INTRODUCTION

Hemispherical resonator gyroscopes (HRG), the operating principle of which is based on the precession of elastic waves in oscillating axisymmetric shells [1], have proven to be highly reliable primary information sensors of inertial navigation systems [2, 3]. The main component part of the HRG is a resonator, whose quality largely determines the accuracy of the gyro. One of the most commonly used materials for such resonators is fused quartz [4], which has low internal friction and high isotropy. Accuracy enhancement of such devices depends directly on the degree of sophistication of the resonator fabrication techniques. Anomalies of mass distribution around the resonator shell circumference and thickness inhomogeneity [5] as well as deviations of the resonator geometry from the ideal axisymmetric shape lead to unwanted precession of the standing wave in the resonator and reduce the accuracy of the gyro.

Various technologies and methods for balancing resonators have been developed to eliminate the mass distribution defect in the HRG resonator [6–8]. One of them is the technology for balancing HRG metal resonators with balancing teeth on the edge of a thin-walled shell [9]. In this case, unbalanced mass is removed from each tooth by an electrochemical method. However, making teeth is a sophisticated technological operation even for metal resonators. Besides, this operation causes additional mass imbalance due to unavoidable errors in the teeth geometry.

Toothless resonators are much easier to fabricate and they have much lower mass disbalance. In [10], for example, a method for balancing toothless fused quartz hemispherical resonators for the HRG is described, in which unbalanced mass is removed from the hemispherical shell by ion milling. The main drawback of this method is low efficiency of the balancing process, which may take a few tens of hours. Another disadvantage is deterioration of the resonator surface quality by ions, therefore, it may reduce the resonator Q-factor.

In [11], it was proposed to balance the first four harmonics of mass distribution defect in a metal cylindrical toothless resonator for a cylindrical resonator gyroscope using a technique based on electrochemical etching of metal. The algorithm for removing mass from the resonator surface is implemented by choosing the angle and depth of the resonator inclined immersion into a chemical bath as well as repeated rotation of the resonator about its axis of symmetry.

In the present paper, this approach is further developed for balancing a fused quartz hemispherical resonator of an HRG. The essential difference is the use of chemical etching of fused quartz to remove unbalanced mass and an algorithm for determining the geometric parameters of the process, which is fundamentally different for the hemispherical shell as compared with the cylindrical one.

CHEMICAL ETCHING OF UNBALANCED MASS

The initial data for balancing are the values of mass distribution defect \( M_k, \phi_k \) predetermined in the Fourier series expansion of the distribution function of the resonator mass \( M \) in the circumferential angle \( \phi \):

\[
M(\phi) = M_0 + \sum_{k=1}^{\infty} M_k \cos k(\phi - \phi_k),
\]  

Balancing of Hemispherical Resonator Gyros by Chemical Etching

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Abstract—A procedure for balancing the four lower harmonics of the mass distribution defect in a fused quartz hemispherical toothless resonator of a hemispherical resonator gyroscope is considered. Chemical etching of unbalanced mass from the surface of a partially immersed resonator is done in accordance with analytically calculated angle of the resonator rotation about the axis of symmetry, inclination and depth of the resonator immersion into a chemical bath, and the time of chemical etching. It is shown that the proposed method significantly reduces the balancing time and labor input as compared with ion plasma etching.

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where $k$ is the harmonic number of the resonator mass distribution; $M_0$ is the mass uniformly distributed in the circumferential angle; $M_k$ is the value of the $k$-th harmonic of the resonator mass distribution; $\varphi_k$ is the orientation of the $k$-th harmonic of the resonator mass distribution with respect to the defined zero of the circumferential angle.

According to [12], nonzero values of $M_1$, $M_2$, or $M_3$ cause oscillations of the resonator center of mass during the gyro operation, additional dissipation of the resonator oscillating energy at the places where the resonator is fixed, and the systematic error of the gyro. Nonzero values of $M_4$ lead to splitting of the resonator natural frequency, resulting in gyro random errors. To prevent such phenomena, the resonator is balanced for four lower harmonics of mass distribution in order to determine parameters $M_k$ and $\varphi_k$ for $k=1,\ldots,4$ and remove unbalanced mass.

