Dissolved oxygen control of the activated sludge wastewater treatment process using model predictive control

B. Holenda a,*, E. Domokos a, Á. Rédey a, J. Fazakas b

a Department of Environmental Engineering and Chemical Technology, Faculty of Engineering, University of Pannonia, P.O. Box 158, 8201 Veszprém, Hungary
b University Babes-Bolyai, College of Sfantu Gheorghe, RO-3400 Cluj-Napoca, Romania

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Abstract
Activated sludge wastewater treatment processes are difficult to be controlled because of their complex and nonlinear behavior, however, the control of the dissolved oxygen level in the reactors plays an important role in the operation of the facility. For this reason a new approach is studied in this paper using simulated case-study approach: model predictive control (MPC) has been applied to control the dissolved oxygen concentration in an aerobic reactor of a wastewater treatment plant. The control strategy is investigated and evaluated on two examples using systematic evaluation criteria: in a simulation benchmark – developed for the evaluation of different control strategies – the oxygen concentration has to be maintained at a given level in an aerobic basin; and a changing oxygen concentration in an alternating activated sludge process is controlled using MPC technique. The effect of some MPC tuning parameters (prediction horizon, input weight, sampling time) are also investigated. The results show that MPC can be effectively used for dissolved oxygen control in wastewater treatment plants.

Keywords: Activated sludge process; Model predictive control; Dissolved oxygen control; ASM1

1. Introduction
Wastewater treatment plants are large non-linear systems subject to significant perturbations in flow and load, together with variation in the composition of the incoming wastewater. Nevertheless, these plants have to be operated continuously, meeting stricter and stricter regulations. The tight effluent requirements defined by the European Union a decade ago (European Directive 91/271 “Urban wastewater”) become effective in 2005 and are likely to increase both operational costs and economic penalties to upgrade existing wastewater treatment plants in order to comply with the future effluent standards. Many control strategies have been proposed in the literature but their evaluation and comparison, either practical or based on simulation is difficult. This is partly due to the variability of the influent, to the complexity of the biological and biochemical phenomena and to the large range of time constants (from a few minutes to several days) but also to the lack of standard evaluation criteria (among other things, due to region specific effluent requirements and cost levels). A benchmark has been proposed by the European program COST 624 for the evaluation of control strategies in the wastewater treatment plants (Copp, 2002; Vrecko, Hvala, & Kocijan, 2002). This study is strictly agreement with the benchmark methodology especially from the viewpoint of control performances.

In the literature several extensive surveys based on simulation can be found on activated sludge process control (Coen, Vanderhaegen, Boonen, Vanrolleghem, & Van Meenen, 1997; Devisscher et al., 2005). Dissolved oxygen concentration, internal recycle flowrate, sludge recycle flowrate and external carbon dosing rate are the frequently investigated manipulated variables in these systems (Barros & Carlsson, 1998; Cho, Sung, & Lee, 2002; Marsi-Libelli & Giunti, 2002; Yuan & Keller, 2002; Yuan, Oehmen, & Ingildsen, 2002). Nevertheless, the dissolved oxygen (DO) control is the most widely-spread in real-life, since the DO level in the aerobic reactors has significant influence on...
the behavior and activity of the heterotrophic and autotrophic microorganisms living in the activated sludge. The dissolved oxygen concentration in the aerobic part of an activated sludge process should be sufficiently high to supply enough oxygen to the microorganisms in the sludge, so organic matter is degraded and ammonium is converted to nitrate. On the other hand, an excessively high DO, which requires a high airflow rate, leads to a high energy consumption and may also deteriorate the sludge quality. A high DO in the internally recirculated water also makes the denitrification less efficient. Hence, both for economical and process reasons, it is of interest to control the DO. Several control strategies have been suggested in the literature. As a basic strategy, a linear PI controller with feedforward from the respiration rate and the flow rate was presented (Carlsson, Lindberg, Hasselblad, & Xu, 1994; Carlsson & Rehnstrom, 2002; Flanagan, Bracken, & Roesler, 1977). Bocken, Braae, and Dold (1989) based their design on a recursively estimated model with a linear oxygen mass transfer coefficient, but the excitation of the process was improved by invoking a relay which increases the excitation. Carlsson et al. (1994) have applied auto-tuning controller based on the on-line estimation of the oxygen transfer rate. A strategy for designing a nonlinear DO controller was developed by Lindberg and Carlsson (1996). Cadet, Betbeau, and Carlos Hernandez (2004) have developed a multicriteria control strategy with Takagi–Sugeno fuzzy supervisor system to decrease the total cost although keeping good performances. In this paper, a model predictive control is depicted to maintain the dissolved oxygen concentration at a certain setpoint based on a linear state-space model of the aeration process.

