A study of optimal state shares in mixed oligopoly: Implications for SOE reform and foreign competition

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Abstract

This article studies the optimal state share in a partially state-owned enterprise (SOE) from the perspectives of a social planner and a transition-economy government that is under pressure to provide employment. In a mixed-oligopoly model, we find that when the SOE is cost inefficient relative to the private firm, the effects of employment burden on the optimal state shares are different between the government and social planner. Whilst the government’s optimal response to increasing employment pressure is to raise state shares, the socially efficient solution implies a reduction in the state share. In effect, the state share that maximizes the government’s payoff is too high from the social perspective. Furthermore, as tariff falls and foreign competition intensifies, the social planner always wants to reduce the state share but the government does not want to do that if the SOE is sufficiently inefficient. Somewhat surprisingly, the SOE’s output may rise as foreign competition intensifies, although such increase is against social efficiency. Finally, our analysis suggests that given the employment burden, the government might resist complete free trade once the initial liberalization is achieved.

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1. Introduction

Under the previous central planning system, the Chinese industrial sector was dominated by state-owned enterprises (SOEs) that basically acted as cost centers to fulfill production quotas and provide lifelong employment (or the “iron rice bowl”). The reform of SOEs had been the centerpiece of China’s economic reform, evolving from the introduction of a “contract system” in the 1980s to the separation of business management from state ownership (“Liang-quan Fen-li”) in the early 1990s. The purpose was to revitalize SOEs through decentralization, improvement of internal managerial and incentive systems, and introduction of market competition, with a view to transforming SOEs from cost centers to economic units responsible for a profit target. Since the mid-1990s, the Chinese government has intensified the SOE reform by restructuring SOEs into limited liability companies or joint stock companies, and converting debt to stock (“Zhai Zhuan Gu”), a process termed “corporatization” or “partial privatization” (e.g., Zhu, 1999). When a joint stock company is listed, it issues three classes of common shares: state, institutional, and individual shares (tradable domestic A shares). At present, more than half of the domestic stock market is held by the government in state-owned shares, with the state holding of many companies at about 60%. Under state dominance, the control rights rest with bureaucrats who have only an indirect interest in profit, thereby leading to inefficiency in many situations (Schleifer & Vishny, 1997; World Bank, 1992; Zhang, 1997). Whilst the Chinese government made clear its intention to retain a controlling stake in the country’s largest SOEs, it has also contemplated various ways to reduce its holding stock in SOEs. In June 2002, the government scrapped a controversial plan to sell state-owned shares in listed companies (see, e.g., Financial Times, 2002).

The main arguments for the sale of state shares were to improve corporate governance in listed companies and to make their objective more align with profit maximization, with the associated benefits of improved efficiency. On the other hand, the decision to back away from the state-shares sale was motivated largely by the concern of social stability. For a long time, SOEs have played an essential role in absorbing urban labor employment and providing a social security network. Whilst the private sector has been developing rapidly and thereby providing many new employment opportunities since China’s economic reform, SOEs remain as the major channel in absorbing urban labors. As argued by Bai, Li, Tao, and Wang (2000), state enterprises still need to be charged with the multitasks of efficient production as well as employment and other social services provision, because independent institutions for social safety are lacking and social stability has the property of

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1 According to the Law on State-owned Industrial Enterprises (1988, People’s Congress of PRC), the property rights of an SOE belong to “all the people,” and the enterprise only has rights to manage these properties based on the principle of the separation between ownership and management rights. The enterprise has the rights to possess, utilize, and dispose of the property put under its management by the state. The enterprise can only change its ownership structure with government approval. The “Law on the Reform of the Operating System of State-owned Enterprises” (1992, People’s Congress of PRC) delegated more decision-making authority to SOEs, but it made no change in terms of property rights.
public good.\(^2\) As a result, parts of SOEs’ assets are “nonproductive” assets that may include the enterprises’ hospital, dormitories, nurseries, and school facilities.\(^3\) Consequently, SOEs may be biased toward increasing sales rather than maximizing profit, and the low profit incentives may contribute to their excessive labor force and poor financial performance. A state-shares sell-off would set these firms free from the heavy employment burden. In the absence of a well-functioning social security system, this could trigger instability in not only the stock market and labor market, but the broad society as well.

In this article, we attempt to investigate the effects of employment burden on the choice of optimal state shares in a partially SOE, and on both government payoffs and total social surplus. Specifically, we examine a two-stage mixed-oligopoly model, where in the first stage, the government chooses a state share within an SOE to maximize its own payoff. As indicated above, the government of a transition economy (such as China) is not a pure social-welfare maximizer; instead, it must pay some attention to employment. Thus, the government’s payoff would consist of a social-welfare term and a term that reflects the employment burden. In the second stage, the SOE competes against a private firm in the product market. Whilst the private firm is a pure profit maximizer, the SOE maximizes a weighted average of its own profit and the government’s payoff, with the weight being the ratio of state holding. Our model formulation has therefore incorporated two important features of a transition economy: first, the government may face an employment burden; and second, the prevalent market structure may be a mixed oligopoly in which state firms compete with nonstate, private firms. The first feature has been discussed above. As for the second feature, in tandem with the restructuring of SOEs, China’s gradual economic reforms have fostered a variety of firm ownership types, including private firms (domestic or foreign). The emergence of the nonstate sector is due in part to the restructuring of SOEs, and in part to industrial deregulations and open-door policies that have allowed the entry of millions of new enterprises. Private-owned firms have emerged to serve the deregulated domestic markets, and have experienced rapid growth for the past two decades.\(^4\)

Our second objective in this article is to consider the impact of increasing foreign competition following China’s entry to the World Trade Organization (WTO) in November 2001. From the mid-1980s, the Chinese government encouraged the establishment of foreign firms in an effort to attract international capital, advanced technology and

\(^2\) In examining the process of privatization in the developing world, Ramamurti (1999) pointed out that, because developing countries have great difficulties in getting successful economic transition in a short term, it is crucial to maintain a long-term stable economic and institutional environment. Hence, gradual reform with state firms continuing to bear certain social welfare burden may be a more realistic and preferable choice for developing countries.