At a first approximation, the removal effect of the mass located at a rather long distance from the resonator edge is not taken into account [7]. Consider the technique used to remove mass from the surface of the hemispherical resonator $I$ with radius $R$, immersed in the inclined position into a chemical bath with the etching solution 2 (Fig. 1).

The geometry is described in the spherical coordinate system $(r,\theta,\varphi)$ with the origin $O$ and $r=R$. Assume that $\delta$ is the inclination angle of the secant plane $E_1DE_2$, limiting the hemisphere etching surface. The inclination angle can vary within $0<\delta<90^\circ$ (at $\delta>90^\circ$, the bottom of the resonator hemisphere is being etched, including the resonator stem at the fixing point $B$; at $\delta=0$, there is no effect of the angular rotation about the $OB$-axis). Let us introduce the notion of depth of immersion $h$ as a perpendicular length from the edge point of hemisphere $F$, corresponding to the polar angle (axis $OF$), to the aforementioned secant plane $E_1DE_2$. The hemisphere is immersed into the liquid until the whole edge of the hemisphere (point $G$, diametrically opposite to $F$) or the fixing point of the stem to the hemisphere pole $B$ gets into contact with the liquid:

$$0 < h \leq \begin{cases} 2R \sin \delta, & \delta \leq \pi/4, \\ R(\sin \delta + \cos \delta), & \delta > \pi/4. \end{cases}$$

Define $2\alpha$ as a double zenith angle between the radius-vectors from origin $O$ to the opposite end points $E_1$ and $E_2$ of the edge being etched [13]:

![Fig. 1. Geometry of etching of a hemispherical resonator at $\alpha < 90^\circ$ (a) and $\alpha > 90^\circ$ (b).](image-url)
\[
\cos \alpha = 1 - \frac{h}{R \sin \delta}.
\]

For the hemisphere points on the border of the surface under treatment, the dependence of angle \(\varphi\) on angle \(\varphi\) and parameters \(\alpha, \delta\) can be written as

\[
\theta(\varphi, \alpha, \delta) = \begin{cases} 
\arcsin \left( \frac{\tan^2 \delta \cos \alpha \cos \varphi + \sqrt{1 + \tan^2 \delta (\cos^2 \varphi - \cos^2 \alpha)}}{1 + \tan^2 \delta \cos^2 \varphi} \right), & |\varphi| \leq \alpha. \\
\pi/2, & |\varphi| > \alpha.
\end{cases}
\]

During chemical etching over time \(t\), a glass layer of mass \(m = St\nu\) is removed from the treated surface \(S\). In this formula, \(\nu\) is an experimentally determined specific dissolution rate of fused quartz, which depends on the composition and temperature of the etching solution.

**BALANCING ALGORITHM**

Given the inclination angle \(\delta = \text{const}\), the angular dependence of the mass removed from the resonator lateral surface has the form:

\[
m(\varphi, \alpha) = \rho d R^2 \cos \theta(\varphi, \alpha), \tag{2}
\]

where \(\rho\) is the resonator material density; \(d\) is the thickness of the layer to be removed.

Expand \(m(\varphi, \alpha)\) into a Fourier series with respect to the circumferential angle \(\varphi\), taking the first four harmonics:

\[
m(\varphi, \alpha) = \rho d R^2 \sum_{k=0}^{4} C_k(\alpha) \cos k \varphi,
\]

where

\[
C_0(\alpha) = \frac{1}{2\pi} \int_0^{2\pi} \cos \theta(\varphi, \alpha) \, d \varphi,
\]

\[
C_k(\alpha) = \frac{1}{\pi} \int_0^{2\pi} \cos \theta(\varphi, \alpha) \cos k \varphi \, d \varphi, \quad k = 1, \ldots, 4.
\]