Model predictive control (MPC) refers to a class of computer control algorithms that utilize an explicit process model to predict the future response of a plant. Originally developed to meet the specialized control needs of power plants and petroleum refineries, MPC technology can now be found in a wide variety of application fields including chemicals, food processing, automotive, and aerospace applications (Bian, Henson, Belanger, & Megan, 2005; Garcia, Prett, & Morari, 1989). In recent years, the MPC utilization has changed drastically, with a large increase in the number of reported applications, significant improvements in technical capability, and mergers between several of the vendor companies. Qin and Badgwell (2003) gives a good overview of both linear and nonlinear commercially available model predictive control technologies. Model predictive control has also been implemented on several complex nonlinear systems (Dowd, Kwok, & Pietr, 2001; Sistu & Bequette, 1991; Weijers, Engelen, Preisig, & van Schagen, 1997; Zhu, Zamamiri, Henson, & Hjortso, 2000), furthermore, Ramaswamy, Cutright, and Qamar (2005) has recently applied MPC to control a non-linear continuous stirred tank bioreactor. Steffens and Lant (1999) already applied model predictive control on an activated sludge system, however, their work has been based on the assumption of a multivariable control problem rather than focusing on the dissolved oxygen control. Consequently, this control method seems to be a good candidate for the oxygen control of wastewater treatment plants, too.

### 2. Modelling aspects

#### 2.1. Modelling the biological reactions

In the simulation studies two internationally accepted models were chosen to simulate the processes in the wastewater treatment plant: the Activated Sludge Model No. 1 (Henze, Grady, Gujer, Marais, & Matsuo, 1987) was chosen to simulate the biological reactions in the aerobic and anoxic reactors and double-exponential settling velocity function of Takacs, Patry, and Nolasco (1991) has been applied to model the clarification and thickening processes in the secondary settler of the wastewater treatment plant.

Since the first introduction of ASM1 several modifications have been suggested (ASM2, ASM2d, ASM3) and there are several limitations with ASM1, however, its universal appeal and practical verification overshadow these limitations. The values used for simulation can be found in Appendix A. The values approximate those that are expected at 15°C.

#### 2.2. Modelling the secondary clarifier

The model of the secondary clarifier is based on a traditional one-dimensional model applying flux-theory. It is assumed that the horizontal velocities profiles are uniform and that horizontal gradients in concentrations are negligible. Consequently, only processes in vertical dimensions are modelled. Biological reactions are also neglected. The transport of solids takes place via the bulk movement of the water and the settling of the sludge relative to the water. The differential conservation equation describing this process is:

$$\frac{\partial X}{\partial t} = V \frac{\partial X}{\partial y} + \frac{\partial v_s X}{\partial y}$$

with $t$ as time, $y$ as vertical coordinate with origin to the surface, $X$ as solids concentration and $V$ as the vertical bulk velocity. The two terms of the right-hand side refer to the bulk flux and the settling flux. Assuming constant horizontal cross-section $A$ over the entire depth, the bulk velocity $V$ depends only on whether the observed cross-section is in the underflow region or in the overflow region above the inlet position. The settling velocity function is related only to the suspended solids concentration according to the double-exponential settling velocity function of Takács et al. (1991):

$$v_s(X) = \max\{0, \min\left( v_0 \left( \exp^{-r_h(X-X_{\min})} - \exp^{-r_p(X-X_{\min})}\right) \right\}$$

(2)

where $v_0$ is the maximum settling velocity, $X_{\min}$ the minimum attainable suspended solids concentration and $r_h$ and $r_p$ are the hindered and flocculant zone settling parameters. The exact parameters used for the simulation can be found in Appendix A.

#### 2.3. Modelling the aeration process

Aeration is a crucial part of the whole activated sludge process, because microorganisms have to be supplied with enough
oxygen so that they have enough electron acceptor capacity for their metabolism process. The equipment used to deliver oxygen to the aeration system is typically provided by surface mechanical type aerators or diffused aeration systems. Diffused aeration systems include a low pressure, high volume air compressor (blower), air piping system, and diffusers that break the air into bubbles as they are dispersed through the aeration tank.