\(^3\) In 1993, SOEs’ social expenses accounted for 6% of their total costs or 40% of wages. The SOEs reform has led to a greater concern for operating efficiency, with welfare outlays being considered nonproductive. For example, when an SOE is selected to become a limited liability stock company for listing on the stock exchange, one condition required for initial public offering is the elimination of most of its welfare assets and outlays.

\(^4\) The establishment of a private ownership system was not contemplated initially, and the dramatic development of a vital nonstate sector is an unintended consequence of the reform process. For an interesting analysis of some of the driving forces behind the unintended rise of a private ownership system in China, see Li, Li, and Zhang (2000).
management expertise, and to boost exports. Two groups of foreign firms have since emerged: foreign-invested enterprises (FIEs) and enterprises owned by overseas Chinese from Hong Kong, Macau, and Taiwan (HMTs). Many FIEs and HMTs are located in special economic zones and are restricted by regulations from participating in domestic markets. Thus, FIEs and HMTs are mainly engaged in the export business: in 1994, for instance, they accounted for 37% of China’s total exports. As part of its WTO accession, China has made substantial market access commitments covering the industrial, services, and agricultural sectors. In the Sino–U.S. agreement, for example, China committed to lowering import tariffs on industrial products from 17.0% to 9.4% by 2005. Import quotas will be eliminated within five years. Foreign companies, for the first time, will be given trading and distribution rights, phased in over three years after accession. In addition to industrial products, China has agreed to relax foreign investment restrictions on many important service industries, including distribution services, telecommunications, financial services, professional services, and tourism.\(^5\) Therefore, China’s accession to the WTO will create significant opportunities for foreign participation in its domestic market. We shall examine how this intensified foreign competition would affect the reform of SOEs and the optimal degree of state ownership.

We find conditions under which complete state ownership or complete privatization is not optimal for the government or from the social perspective. Basically, when the fully state-owned firm is not as cost efficient as its private counterpart, privatization would enhance both government payoff and social welfare. On the other hand, as long as the SOE is reasonably cost efficient, then given the employment burden, complete privatization is suboptimal, and adding some state shares would improve both government payoff and social welfare. We further show that the higher the employment pressure, the more sales the government would prefer its SOE to produce, which can be achieved by increasing the state share. In the Chinese context, however, it is more likely that state firms are less efficient than their private counterparts. We find, for this case, that the effects of employment burden on the optimal state shares are different between the government and welfare-maximizing solutions. Whilst the government’s optimal response to increasing employment pressure is to raise state shares, the socially efficient solution requires a reduction in the state share.\(^6\) This suggests that for the likely cases of relatively inefficient SOEs, the government objective and social efficiency may not be fulfilled simultaneously. In effect, we show that the state share that maximizes the government’s payoff is too high from the social perspective.

Our basic results extend to the case of foreign competition. In addition, we find that as the tariff rate falls, reducing state shares is socially efficient irrespective of relative cost efficiency.
efficiency of the state firm, but that it is in the government’s interest to reduce state share only when the state firm is relatively efficient. If not, then there is an incentive for the government to raise the state share in the SOE. Thus, for the likely cases where the SOE is relatively inefficient, the WTO entry may actually result in a slowdown in the SOE privatization, although such slowdown is against social efficiency. When the state firm is sufficiently inefficient relative to the private firms, it is possible, somewhat surprisingly, that its output rises post-WTO entry, owing to the fact that the WTO entry may result in an increase in state ownership. This output increase is, however, against social efficiency. Finally, our analysis suggests that given the employment burden, the government might resist complete free trade once the initial liberalization is achieved.

A large number of academic studies have assessed China’s SOE reforms in terms of productivity, profitability, and corporate governance (e.g., Bai, Li, & Wang, 1997; Chen et al., 1988; Dollar, 1990; Gordon & Li, 1995; Groves, Hong, McMillan, & Naughton, 1994; Jefferson & Xu, 1991; Jefferson et al., 1992; Kalirajan & Zhao, 1997; Li, 1997; Lin, 2000; Wang, 1999; Woo et al., 1994; World Bank, 1992; Wu, 1999; Zhang, 1997; Zhang, Zhang, & Zhao, 2001, 2002, 2003). The present article contributes to this line of research by examining the SOE reform from the perspective of state-owned shares within an SOE and the impact of employment burden on the choice of state shares, which currently is a major policy concern in the deepening of China’s SOE reform. Furthermore, the impact of increasing foreign competition following the WTO entry on the SOE reform is itself an important topic to researchers, managers, and policymakers.

Our work is also related to studies of mixed markets, in which state-owned public firms compete against profit-maximizing private firms. Mixed oligopolies are common in developed and developing economies, as well as in transition economies. Studies on mixed oligopolies have become increasingly popular in recent years; for pioneering work, see Harris and Wiens (1980) and Merrill and Schneider (1966). See also Bos (1986, 1991), De Fraja and Delbono (1990), and Vickers and Yarrow (1988) for excellent surveys. In these studies, the government maximizes social welfare and the SOEs have social welfare as an objective. One of the primary contributions of the present paper is that it allows the government to have a preference for employment in addition to social welfare. We have argued above that this is an important feature for a transition economy, such as China.

7 For example, Zhang (1997) argued that the SOE reform has been relatively successful in solving the short-term managerial incentive problem but not the long-term managerial incentive problem. Fundamentally, the managers of SOEs are selected by government officials and bureaucrats who need not be responsible for the consequences of their selection. To solve this problem, the authority of selecting management must be transferred from bureaucrats to capitalists. Chen et al. (1988), Dollar (1990), Gordon and Li (1995), Groves et al. (1994), Jefferson and Xu (1991), Jefferson et al. (1992), Kalirajan and Zhao (1997), Li (1997), and World Bank (1992), found significant improvements in the productivity of SOEs. The estimates of annual total factor productivity (TFP) growth in the late 1970s and 1980s in these studies range from 2% to 5%, compared with almost no growth prior to reforms. Bai et al. (1997) contained a very interesting analysis on the validity of using productivity growth as an index of efficiency improvement in SOEs. Using a simple model in the spirit of a counterexample, they showed that measured growth of TFP may be a misleading indicator of SOE performance given significant nonprofit objectives of SOEs.