Coefficient \(C_0\) corresponds to the uniformly distributed mass being removed from the resonator surface; its value does not affect the gyro accuracy. In the process of balancing, the first four harmonics of the resonator mass distribution defect are removed alternately and independently of each other. Balancing of any \(k\)-th harmonic gives rise to multiple harmonics with numbers \(nk\) \((n = 2, 3, 4, \ldots)\). The balancing procedure consists in the following: first, the 1st and 2nd harmonics are balanced sequentially, then the 3rd and 4th harmonics are balanced in a random order. This is due to the fact that after the 1st harmonic has been removed, there arise harmonics numbered 2, 3, 4, ..., and the removal of the 2nd harmonic gives rise to all even-numbered harmonics. Balancing of the 3rd and 4th harmonics gives rise only to higher harmonics, with numbers higher than 4, which do not significantly affect the gyro performance.
The values of coefficients $C_k^{(i)} = C_k^{(i)}(\alpha_1)$, $k = 0, 1, 2, 4$ are given in the table below.

Thus, in order to eliminate the 1st harmonic of the mass distribution defect with parameters $M_1$, $\phi_1$, the hemispherical resonator must be installed in a position where $\Delta \phi = -\phi_1$ and the double value of the zenith angle $2\alpha_1 = 134^\circ$. The time of resonator chemical etching needed to remove the 1st harmonic of the mass distribution defect is $t_1 = 0.973 M_1^{(i)} / (R^2 \nu)$. The resulting additional defects will be removed in a subsequent balancing of the 2nd and 4th harmonics.

To compensate for the 1st harmonic of the defect, it is assumed that

$$d = d_i = \frac{M_1}{\rho R^2 C_1^{(i)}}.$$

The result is a new mass distribution in the circumferential angle:

$$M'(\phi) = M(\phi) - m_i(\phi) = M_0^{(i)} + M_2^{(i)} \cos 2(\phi - \varphi_2^{(i)})$$

$$+ M_3 \cos 3(\phi - \phi_3) + M_4^{(i)} \cos 4(\phi - \varphi_4^{(i)}),$$

where

$$M_0^{(i)} = M_0 - M_1 C_1^{(i)} / C_1^{(i)};$$

$$M_2^{(i)} = \sqrt{M_2 \cos 2\phi_2 - M_1 C_2^{(i)} / C_1^{(i)}} + M_2^{(i)} \sin 2\varphi_2;$$

$$\varphi_2^{(i)} = \frac{1}{2} \arctan \frac{M_2 \sin 2\phi_2}{M_2 \cos 2\phi_2 - M_1 C_2^{(i)} / C_1^{(i)}};$$

$$M_3^{(i)} = \sqrt{M_4 \cos 4\phi_4 - M_1 C_3^{(i)} / C_1^{(i)}} + M_4^{(i)} \sin 2\varphi_4;$$

$$\varphi_4^{(i)} = \frac{1}{4} \arctan \frac{M_4 \sin 4\phi_4}{M_4 \cos 4\phi_4 - M_1 C_4^{(i)} / C_1^{(i)}};$$

In eliminating the 2nd harmonic of the mass distribution defect with parameters $M_2$, $\varphi_2$, the optimal value of the zenith angle of the spherical segment is $\alpha = \alpha_2 = 52^\circ$, when $C_1 = 0$. The treatment of the resonator surface is carried out in two stages. First, we set a double zenith angle: $2\alpha_2 = 104^\circ$ and $\Delta \phi = -\varphi_2^{(i)}$, perform chemical etching over time $t_2 = 0.506 M_2^{(i)} / (R^2 \nu)$, then rotate the resonator about its axis by $180^\circ$ and perform chemical etching over time $t_2$. The values of $M_2^{(i)}$ and $\varphi_2^{(i)}$ are determined as follows:

$$M_2^{(i)} = \sqrt{(M_2 \cos 2\phi_2 - 0.478 M_1)^2 + M_2^2 \sin^2 2\varphi_2};$$

$$\varphi_2^{(i)} = \frac{1}{2} \arctan \frac{M_2 \sin 2\phi_2}{M_2 \cos 2\phi_2 - 0.478 M_1}.$$

In this case, only coefficients $C_0$ are $C_2$ are nonzero, i.e., the 2nd harmonic of the resonator mass distribution effect may be eliminated selectively:

Fig. 2. Harmonic amplitudes at $2\alpha = 134^\circ$ (a), $2\alpha = 104^\circ$ (b).