The whole process while oxygen transports from the air bubbles to the cells of the microorganisms is complex, which can be divided into several subprocesses: convective mass transfer within the air bubble to the gas–liquid border surface; getting through the phase border; mass transfer within the liquid phase to the microbial flocs. Within the flocs, after getting to the cell walls the oxygen has to diffuse through the cell wall. Nevertheless, the slowest of these processes is the second one (transfer through the phase border), so it soon becomes the determining factor for the whole transfer process. This complex process can be divided into several subprocesses: convective mass transfer (blower), air piping system, and diffusers that break the air into bubbles as they are dispersed through the aeration tank.

The aeration details of the model are introduced as a dissolved oxygen mass balance around a complete stirred tank reactor. This is shown by the following equation:

\[ \frac{dS_O}{dt} = \frac{Q \times S_{O,in} - Q \times S_O}{V} + K_{L,a}(S_{sat} - S_O) + r_{SO} \]  

(3)

where \( V \) is the reactor volume, \( S_O \) the concentration of dissolved oxygen in the reactor, \( Q \) the flow rate, \( S_{O,in} \) the DO concentration entering the reactor, \( K_{L,a} \) the overall mass transfer coefficient, \( S_{sat} \) the DO saturation concentration and \( r_{SO} \) is the rate of use of DO by biomass.

2.3.1. Control of the dissolved oxygen concentration

In order to maintain the dissolved oxygen concentration at a given level, the following process model is used. The dissolved oxygen concentration is measured by an ideal sensor in the reactor; the concentration value is processed by the control method to calculate \( K_{L,a} \); the \( K_{L,a} \) is corrected according to the temperature if needed; finally \( K_{L,a} \) is applied to change the oxygen concentration level in the biological reactor. Using this value, the cost for the aeration and the volume of air blown by the diffusers can also be calculated (Fig. 1).

3. Model predictive control

Model predictive control refers to a class of algorithms that compute a sequence of manipulated variable adjustments in order to optimize the future behavior of a plant. At each control interval the MPC algorithm attempts to optimize future plant behavior by computing a sequence of future manipulated variable adjustments. The first input in the optimal sequence is then sent into the plant, and the entire calculation is repeated at subsequent control intervals (Fig. 2).

For any assumed set of present and future control moves \( \Delta u(k), \Delta u(k+1), \ldots, \Delta u(k+m-1) \) the future behavior of the process outputs \( y(k+1|k), y(k+2|k), \ldots, y(k+p|k) \) can be predicted over a horizon \( p \). The most present and future control moves \( (m < p) \) are computed to minimize a quadratic objective of the form:

\[
\min_{\Delta u(k),\Delta u(k+1),\ldots,\Delta u(k+m-1)} \sum_{i=1}^{p} \left| y_i^y[k+1|k] - r(k+1) \right|^2 \\
+ \sum_{i=1}^{m} \left| y_i^u[k+l-1] \right|^2
\]

(4)

subject to inequality constraints:

\[
y \leq y(k+j) \leq \bar{y}, \quad j = 1, \ldots, p
\]

\[
u \leq u(k+j) \leq \bar{u}, \quad j = 0, \ldots, m-1
\]

\[
\Delta u \leq \Delta u(k+j) \leq \Delta \bar{u}, \quad j = 0, \ldots, m-1
\]

Here \( y_i^y \) and \( y_i^u \) are weighting matrices to penalize particular components of \( y \) or \( u \) at certain future time intervals. \( r(k+l) \) is the (possibly time-varying) vector of future reference values (setpoints). Though \( u \) control moves \( \Delta u(k), \Delta u(k+1), \ldots, \Delta u(k+m-1) \) are calculated, however, only the first one (\( \Delta u(k) \)) is implemented. At the next sampling interval, new values of the measured output are obtained, the control horizon is shifted forward by one step, and the same computations are repeated. The predicted process outputs \( y(k+1|k), \ldots, y(k+p|k) \) depend on the current measurement \( y(k) \) and assumptions we make about the unmeasured disturbances and measurement noise affecting the outputs.