8 Formal modeling studies on the impact of WTO entry are relatively rare; see Li and Zhang (2003) and references cited therein.
Because of this preference for employment, the government’s and social planner’s perspectives diverge and, as indicated above, this divergence results in important implications for China’s SOE reform and WTO entry. These results are qualitatively different from ones derived from a standard mixed-oligopoly model in which the effect of employment burden is absent.

The article is organized as follows. Section 2 sets up the basic model, and Sections 3 and 4 provide the analysis. Section 5 extends the analysis to the situation of foreign competition, and Section 6 contains concluding remarks. The proofs of our results are, unless indicated otherwise, collected in Appendix A.

2. Basic model

Consider a mixed oligopoly in which one of the firms is partially state owned (Firm 1), the other is private owned (Firm 2). The two firms produce a homogenous product, with outputs being \( Q_1 \) and \( Q_2 \) and the inverse demand curve being \( P(Q) \), where \( Q = Q_1 + Q_2 \) and \( P'(Q) < 0 \). Using \( C_i(Q_i) \) and \( \pi_i \) to denote, respectively, the cost and profit of each firm \((i=1,2)\) and CS the consumer surplus, we have:

\[
\begin{align*}
\pi_1(Q_1, Q_2) &= PQ_1 - C_1, \quad \pi_2(Q_1, Q_2) = PQ_2 - C_2, \\
&= \int_0^{Q_1+Q_2} Pdq - PQ.
\end{align*}
\]

As usual, we use the social-welfare function \( W \) to measure the society’s economic efficiency and define it as the sum of producer surplus and consumer surplus:

\[
W(Q_1, Q_2) = \pi_1 + \pi_2 + CS = \int_0^{Q_1+Q_2} Pdq - C_1(Q_1) - C_2(Q_2). \tag{1}
\]

The governments in transition economies may not be social-welfare maximizers. In the Chinese context, state banks often pump “soft loans” into SOEs to avoid their possible closure (Lu & Tang, 1997; Steinfeld, 1998). This is because SOEs play an important role in providing a wide range of social services to employees and their families and in maintaining social stability (Bai et al., 2000). Because SOEs provide a major share of employment to Chinese population, failures of SOEs might cause social unrest. It is not surprising, therefore, that state banks continue to extend loans to SOEs, although their ability to pay back the loans is questionable. To reflect the government’s concern for employment (and hence social stability), we assume that its objective is a linear combination of the social welfare and the revenue of the SOE:

\[
G(Q_1, Q_2) = W + \beta PQ_1 = \int_0^{Q_1+Q_2} Pdq - C_1(Q_1) - C_2(Q_2) + \beta PQ_1, \tag{2}
\]
where parameter $\beta$ ($0 \leq \beta \leq 1$) may be used to measure the degree of employment pressure. As $\beta$ increases, the government is biased more toward increasing sales, thereby deviating further from the social-welfare maximization.\footnote{There are potentially two issues about this government’s objective function that are worth discussing. First, the government’s preference for employment is modeled as a preference for greater SOE’s revenue. We discuss the issue further in the concluding remarks where the output of the SOE rather than revenue is used as a proxy for employment. Second, the objective function contains only the revenue of the SOE because the government is under pressure to provide employment. It is noted that total employment may be more important for social stability than simply the SOE’s employment. While we acknowledge this point, the modeling in the present article limits the government’s policy instrument to only the state share within the SOE. As a result, the government cannot directly influence the production decision of the private firm. In addition, the emphasis of this article is the social employment burden in SOEs rather than total employment in the society. In China, whilst the private sector has been developing rapidly and thereby providing many new employment opportunities, SOEs remain as the major channel in absorbing urban labors. We consider an extended model that addresses the total employment issue as a future research area.}

To address the choice of optimal state shares, we consider the SOE as a “joint stock” company with its shares consisting of both state- and private-owned stocks. We use parameter $\alpha$ ($0 \leq \alpha \leq 1$) to represent the ratio of state stock: $\alpha=0$ means that all the stocks are held in the hands of private investors, whilst $\alpha=1$ means that all the stocks are held in the hands of the government. Consequently, the degree to which the SOE takes the government’s payoff into account depends on $\alpha$. Specifically, the objective function of the SOE is a linear combination of its profit and the government objective, with the weight of the government objective proportionate to its state share:

$$S_1(Q_1, Q_2; \alpha) = (1 - \alpha)\pi_1(Q_1, Q_2) + \alpha G(Q_1, Q_2).$$  \hspace{1cm} (3)

In the Chinese context, Bai et al. (2000) argued, for instance, that because SOEs play an important role in providing a wide range of social services to employees (e.g., lifelong employment) and their families and in maintaining social stability, the managers of SOEs may be biased toward increasing sales rather than maximizing profit. It is noted that the empirical evidence presented in Zhang et al. (2002) is consistent with nonprofit maximizing behavior by SOEs. Whilst the SOE maximizes a weighted average of profit and government payoff, the private firm maximizes its profit $\pi_2(Q_1, Q_2)$.