Fig. 3. Geometry of hemispherical resonator etching at $\delta = 90^\circ$: (1) resonator, (2) etching solution.
The calculated values of harmonic amplitudes

<table>
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<tr>
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<th>(C_k^{(1)})</th>
<th>(C_k^{(2)})</th>
<th>(C_k^{(3)})</th>
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<tr>
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<td>0</td>
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<td>0.037</td>
</tr>
</tbody>
</table>

* No effect on the HRG performance.

\[ m_2(\varphi) = 2\rho d R^2 \left[ C_0^{(2)} + C_2^{(2)} \cos 2(\varphi - \varphi_2^{(1)}) \right], \]

where \(C_k^{(2)} = C_k(\alpha_2), k = 0, 2\) (see Table).

The 3rd harmonic of the hemispherical resonator mass distribution effect with parameters \(M_3, \varphi_3\) is eliminated in three stages. After setting \(2\alpha_3 = 86^\circ, \Delta\varphi = -\varphi_3\), we perform chemical etching over time \(t_1 = 0.422 M_3/(R^2\nu)\), then the resonator is rotated by 120° about the axis of symmetry two times successively, in the same direction. In each angular position, etching is performed over time \(t_1\). In each resonator position, a third part of the total mass to be removed is removed:

\[ m_3(\varphi) = 3d^2 R^2 \left[ C_0^{(3)} + C_3^{(3)} \cos 3(\varphi - \varphi_3) \right], \]

where \(C_k^{(3)} = C_k(\alpha_3), k = 0, 3\) (see Table).

The 4th harmonic of the mass distribution effect with parameters \(M_4, \varphi_4\) is eliminated in four stages. After setting \(2\alpha_4 = 66^\circ, \Delta\varphi = -\varphi_4\), we perform chemical etching over time \(t_4 = 0.405 M_4/(R^2\nu)\). Then the resonator is rotated by 90° about the axis of symmetry three times successively, in the same direction. In each angular position, etching is performed over time \(t_4\). At each stage, a fourth part of the total mass to be removed is removed:

\[ m_4(\varphi) = 4d^2 R^2 \left[ C_0^{(4)} + C_4^{(4)} \cos 4(\varphi - \varphi_4) \right], \]

where \(C_k^{(4)} = C_k(\alpha_4), k = 0, 4\) (see Table). Values \(M_4^{(l)}\), \(\varphi_4^{(l)}\) are determined by the formulas:

\[ M_4^{(l)} = \sqrt{(M_4 \cos 4\varphi_4 + 0.217 M_1)^2 + M_4^2 \sin^2 2\varphi_4}, \]

\[ \varphi_4^{(l)} = \frac{1}{2} \arctan \frac{M_4 \sin 4\varphi_4 + 0.217 M_1}{M_4 \cos 4\varphi_4}. \]

**CONCLUSIONS**

The advantage of the proposed balancing of HRG toothless fused quartz hemispherical resonators for the 1st–4th harmonics of mass distribution defect is short time and low labor input. From the example discussed above it follows that the whole cycle of treatment in accordance with the described procedure lasts about 30 minutes, whereas according to the authors’ estimates, it takes about 20–30 hours to remove the same unbalanced mass by ion plasma etching. Thus, the time and labor input of the balancing process is significantly reduced. Another advantage is the simplicity and low cost of the equipment needed to implement the proposed process as compared with the techniques used in the ion-plasma method of balancing. It is pertinent to note that the proposed balancing technology...

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eliminates the possibility of surface defects, ensuring a high quality of the resonator.

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REFERENCES


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