3.1. Controller design

The state-space model for the controller design model is generated by the linearization of the aeration process in the ASM1 model at a steady-state operating point of the wastewater treatment plant. The steady-state is reached by applying constant concentration parameters for the influent for 100 days, which can be also used as a starting point for later simulations. The exact parameters can be found in the simulator.
From the point of view of process modeling for model predictive control, the following input variables can be separated: manipulated variables, unmeasured disturbances and measured disturbances. Moreover, measurement noise can also be added to the plant output. In the investigated example, the concentration of the dissolved oxygen is considered as the plant output, the manipulated variable is the oxygen mass transfer coefficient ($K_{L,a}$, [day$^{-1}$]), all the other inputs to the reactor are considered as unmeasured disturbances. No noise on the value of the measured dissolved oxygen concentration is supposed which is also falls in with the recommendations of the benchmark: the oxygen sensor is ideal, neither sampling, nor delay time, the low detection limit is zero and no measurement noise is taken into consideration.

Using sampling time low enough to capture the dynamic properties of the system, the dissolved oxygen concentration has been determined around the steady-state at different aeration intensity. This resulted in the following continuous-time state-space model:

$$ \frac{dx}{dt} = Ax + Bu, \quad y = Cx + Du \quad (5) $$

where $x$ is the state vector, $u$ and $y$ are the input and output vectors and $A$, $B$, $C$ and $D$ are the state-space matrices. A second-order model proved to be a good representation of the aeration process.

State-space models of the aeration process have been set up around different steady states of the wastewater treatment plant using prediction error method based on iterative minimization. State-space models can be characterized by their step response: step response at high dissolved oxygen level is depicted by the dashed line (Step response 2) in Fig. 3. Responses at lower dissolved oxygen level gave results of lower amplitude (Step response 1 at 1.5 mg/l, response 2 at < 1 mg/l). Since in the dissolved oxygen concentration generally has to be maintained about 2 mg/l, the following continuous state-space matrices were selected for the simulation:

$$ A = \begin{bmatrix} -100.03 & 115 \\ 167.77 & -211.47 \end{bmatrix}, \quad B = \begin{bmatrix} 0.87 \\ -1.55 \end{bmatrix}, $$

$$ C = \begin{bmatrix} 7.55 & 0.32 \end{bmatrix}, \quad D = 0 \quad (6) $$

A number of tuning parameters such as control and prediction horizons, weight matrices, influence the performance of the controller. Trial-and-error method was used for the identification of these parameters.

For the tuning process a setpoint-change at $t = 0.03$ day and an input disturbance (reducing the input dissolved oxygen concentration with 1 mg/l) at $t = 0.07$ day were used. In Fig. 4 the responses of the controlled and manipulated variables to the setpoint change and the input disturbance can be seen at different tuning parameters. The setpoint can be seen in the upper figure marked with dashed line. The continuous line represents the response of a controller with sampling time $\Delta t = 2.5 \times 10^{-4}$ day and controller tuning parameters: $\Gamma^m = 1$, $\Gamma^n = 0.01$, $m = 1$ and $p = 10$. Reducing the prediction horizon gave the response marked with dotted line if Fig. 4 and increasing the input weight resulted in the line marked with dashed-dotted line. The simulation studies show the lower prediction horizon gave faster responses but significantly increasing the overshoot amplitude, while larger input weight increased both response time and overshoot.

### 4. Performance assessment

The process assessment is performed at two different levels: IAE (integral of absolute error) and ISE (integral of square error), maximal deviation from setpoint and error variance serve as a proof that the proposed control strategy has been applied properly. In this paper emphasis is placed on the first level of assessment, however, assessment of a activated sludge treatment process (effluent quality, cost factor for operation) in the bench-
mark example is also carried out for the sake of comparison. Length of the observation period is 7 days in the first example as defined in the benchmark and 12 h in the second example.

At the second level of the controller assessment, effluent quality operating cost is defined in the simulation benchmark. Effluent quality index represents the levies or fines to be paid due to the discharge of pollution in the receiving bodies. The effluent quality is averaged in the first example over a 7-day observation period based on a weighting of the effluent loads of compounds:

\[
EQ = \frac{1}{1000T} \int_{t_1}^{t_2} \left( B_{SS} \times SS_e(t) + B_{COD} \times \text{COD}_e(t) + B_{NK_j} \times S_{NK_j,e}(t) + B_{NO} \times S_{NO,e}(t) + B_{BOD} \times \text{BOD}_e(t) \right) dt
\]