We consider a two-stage mixed-oligopoly model: in Stage 1, the government chooses $\alpha$ to maximize its own payoff; and in Stage 2, the two firms compete in the product market by choosing their quantities. To solve for the subgame perfect Nash equilibrium, we start with the second-stage competition. In this stage, the firms simultaneously choose their outputs, taking the state-owned share $\alpha$ as given.\footnote{This setup is similar in spirit to the multistage model in Li et al. (2000), which considers how product market competition induces institutional change through the interaction between bureaucrats and managers in regional government-controlled economies.} The Cournot equilibrium is characterized by first-order conditions,

$$P_1' Q_1 + (1 + \alpha \beta) P - C_1' = 0, \hspace{1cm} (4)$$

$$P_2' Q_2 + P - C_2' = 0, \hspace{1cm} (5)$$

$$P_1' Q_1 + (1 + \alpha \beta) P - C_1' = 0, \hspace{1cm} (4)$$

$$P_2' Q_2 + P - C_2' = 0, \hspace{1cm} (5)$$
and second-order conditions $\frac{\partial^2 S_1}{\partial Q_1^2} < 0$ and $\frac{\partial^2 \pi_2}{\partial Q_2^2} < 0$. We assume stable equilibrium so that

$$\Delta = \left| \frac{\partial^2 S_1}{\partial Q_1^2} \right| - \left| \frac{\partial^2 \pi_2}{\partial Q_2^2} \right| > 0 \quad \text{(e.g., Dixit, 1986)}$$

and nondecreasing marginal costs, that is, $C_i''(Q_i) \geq 0$.

Eqs. (4) and (5) define, respectively, the reaction functions for the state and private firms as,

$$R_1(Q_2) = \arg \max_{Q_1 \geq 0} S_1(Q_1, Q_2; \alpha), \quad R_2(Q_1) = \arg \max_{Q_2 \geq 0} \pi_2(Q_1, Q_2).$$

We assume that the industry demand curve satisfies:

$$P + P' Q_2 < 0, \quad (1 + \alpha \beta) P' + (1 - \alpha + \alpha \beta) P'' Q_1 < 0,$$

for $0 \leq \beta \leq 1$ and $0 \leq \alpha \leq 1$. Condition (6) is equivalent to the outputs of the two firms being “strategic substitutes”: that is, one firm’s marginal revenue declines when the output of the other firm rises. It is satisfied, for example, if the demand function is linear, and is a standard condition in Cournot analysis (see Dixit, 1986; Shapiro, 1989). The condition ensures that $R_i'(Q_i) < 0$, that is, each reaction function is downward sloping, and that $|R_i'(Q_i)| < 1$. For example,

$$-1 < R_1'(Q_1) = - \frac{P' + P'' Q_2}{2P' + P'' Q_2 - C_2''} < 0,$$

because $C_i''(Q_i) \geq 0$, $P' + P'' Q_2 < 0$ by Condition (6) and $2P' + P'' Q_2 - C_2'' < 0$ by the second-order condition. These are standard results for a Cournot-quantity game. Furthermore, with nondecreasing marginal costs, the demand curve intersects each firm’s marginal cost curve from above, which ensures that the Cournot equilibrium is stable.11

3. Analysis

The comparative static effects of the state share on the equilibrium outputs, denoted $E_1(\alpha)$ and $E_2(\alpha)$, are reported in Proposition 1:

**Proposition 1.** An increase in the state share will increase the state firm’s output, decrease the private firm’s output, but nevertheless increase total output and, hence, reduce price: that is, $\frac{dE_1}{d\alpha} > 0$, $\frac{dE_2}{d\alpha} < 0$, $\frac{d(E_1 + E_2)}{d\alpha} > 0$, and $\frac{dP}{d\alpha} < 0$.

The intuition associated with Proposition 1 is clear. As $\alpha$ rises, the weight on government payoff becomes heavier in the state firm’s objective function. Because the government-payoff maximizing output is greater than the pure profit-maximizing level, an increase in $\alpha$ allows the state firm to commit to greater output. Given that the firms’ products are strategic substitutes, such a commitment would induce a contraction in the private firm’s output. Moreover, because $|dR_2/dQ_1| = |R_2'(Q_1)| < 1$, the output decrease of the private firm

11 It is among the weaker known stability conditions for Cournot equilibrium (see, e.g., Dixit, 1986).
will be smaller than the output increase by the state firm, thus leading to the increase in total output. The increase in total output (hence, the reduction in market price) is the “procompetitive effect” of state ownership discussed in Harris and Wiens (1980) and Merrill and Schneider (1966). This effect corrects the oligopoly distortion of too little output (from the social point of view), and is a major reason for having SOEs in a mixed oligopoly.

State-owned shares, therefore, influence the subsequent product market competition, which in turn affects price, firm profitability, and government payoff. The choice of the degree of state ownership takes place in the first stage. Taking the second-stage equilibrium outputs into account, the government chooses $a$ to maximize its payoff $GE(a) = G(E_1(a), E_2(a))$, where function $G$ is given by Eq. (2). We obtain the following result regarding this choice:

**Proposition 2.** When the government chooses state share $a$ to maximize its payoff $GE$,

1. if $C_1 > C_1' + \beta P$ at $a=1$, then $\frac{dG^E}{da}|_{a=1} < 0$, that is, full state ownership is not optimal;
2. if $P - C_1 > (P - C_2')R_2$ at $a=0$, then $\frac{dG^E}{da}|_{a=0} > 0$, that is, complete privatization is not optimal.

**Proof.**

1. Differentiating $G^E(a) = G(E_1(a), E_2(a))$ with respect to $a$ and rearranging the terms give:

$$\frac{dG^E}{da} = \frac{dE_1}{da} \left[ \frac{\partial G}{\partial Q_1} + \frac{\partial G}{\partial Q_2} \frac{dE_2}{da} \right] = \frac{dE_1}{da} \left[ \beta P' E_1(1 + R_2') + (1 + \beta)P - C_1' + (P - C_2')R_2' \right].$$

Substituting $a=1$ into the first-order Condition (4) yields:

$$\beta P' E_1 + (1 + \beta)P - C_1' = 0.$$

Substituting this equation into the expression for $(dG^E)/(dx)$ then gives:

$$\left. \frac{dG^E}{dx} \right|_{a=1} = \frac{dE_1}{dx} \left( C_1' - C_2' - \beta P \right) R_2'.$$

Hence, by Eq. (7) and Proposition 1, $\left. \frac{dG^E}{dx} \right|_{a=1} < 0$ if $C_1' > C_2' + \beta P$ at $a=1$.

