
<td>4.382</td>
<td></td>
</tr>
<tr>
<td>13.9</td>
<td>3.9009</td>
<td>2.3461</td>
<td>2.5417</td>
<td>4.2852</td>
<td></td>
</tr>
</tbody>
</table>

*: optimal.
For the optimal values of $\omega_{\text{sat}}$ and $D_0$, Fig. 3 shows the variation of height of the porous block at different times. The height is observed to reduce with width at any time. The gradient of height with respect to width decreases considerably with time as can be seen from the figure. The gradient is the most negative in the end. The reason for this behavior is the implementation of Vapex under the constraint of initial and boundary conditions given by Eqs. (7)–(10). Initially, the concentration of gas and resulting live oil production are higher, closer to the exposed surface of block on right hand side where the consequent reduction of height begins. With the passage of time, this reduction increasingly exposes the block to gas from the top, beginning from the top right corner. This exposure, which progressively increases with time, results in the increasingly negative gradient of height of block with respect to its width. The rate of production with time, however, remains almost constant for most of the operation time as can be seen in Fig. 2.

The Cold Lake bitumen used in Vapex experiments had 40.55 Pa·s viscosity, and 0.9703 g/cm³ density at room temperature (Oduntan, 2001). Experimental temperature varied in the range, 19–22 °C with pressure close to dew point, i.e., in the range, 0.21–0.23 MPa. Under these conditions on the basis of the above calculations, the saturation mass fraction or solubility of butane is 0.87. The dispersion of butane in Cold Lake bitumen (held in a homogenous porous medium) during its vapor extraction is expressed by the following relation:

$$D = 0.556\omega \ [\text{cm}^2/\text{s}]$$

(25)

where $\omega$ is the mass fraction of butane in Cold Lake bitumen.

4. Conclusion

A mathematical model was developed and simulated to describe Vapex in a rectangular block of homogenous porous medium at specified temperature and pressure. This model was used to determine the dispersion of butane along with its solubility in Cold Lake bitumen. Both gas dispersion, and heavy oil and bitumen viscosity were considered to be dependent on composition. The shrinkage of block during Vapex, i.e., its moving boundary with time and space was taken into account. The simulations of model were based on the previously reported experimental data of Vapex implementation in a laboratory scale physical model. Different values for the solubility and dispersion of butane were used in the simulations. The optimum solubility and dispersion were determined for which the root mean square fractional error between the simulated and experimental values of live oil production is minimum. The corresponding dispersion of butane in Cold Lake bitumen was found to be four orders of magnitude higher than previously reported molecular diffusion of butane in heavy oil and bitumen.

Notation

- $D$ dispersion coefficient of gas in heavy oil and bitumen, cm²/s
- $D_0$ $D$ when $\omega = 1$, cm²/s
- $e$ root mean square error given by Eq. (24)
- $g$ gravity, cm/s²
- $K$ permeability of physical model, cm²
- $K_r$ relative permeability of physical model
- $N$ number of experimental data points
- $N_i$ number of grid points along $x$-direction
- $N_j$ number of grid points along $y$-direction
- $t$ time, s
- $t_f$ operation time, h
- $V$ Darcy velocity given by Eq. (2), cm/s
- $V_{\text{cal}}$ predicted volume of produced live oil, cm³
- $V_{\text{exp}}$ experimental volume of produced live oil, cm³
- $X$ width of physical model, cm
- $x$ direction along $X$, cm
- $Y$ height of physical model, cm
- $Y_o$ initial $Y$, cm
- $y$ direction along $Y$, cm
- $Z$ thickness of physical model, cm

Greek symbols

- $\alpha$ volume correction coefficient of Eq. (12)
- $\theta$ angle between the base of physical model and the horizontal, rad
- $\mu$ viscosity of live oil, g/cm s
- $\Delta x$ distance between grids along $x$-direction
- $\Delta y_i$ distance between grids along $y$-direction for a given $x$-location, $i$
- $\mu_o$ viscosity coefficient for live oil, g/cm s
- $\rho$ density of live oil, g/cm³
- $\phi$ porosity of physical model
- $\omega$ mass fraction of gas in heavy oil and bitumen
- $\omega_{\text{sat}}$ equilibrium saturation value of $\omega$, or gas solubility

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References


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