(7)

where \( EQ \) is the effluent quality index (kg poll. unit/day), \( B_i \) are weighting factors, \( SS \) the suspended solids concentration, \( COD \) and \( BOD \) the chemical and biological oxygen demands, \( S_{NO} \) is the nitrite- and nitrate-concentration and \( S_{TNK} \) is the total N (all concentrations are in g/m³). The energy needed for the aeration is of special interest in this study, which is determined by the following formula:

\[
AE = \frac{24}{T} \sum_{i=1}^{n} [0.4032(Kt_a(t))^2 + 7.8408Kt_a(t)] dt
\]

(8)

where \( Kt_a \) is the mass transfer coefficient in h⁻¹ of the \( i \)-th compartment. The sludge production to be disposed \( (P_{sludge}) \) is calculated from the total solid flow from wastage and the solids accumulated in the system over the 7-day period. The pumping energy is calculated as:

\[
PE = \frac{0.04}{T} \int_{t_1}^{t_2} (Q_a(t) + Q_t(t) + Q_w(t)) dt
\]

(9)

where \( Q_a \) is the internal recirculation flow rate, \( Q_t \) the sludge recirculation and \( Q_w \) is the wasteage flow rate, all expressed in m³/day.

5. Application example I: control of the simulation benchmark

The COST 682 Working Group No. 2 has developed a benchmark for evaluating by simulation, control strategies for activated sludge plants (Copp, 2002). The benchmark is a simulation environment defining a plant layout, a simulation model, influent loads, test procedures and evaluation criteria.

The layout is relatively simple: it combines nitrification with pre-denitrification, which is most commonly used for nitrogen removal. The benchmark plant is composed of a five-compartment reactor with an anoxic zone and a secondary settler. A basic control strategy is proposed to test the benchmark: its aim is to control the dissolved oxygen level in the final compartment of the reactor by manipulation of the oxygen transfer coefficient and to control the nitrate level in the last anoxic compartment by manipulation of the internal recycle flow rate. In this paper, only the control of the dissolved oxygen level is selected for the demonstration of the efficiency of the MPC controller.

The plant layout can be seen in Fig. 5. The first two compartments make up the anoxic zone with individual volume of 1000 m³, and 3 compartments create the aerobic zone with individual volume of 1333 m³. The oxygen mass transfer coefficient rate is set to 240 day⁻¹, while the \( Kt_a \) at the last compartment is controlled in order to maintain the dissolved oxygen concentration at 2 mg/l. The flowrate of the internal recirculation is kept at 55338 m³/day. The secondary settler has a conical shape with the surface of 1500 m² and the depth of 4 m. The flowrate of the sludge recirculation is 18446 m³/day and the excess sludge is removed from the settler at 385 m³/day.

Since disturbances play an important role in the evaluation of controller performances, influent disturbances are defined for different weather conditions. In this paper, dry-weather data are considered containing 2 weeks of influent data at 15 min sampling interval. Parameters for the second week influent are depicted in Fig. 6. Diurnal variations and weekly trends (lower peaks in weekend data) are also depicted by these data. The primary goal of the control is to maintain the dissolved oxygen concentration at the 2 mg/l level in the last compartment.

The controller tuning process in described in Section 4, but it is emphasized that sampling time has a significant effect on the effectiveness of the controller. Sampling time was selected at \( \Delta t = 10^{-3} \) day ≈ 1 min 25 s, later simulations were carried out
at $\Delta t = 2.5 \times 10^{-4}$ day $\approx 20$ s what resulted in considerable effect on the performance of the controller. Parameters of the controller were tuned by trial-and-error method. On one hand, the main goal was to maintain the dissolved oxygen concentration at the desired level, on the other hand, high energy consumption and rapid changes in the air flow rate should be avoided.

Data of the second week of a 2-week dry weather dynamic simulation are of interest, preceding days are used for stabilization of the system. The assessment – as described in Section 4 – can be seen in Figs. 7 and 9 and in Tables 1 and 2 compared to the PI controller described originally in the benchmark for process control. It has to be noted, that internal recycle flow control was also applied in the benchmark besides the DO control, however, for the sake of direct evaluation only DO control has been applied in this simulation, recycle flow rate is kept at constant flowrate. Using this setting, better effluent quality index was achieved, nevertheless, pumping energy is almost double of that achieved with control. The energy consumptions for the aeration are approximately the same using either control strategy.