2. When $a=0$, the first-order Condition (4) becomes $P' Q_1 + P - C_1' = 0$. Substituting this into the above expression for $(dG^E)/(dx)$ yields:

$$\left. \frac{dG^E}{dx} \right|_{a=0} = \frac{dE_1}{dx} \left[ (P - C_1')(1 - \beta R_2') + \beta C_1' + (P - C_2')R_2' \right].$$

Since $1 - \beta R_2' > 1$ and $P - C_1' = -P' Q_1 > 0$ we have, again using Proposition 1, that $\left. \frac{dG^E}{dx} \right|_{a=0} > 0$ if $P - C_1' > -(P - C_2')R_2'$. 

\[ \square \]
Proposition 2.1 provides a sufficient condition for suboptimality of full state ownership in the government maximization problem. Essentially, when the marginal cost of the SOE exceeds, at equilibrium, that of the private firm plus $\beta P$, it would improve government payoff if the government privatizes a fully state-owned firm, even after taking the employment burden into account. On the other hand, Proposition 2.2 says that if $P-C_1'>(P-C_2')R_2'$ at $x=0$, then complete privatization is not optimal for the government. Because $C_1'\geq C_2'$ implies, using Eq. (7), $P-C_1'>(P-C_2')R_2'$, it follows that as long as the marginal cost of Firm 1 is not greater than that of Firm 2 when both firms are pure private firms, then adding state shares to Firm 1 will improve government payoff. As is to be seen below, even when $C_1'>C_2'$, adding state shares to Firm 1 could still improve government payoff. This is due in part to the procompetitive effect of state ownership discussed earlier: The increase in total output beyond the private duopoly level (and hence the price reduction) improves social welfare, and social welfare is part of the government objective function.

Proposition 2 has established that under certain conditions, neither full state ownership nor full privatization is optimal for the governments of transitional economy that bear an employment burden. Will this result still hold when the objective function is social welfare rather than government payoff? In this case, state share $z$ is chosen to maximize total social surplus $W^E(z)=W(E_1(z), E_2(z))$.

**Proposition 3.** From the social-welfare point of view,

1. if $C_1'>C_2'$ at $x=1$, then $\frac{dW^E}{dz}|_{z=1}<0$, that is, full state ownership is not optimal;
2. if $P-C_1'>-(P-C_2')R_2'$ at $x=0$, then $\frac{dW^E}{dz}|_{z=0}>0$, that is, complete privatization is not optimal.

**Proof.**

1. Differentiating $W^E(z)=W(E_1(z), E_2(z))$ with respect to $z$ and rearranging the terms give:
   \[
   \frac{dW^E}{dz} = \frac{dE_2}{dz} \left[ \frac{\partial W}{\partial Q_1} \frac{dE_1}{dz} + \frac{\partial W}{\partial Q_2} \right] = \frac{dE_2}{dz} [(P-C_1')R_1' + (P-C_2')].
   \]
   Notice that when $C_1'>C_2'$, the bracketed term is positive: (1) If $P-C_1'\leq 0$, then $(P-C_1')R_1'\geq 0$ and $P-C_2'=-PQ_2>0$ by Eq. (5), so this is true; and (2) If $P-C_1'>0$, then $P-C_2'=-R_1' (P-C_1')$ because $P-C_2'>P-C_1'$ and $-1<R_1'<0$. Hence, by Proposition 1, $\frac{dW^E}{dz}|_{z=1}<0$ if $C_1'>C_2'$ at $z=1$.

2. Differentiating $W^E(z)=W(E_1(z), E_2(z))$ with respect to $z$ and rearranging the terms give:
   \[
   \frac{dW^E}{dz} = \frac{dE_1}{dz} \left[ \frac{\partial W}{\partial Q_1} + \frac{\partial W}{\partial Q_2} \right] = \frac{dE_1}{dz} [(P-C_1') + (P-C_2')R_2'].
   \]
   Thus, by Proposition 1, $\frac{dW^E}{dz}|_{z=0}>0$ if $P-C_1'=-R_2' (P-C_2')>0$ at $z=0$. \qed
Proposition 3.1 shows that when the fully state-owned firm has a higher marginal cost than the private firm, privatization (from the full state ownership) would enhance social welfare. Notice that, as compared to the government-payoff maximization (Proposition 2.1), the condition on the SOE’s relative efficiency for suboptimality becomes stricter here, because \( C_1^r > C_2^r + \beta P \) is sufficient, but not necessary, for \( C_1^r > C_2^r \). This suggests, as expected, that a government with an employment burden would be biased towards more state ownership than would be warranted by social efficiency. On the other hand, a comparison of Propositions 3.2 with 2.2 shows that the government problem and social welfare maximization share the same form of conditions for the suboptimality of full privatization. In particular, when the marginal cost of Firm 1 is not greater than that of Firm 2 with both firms being pure private firms (recall that \( C_1^r \leq C_2^r \) implies \( P - C_1^r > -(P - C_2^r)R^2 \)), adding state shares to Firm 1 would improve social welfare. This is due to the procompetitive effect of state ownership in oligopoly. Essentially, adding state shares would increase total output, reduce price and hence improve social welfare.

To illustrate the above results, consider the following numerical example: \( P = a - Q_1 - Q_2 \), \( C_1 = (3/8)Q_1^2 \), \( C_2 = (1/2)Q_2^2 \), and \( \beta = 0.25 \). When \( a = 1 \), it can be shown that \( C_1^r > C_2^r \) and \( dW^E/dx < 0 \), so Proposition 3.1 holds. In this example, full state ownership does not maximize social welfare and thus is socially inefficient. It is interesting to note that for this example, if the SOE were to produce the same quantity as the private firm, it would be more efficient in the sense that \( C_1^r < C_2^r \). However, when \( a = 1 \) (full state ownership), the equilibrium output of the SOE is much higher than that of the private firm (5a/11 vs. 2a/11). With increasing marginal costs, its marginal cost will thus be much higher than the private firm’s (15a/44 vs. 8a/44). Privatization would alleviate excessive production and hence reduce the SOE’s marginal cost. At the margin, because the high marginal-cost effect outweighs the procompetitive effect, full state ownership is suboptimal and privatization would improve social welfare.