The performance of the model predictive controller – largely determined by the parameters of the controller, like sampling time, prediction horizon and input weight – is compared to the benchmark results. PI controller performance is also influenced by the parameters, the values presented here are the average results taken from the simulator manual. In this simulation, two sampling times were used for evaluation. It can be seen from Table 2 that reducing the sampling time to its one-fourth, (from $10^{-3}$ to $2.5 \times 10^{-4}$ day) reduced the integral of absolute error with more than 50% and reduced the integral of square error with more than 80%. Maximum deviation from setpoint and variance also decreased as the absolute error is significantly less during the whole observation period.

### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PI control benchmark</th>
<th>DO MPC, $\Delta t = 10^{-3}$ day</th>
<th>DO MPC, $\Delta t = 2.5 \times 10^{-4}$ day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Influent quality (kg poll. unit/day)</td>
<td>42,042</td>
<td>42,042</td>
<td>42,042</td>
</tr>
<tr>
<td>Effluent quality (kg poll. unit/day)</td>
<td>7,605</td>
<td>7,560</td>
<td>7,560</td>
</tr>
<tr>
<td>Sludge production (kg SS)</td>
<td>17,100</td>
<td>17,117</td>
<td>17,116</td>
</tr>
<tr>
<td>Aeration energy (kWh/day)</td>
<td>7,248</td>
<td>7,277</td>
<td>7,277</td>
</tr>
<tr>
<td>Pumping energy (kWh/day)</td>
<td>1,458</td>
<td>2,966</td>
<td>2,966</td>
</tr>
</tbody>
</table>

#### 6. Application example II: control of an alternating sludge process

Most municipal wastewater treatment plants use an activated sludge process. More specifically, for small-size treatment facilities the process generally consists of a single aeration basin configuration in which oxygen is either supplied by surface turbines or diffusers, and is known as the alternating activated sludge (AAS) process. Nitrogen removal is realized by simply switching the aeration system on and off to create continuous alternating aerobic and anoxic conditions, respectively. During switched-on periods, ammonium is converted into nitrate which is subsequently used to remove organic carbon in switched-off periods. An important feature of the AAS process is its flexible control ability which makes it suitable for optimization of operating costs. Since the process consists of alternating aerated and nonaerated periods and the aeration induces 60–80% of the global energy consumption (and subsequently operating costs) of a treatment plant, oxygen control is therefore of great importance.

In this study, an industrial-scale AAS treatment plant is considered described by Chachuat, Roche, and Latifi (2005). The process consists of a unique aeration tank ($V = 2050$ m$^3$) equipped with three mechanical surface aerators (turbines) which provide oxygen ($P = 3 \times 30$ kW, $K_{L,a} = 4.5$ h$^{-1}$) and mix the incoming wastewater with biomass (Fig. 8). The settler is a cylindrical tank where the solids are either recycled...
Table 2
Performance of the oxygen controller

<table>
<thead>
<tr>
<th>Controlled variables ($S_{O,5}$)</th>
<th>PI control benchmark</th>
<th>DO MPC, $\Delta t = 10^{-3}$ day</th>
<th>DO MPC, $\Delta t = 2.5 \times 10^{-4}$ day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setpoint (gCOD/m$^3$)</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Integral of absolute error (gCOD/(m$^3$ day))</td>
<td>0.15</td>
<td>0.1950</td>
<td>0.0892</td>
</tr>
<tr>
<td>Integral of square error ((gCOD/(m$^3$ day)$^2$))</td>
<td>0.02</td>
<td>0.0128</td>
<td>0.0026</td>
</tr>
<tr>
<td>Max deviation from setpoint (gCOD/m$^3$)</td>
<td>0.21</td>
<td>0.1648</td>
<td>0.0781</td>
</tr>
<tr>
<td>Variance of error (gCOD/m$^3$)</td>
<td>0.04</td>
<td>0.0427</td>
<td>0.0196</td>
</tr>
</tbody>
</table>

Manipulated variable ($K_{La}$)
- Max deviation of MV (day$^{-1}$) | 204.5                | 187.39                           | 187.19                         |
- Max deviation of $\Delta$ MV (day$^{-1}$) | 28.71                | 33.12                            | 18.89                          |
- Variance of MV | 59.85                | 59.79                            | 59.76                          |

Table 3
Performance of the oxygen controller in the alternating activated sludge process

<table>
<thead>
<tr>
<th>Prediction horizon</th>
<th>$p = 3$</th>
<th>$p = 5$</th>
<th>$p = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controlled variables ($S_{O}$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Setpoint (gCOD/m$^3$)</td>
<td>0/2</td>
<td>0/2</td>
<td>0/2</td>
</tr>
<tr>
<td>Integral of absolute error (gCOD/(m$^3$ day))</td>
<td>$2.08 \times 10^{-2}$</td>
<td>$2.18 \times 10^{-2}$</td>
<td>$3.48 \times 10^{-2}$</td>
</tr>
<tr>
<td>Integral of square error ((gCOD/(m$^3$ day)$^2$))</td>
<td>$9.46 \times 10^{-3}$</td>
<td>$5.99 \times 10^{-2}$</td>
<td>$1.33 \times 10^{-2}$</td>
</tr>
<tr>
<td>Max deviation from setpoint (gCOD/m$^3$)</td>
<td>$2.32 \times 10^{-2}$</td>
<td>$2.73 \times 10^{-2}$</td>
<td>$4.55 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Manipulated variable ($K_{La}$)
- Max deviation of MV (day$^{-1}$) | 240     | 240     | 240      |
- Max deviation of $\Delta$ MV (day$^{-1}$) | 157.28   | 126.05  | 45.38    |

to the aeration tank ($Q_{rec} = 7600$ m$^3$/day) or extracted from the system ($Q_{w} = 75$ m$^3$/day). During the simulation constant influent flow rate and composition were supposed in order to evaluate the efficiency of the controller subject to rapid setpoint changes.

In this simulation the alternating sludge process is realized by changing the dissolved oxygen setpoint between 0 and 2 mg/l in the bioreactor at 72 min (0.05 day). The manipulated variable (oxygen mass transfer coefficient) is varied between 0 and 240 day$^{-1}$ to reach the desired DO-level using model predictive control. The controller is based on a linear state-space model of the aeration process assuming ideal controller and measurement described in Section 4. The changing dissolved oxygen concentration can be seen in Fig. 9 and in Table 3 with different prediction horizons of the controller.

Simulations were carried out at several parameter settings to evaluate the performance of the controller during the 0.5 day observation period. Sampling time was $2.5 \times 10^{-4}$ day ($\approx 20$ s). The output weight was fixed to 1, while the input weight was varied between 0.001 and 0.01. The control horizon was also fixed to 1, the prediction horizon was changed between 3 and 100. The results showed that lower prediction horizon reduced significantly the integral of absolute and square error, however, input weight had insignificant effect on the error according

Fig. 9. Dissolved oxygen control in the alternating activated sludge process (solid line: $p = 3$; dashed line: $p = 10$; dotted line: $p = 20$).

Fig. 10. Integral of absolute error over the 12-h simulation period.
the prediction horizon (Fig. 10). Reducing the prediction horizon from 10 to 3 moves \( \Gamma_\mu = 0.005 \), decreased the integral of absolute error with more than 40%, nevertheless, maximal change in the manipulated variable between two sampling times increased from 45 to 157 \( \text{day}^{-1} \). It can be observed in Fig. 11 that both lower prediction horizon and lower input weight can significantly increase the maximum deviation in the change of \( K_{La} \), at \( \Gamma_\mu = 0.001 \) and \( p = 3 \) the change in the value of the \( K_{La} \) reaches 240 \( \text{day}^{-1} \), which is near to its maximal value (270 \( \text{day}^{-1} \)).

7. Conclusion

Model predictive control strategy of the dissolved oxygen concentration has been quantitatively investigated on two simulated case-studies: the dissolved oxygen concentration has to be maintained at 2 mg/l in the an aerobic basin of a pre-denitrification process with influent disturbances and an alternating dissolved oxygen level has to be kept up in an alternating activated sludge process. To evaluate the results systematic performance criteria were set up and calculated during the simulations concerning the performance of the controller. Several tuning parameters of the controller (input weight, prediction horizon, sampling time) were also investigated. According to the results of the paper, model predictive control can be effectively applied in the control of dissolved oxygen concentration of wastewater treatment plants.

Results from the first case-study show that the performance of the controller can be considerably enhanced by decreasing the sampling time, however, this improvement has no significant impact either on the the whole activated sludge process, or the energy consumption used for the aeration process. The integral of absolute error decreased with 40% by reducing the sampling time from 1 min 25 s to 20 s, however, the effluent quality index remained at 7560 kg (pollution unit)/day and the energy for the aeration remained at 7277 kWh/day.