Similarly, Proposition 3.2 holds for the case of \( P = a - Q_1 - Q_2 \), \( C_1 = (5/8)Q_1^2 \), \( C_2 = (1/2)Q_2^2 \), and \( 0 \leq \beta \leq 1 \). Because \( P - C_1^r > -(P - C_2^r)R^2 \) at \( a = 0 \), complete privatization of the SOE will not maximize social welfare. In this second numerical example, if Firm 1 were to produce the same quantity as Firm 2, its marginal cost would be higher than that of Firm 2, implying that the state firm has lower production efficiency than the private firm. However, when \( a = 0 \), the equilibrium outputs of the state and private firms are 8a/35 and 9a/35, respectively; as a consequence, their marginal production costs are 10a/35 and 9a/35, respectively. Obviously, although the equilibrium output of the state firm is less than that of the private firm, the discrepancy of their marginal costs is not so large as to outweigh the gain of adding state shares to Firm 1 arising from the procompetitive effect. Here, adding state shares would be welfare enhancing.12

4. Effects of employment burden

As indicated earlier, one feature that distinguishes the present model from others in the literature on mixed oligopoly is that it allows the government to have a preference for

12 It can be easily shown that Proposition 2.1 holds for the first numerical example, whereas Proposition 2.2 holds in the second numerical example.
employment in addition to social welfare. The preference for employment is captured by parameter $b$. When $b \to 0$, the SOE’s reaction function reduces to those derived in other papers on mixed oligopoly. It is therefore important to investigate the results that would emerge when $b > 0$. In this section, we investigate the effect of employment burden on the optimal degree of state ownership in a transition economy, and our discussion will be based on the following linear demand and quadratic cost functions:

$$
\begin{align*}
P &= a - Q_1 - Q_2, \\
C_1 &= \frac{k}{32} Q_1^2, \\
C_2 &= \frac{1}{2} Q_2^2,
\end{align*}
$$

where $a$ and $k$ are positive parameters, with $a$ capturing the demand level. It is noted that parameter $k$ captures the cost efficiency of the state firm relative to the private firm. If the two firms produce same quantities, then the state firm has higher, the same, and lower relative efficiency when $k < 16$, $= 16$, and $> 16$, respectively. Thus, as $k$ rises, the state firm becomes relatively more inefficient. It turns out that $k$ must be less than 80 to ensure positive optimal state shares. Obviously, the specific functions in Eq. (8) satisfy all the regularity conditions discussed in Section 2.

We use $\bar{x}_G$ to denote the optimal degree of state ownership when $x$ is chosen to maximize the government payoff. From first-order Conditions (4) and (5), the second-stage equilibrium outputs can be calculated as:

$$
E_1(x) = \frac{2 + 2x \beta}{5 - 3x + 5x \beta + \frac{3}{16} k} a, \\
E_2(x) = \frac{1 - x + x \beta + \frac{1}{16} k}{5 - 3x + 5x \beta + \frac{3}{16} k} a.
$$

Taking Eq. (9) into account, the government maximizes $G^E(x) = G(E_1(x), E_2(x))$, giving rise to:

$$
\bar{x}_G(k, \beta) = \frac{k - 6 \beta k - 32 \beta - 80}{32 \beta^2 - 6k \beta - 16 \beta - 80}.
$$

Differentiating Eq. (10) with respect to $\beta$ yields:

$$
\frac{\partial \bar{x}_G}{\partial \beta} = \frac{32(3k + 16) \beta^2 + 32(80 - k) \beta + (3k^2 + 8k + 640)}{2(16\beta^2 - 3k \beta - 8\beta - 40)^2} > 0,
$$

for $0 < k < 80$. Thus, as the employment burden gets heavier, it is in the government’s interest to increase state shares in Firm 1.

How would this result be compared to the welfare-maximizing benchmark? The expression for the optimal state ownership that maximizes social welfare, denoted $\bar{x}_W$, is:

$$
\bar{x}_W(k, \beta) = \frac{k - 80}{2\beta(40 - 3k) - 80}.
$$
Differentiate Eq. (12) with respect to $\beta$ yields:

$$\frac{\partial z^*_W}{\partial \beta} = - \frac{(k - 80)(40 - 3k)}{2\left[\beta(40 - 3k) - 40\right]^2}. \quad (13)$$

It can be easily shown that when $k < 40/3$, $k = 40/3$, and $> 40/3$, then $(\partial z^*_W/\partial \beta) > 0$, $= 0$, and $< 0$, respectively. The foregoing discussion leads to:

**Proposition 4.** Given the demand and cost specifications in Eq. (8),

1. when state share $\alpha$ is chosen to maximize the government payoff, then $(\partial z^*_W/\partial \beta) > 0$. That is, as the employment burden gets heavier, it is in the government’s interest to increase state shares;
2. when $\alpha$ is chosen to maximize social welfare, then $(\partial z^*_W/\partial \beta) > 0$, $= 0$, and $< 0$ if $k < 40/3$, $= 40/3$, and $> 40/3$, respectively. That is, as the employment burden gets heavier, it is socially efficient to increase state shares only if the state firm is relatively cost efficient.

Simulations are carried out to solve for the optimal degree of state shares for specific values of $\beta$ and $k$. It can be seen from Table 1 that for $k = 12$, 16, 24, and 32, $z^*_W$ exists and increases in $\beta$. Table 2 gives the state shares that maximize social welfare. It is clear that $z^*_W$ increases in $\beta$ for $k = 12(< 40/3)$, but decreases in $\beta$ when $k = 16$, 24, and 32. These simulation results are consistent with Proposition 4.