The goal of the alternating sludge process simulation was to investigate how efficiently model predictive control can follow the rapidly changing dissolved oxygen setpoint. From the results it can be concluded that lower prediction horizon and input weight can decrease the error between the setpoint and the dissolved oxygen concentration, however, this will increase overshot and cause rapid moves of the manipulated variable what can be avoided imposing constraints on the manipulated variable.

Appendix A

See Tables A.1–A.3.

Table A.1

Double-exponential settling velocity parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v'_0 )</td>
<td>( \text{m day}^{-1} )</td>
<td>250</td>
</tr>
<tr>
<td>( v_0 )</td>
<td>( \text{m day}^{-1} )</td>
<td>474</td>
</tr>
<tr>
<td>( r_h )</td>
<td>( \text{m}^3 \text{ (g SS)}^{-1} )</td>
<td>( 5.76 \times 10^{-4} )</td>
</tr>
<tr>
<td>( r_p )</td>
<td>( \text{m}^3 \text{ (g SS)}^{-1} )</td>
<td>( 2.86 \times 10^{-3} )</td>
</tr>
<tr>
<td>( f_{ss} )</td>
<td>–</td>
<td>( 2.28 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

Table A.2

Weighting factors for the different types of pollution

<table>
<thead>
<tr>
<th>Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{SS} )</td>
<td>2</td>
</tr>
<tr>
<td>( B_{COD} )</td>
<td>1</td>
</tr>
<tr>
<td>( B_{NK} )</td>
<td>20</td>
</tr>
<tr>
<td>( B_{NO} )</td>
<td>20</td>
</tr>
<tr>
<td>( B_{BOD_5} )</td>
<td>2</td>
</tr>
</tbody>
</table>

Table A.3

Stoichiometric and kinetic parameters of the activated sludge model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_A )</td>
<td>g cell COD formed (g N oxidized)(^{-1} )</td>
<td>0.24</td>
</tr>
<tr>
<td>( Y_H )</td>
<td>g cell COD formed (g COD oxidized)(^{-1} )</td>
<td>0.67</td>
</tr>
<tr>
<td>( f_p )</td>
<td>dimensionless</td>
<td>0.08</td>
</tr>
<tr>
<td>( f_{XB} )</td>
<td>g N (g COD)(^{-1} ) in biomass</td>
<td>0.08</td>
</tr>
<tr>
<td>( f_{X_P} )</td>
<td>g N (g COD)(^{-1} ) in endogenous mass</td>
<td>0.06</td>
</tr>
<tr>
<td>( \mu_H )</td>
<td>day(^{-1} )</td>
<td>4</td>
</tr>
<tr>
<td>( K_S )</td>
<td>g COD m(^{-3} )</td>
<td>10.0</td>
</tr>
<tr>
<td>( K_{O,H} )</td>
<td>g O(_2) m(^{-3} )</td>
<td>0.2</td>
</tr>
<tr>
<td>( K_{NO} )</td>
<td>g NO(_3)-N m(^{-3} )</td>
<td>0.5</td>
</tr>
<tr>
<td>( b_H )</td>
<td>day(^{-1} )</td>
<td>0.3</td>
</tr>
<tr>
<td>( \eta_i )</td>
<td>dimensionless</td>
<td>0.8</td>
</tr>
<tr>
<td>( \eta_b )</td>
<td>dimensionless</td>
<td>0.8</td>
</tr>
<tr>
<td>( K_X )</td>
<td>(g cell COD)(^{-1} )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \mu_A )</td>
<td>day(^{-1} )</td>
<td>0.5</td>
</tr>
<tr>
<td>( K_{NH} )</td>
<td>g NH(_3)-N m(^{-3} )</td>
<td>1.0</td>
</tr>
<tr>
<td>( b_A )</td>
<td>g day(^{-1} )</td>
<td>0.05</td>
</tr>
<tr>
<td>( K_{O_A} )</td>
<td>g O(_2) m(^{-3} )</td>
<td>0.4</td>
</tr>
<tr>
<td>( k_a )</td>
<td>m(^3)COD (g day(^{-1} ))</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Fig. 11. Maximum deviation in the change in the oxygen mass transfer coefficient over the 12-h simulation period.
References