Proposition 4 shows that the higher the employment pressure, the more output the government would prefer its SOE to produce, which can be achieved by further raising the state share. If the state firm is more cost efficient than the private firm in the sense that $k < 40/3$, then the optimal degree of state shares increases in employment-burden parameter $\beta$, whether state share $\alpha$ is chosen to maximize social welfare, or to maximize the government payoff (which has an employment component). In this case, the adverse effect of excessive production following an increase in $\alpha$ would, owing to a relatively efficient SOE, be alleviated, and would be outweighed by the procompetitive effect of state ownership. As a result, raising state ownership in response to increasing employment pressure is consistent with social-welfare maximization.

In the Chinese context, however, it is more likely that state firms are less efficient than their private counterparts. For example, empirical results obtained in Zhang et al. (2001, 2002, 2003) showed that SOEs are less efficient than private firms, whether the latter are domestic or foreign-owned enterprises. In our model, the situation may be represented by $k > 40/3$. Proposition 4 then indicates that the effects of $\beta$ on the optimal state shares are different between the government and welfare-maximizing solutions. Whilst the government’s optimal response to increasing employment pressure is to raise state shares, the socially efficient solution implies a reduction in the state share. This is because the social objective function does not include the employment component; as a result, social efficiency is the sole concern. When the SOE is relatively inefficient, the adverse effect of excessive production by the SOE would outweigh the procompetitive effect of state ownership.
ownership. In this case, raising state ownership in response to increasing employment pressure is socially inefficient.

This suggests that for the likely case of relatively inefficient SOEs, achieving the government objective would be at the expense of social efficiency, so the government objective and social efficiency may not be fulfilled simultaneously. Furthermore, our analysis may provide an explanation for why it appears more difficult for governments of transitional economy to privatize SOEs than for governments of market economy. Whilst the latter have little or no employment pressure and hence are able to focus on social-welfare maximization, the former have an employment burden to shoulder, owing to the historical legacy of command economy and to the lack of social security network during their transition to a market economy. This then points to the importance of establishing social security network in a transition economy, which may be viewed as an integral part of privatization process.

The above discussion suggests that $x_G^*$ and $x_W^*$ may move in opposite directions as $\beta$ increases. This is because, given $\beta>0$, the government objective is only partially consistent with the goal of social efficiency. In effect, using Eqs. (10) and (12), we can easily show:

**Proposition 5.** Given the demand and cost specifications in Eq.(8), then $x_G^*<x_W^*$, $x_G^*=x_W^*$, and $x_G^*>x_W^*$ if $k<80/9$, $k=80/9$, and $k>80/9$, respectively. Thus, for the most likely case of $k>80/9$, the state share that maximizes the government’s payoff is too high as compared to the socially efficient level.
Proposition 5 shows that in general, \( G^* \neq W^* \). Recall that if the two firms produce same quantities, then the state firm has higher, the same, and lower relative efficiency when \( k \leq 16 =16, \) and \( >16, \) respectively. Consequently, the case of \( k > 80/9 = 8.9 \) would be very rare in the context of a transition economy, such as China. Here, because the SOE is far more efficient than its private counterpart, in a Cournot rivalry, it would have a much larger market share and so the employment pressure would be taken care of even with a small \( \beta. \) In this case, the government’s privatization would be too aggressive judging from social-welfare maximization. The case of \( k < 80/9 \) is more interesting; this is when the SOE is either less efficient than the private firm or slightly more efficient. As mentioned earlier, this case is more relevant in the Chinese context. In addition, since the government’s objective and the social planner’s objective are more consistent with each other for \( k > 80/9 \) than for \( k < 80/9 \), the discrepancy between \( G^* \) and \( W^* \) is smaller in the former than in the latter. This observation is confirmed by examining and comparing Tables 1 and 2. Thus, for the more interesting, and certainly more likely, case of \( k < 80/9 \), the state share that maximizes the government’s payoff is too high from social point of view.

\[ \frac{B_{G^*}}{B_k} = \frac{0.851703}{0.796813} = 0.851703 \]

Specifically, \( \frac{\partial x_G}{\partial k} < 0 \) and \( \frac{\partial x_W}{\partial k} < 0. \) That is, as the state firm gets more and more inefficient

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relative to the private firm, reducing state shares is optimal both for the social planner and for the government, whose objective includes employment as a component. This result is expected. It can also be seen from Tables 1 and 2: for each $\beta$ (each row), both $z_i^*$ and $z^*_W$ decrease in $k$.

Finally, we consider the impact of employment burden on profits. Suppose for simplicity that the state and private firms have the cost functions, $C_1 = C_2 = Q_i^2/2$, $i=1,2$ (this is the case when $k=16$ in Eq. (8)). Then it follows from Eq. (9) that

$$\frac{\partial E_1}{\partial \beta} = \frac{6\alpha x(1-x)}{(8 - 3x + 5x\beta^2)} > 0, \quad \frac{\partial E_2}{\partial \beta} = \frac{-2\alpha x(1-x)}{(8 - 3x + 5x\beta^2)} < 0,$$

and $(\partial E_i)/(\partial \beta)=0$ only when $x=1$. Thus, for $x<1$, the government’s preference for employment makes the partially state-owned firm more aggressive, and the private firm less aggressive, in the output market. This output effect of employment preference has implications for equilibrium profits. Specifically, using this effect, we have:

$$\frac{\partial \pi_1}{\partial \beta} = \frac{\partial \pi_1}{\partial Q_1} \frac{\partial E_1}{\partial \beta} + \frac{\partial \pi_1}{\partial Q_2} \frac{\partial E_2}{\partial \beta}, \quad \frac{\partial \pi_2}{\partial \beta} = \frac{\partial \pi_2}{\partial Q_1} \frac{\partial E_1}{\partial \beta} + \frac{\partial \pi_2}{\partial Q_2} \frac{\partial E_2}{\partial \beta}$$

$$= (P' E_2) \frac{\partial E_1}{\partial \beta} < 0.$$

Thus, the preference for employment would reduce the private firm’s profit, where the second equality in the expression $(\partial \pi_2)/(\partial \beta)$ follows by using the first-order condition $(\partial \pi_2)/(\partial Q_2)=0$. Whilst the effect on the private firm’s profit is straightforward, the effect on the SOE’s profit is not obvious: Whilst the second term in $(\partial \pi_1)/(\partial \beta)$ is positive, the first term can be either positive or negative so its overall sign is ambiguous. Nevertheless, for small $x$, it can be shown that the first term is small and the second term dominates. So in this case, the government’s preference for employment would yield higher equilibrium profits for the SOE. This result is similar to the well-known result obtained by Fershtman and Judd (1987), Ross (1987), and Sklivas (1987): that is, owners of a firm can increase profits by writing management contracts that reward revenue, not profits. Assuming contracts to be linear in profits and sales and using linear demand and constant marginal costs, these authors found that rivalry in managerial incentives in an oligopolistic market can result in sales maximizing behavior even if owners maximize profits. In our case, the owner of the SOE is the state. But rather than maximizing the SOE’s profit, the government maximizes a linear combination of social welfare and the sales of the SOE.¹⁴

5. Extended analysis: effects of WTO entry

As discussed in the Introduction, China has, as part of its WTO accession, made substantial market access commitments covering the industrial, services, and agricultural

¹⁴ Using linear contracts, Ferguson and Zhang (1993) examined the strategic implications of labor contracts for worker welfare and level of employment in an oligopolistic industry.
sectors. In the Sino–U.S. agreement, for example, China committed to lowering import tariffs on industrial products from 17.0% to 9.4% by 2005. Import quotas will be eliminated within five years. As a result, foreign firms will compete more directly with SOEs and other domestic firms in the domestic market than ever before. In this section, we consider the effect of foreign competition on the optimal choices of the state share and the output levels of the firms.

We extend our basic model now to three firms: a state firm, a private firm, and a foreign firm, denoted as Firms 1, 2, and 3, respectively; and as before, the market demand is given by \( P(Q) = Q_1 + Q_2 + Q_3 \). The foreign firm is assumed to maximize its profit, which may be written as:

\[
\pi_3 = P(Q)Q_3 - C_3(Q_3) - tQ_3,
\]

whereas the objective functions for Firms 1 and 2 take the same form as before. In Eq. (14), \( t(\geq 0) \) denotes the unit tariff on Firm 3’s output sold in the domestic market. We shall capture the effect of WTO entry through a reduction in trade barriers (broadly defined tariff or nontariff barriers); in our model, it is represented by (exogenous) reductions in tariff rate \( t \). Whilst the government’s payoff function remains the same as expression (2), the total domestic surplus is now the sum of domestic producer surplus, consumer surplus, and tariff revenue \( tQ_3 \):

\[
W = \pi_1 + \pi_2 + CS + tQ_3 = \int_0^{Q_1 + Q_2 + Q_3} Pdq - C_1(Q_1) - C_2(Q_2) - PQ_3 + tQ_3.
\]

Proposition 6. In the extended model with foreign competition,

1. an increase in the state share will increase the state firm’s output, decrease the outputs of both the domestic private firm and the foreign firm, but nevertheless increase total output and, hence, reduce domestic price: that is, \( \frac{\partial E_1}{\partial x} > 0, \frac{\partial E_2}{\partial x} < 0, \frac{\partial E_3}{\partial x} < 0 \) and \( \frac{\partial E}{\partial t} < 0 \);

2. holding \( x \) constant, \( \frac{\partial E_1}{\partial t} > 0, \frac{\partial E_2}{\partial t} < 0, \) but the sign of \( \frac{\partial E_3}{\partial t} \) is undetermined. Thus, a decrease in tariff rate will increase the foreign firm’s output, reduce the domestic private firm’s output, and may or may not reduce the state firm’s output.

Proposition 6.1 is similar to Proposition 1; it shows that the procompetitive effect of state ownership continues to hold in the three-firm case. Based on this result, we can further obtain results similar to Propositions 2 and 3 of Section 3. Basically, when the fully state-owned firm is not as cost efficient as the domestic private firm and the foreign firm under the setting of free trade (i.e., \( t=0 \)), privatization would enhance both the government payoff and social welfare. At the other extreme, as long as Firm 1 is reasonably cost
efficient relative to the private firms, then complete privatization is suboptimal and adding state shares to Firm 1 would improve both the government payoff and social welfare. The economic intuitions behind these results are the same as ones in our earlier analysis.\footnote{The results, their proofs, and corresponding numerical examples are available upon request from the authors.}

Proposition 6.2 is concerned with how changes in tariff rate affect the equilibrium outputs $E_i$, holding $\alpha$ constant. The effects on the domestic private firm’s output and the foreign firm’s output are as expected. The impact on the SOE’s output is less clear, however. A closer look at the proof shows that this impact depends on the state share $a$ and the employment parameter $b$. More specifically, a tariff reduction affects firms’ profits. Because the state firm also needs to take employment burden into account when making its production decision, the profit-maximization goal has relatively less weight in this decision than would be for nonstate firms. Consequently, the output level of the SOE is dependant to certain degree upon how heavy its employment burden is, which is captured by specific values of $\alpha$ and $b$, and the sign of $(\partial E_1)/(\partial t)$ is in general undetermined as a result. When $\alpha$ is sufficiently small, the production decision of the SOE is sufficiently closer to that of the private firms pursuing profit maximization. In this case, we can demonstrate that the “normal” result of $(\partial E_1)/(\partial t)>0$ holds: that is, the SOE’s output will decline as tariff falls.

The above analysis treats state share $\alpha$ as an exogenous parameter. However, $\alpha$ is a first-stage choice variable: for a given tariff rate, $\alpha$ is chosen to maximize the government payoff. As a result, the optimal $\alpha$ is a function of tariff rate $t$. Thus, the endogenous change in $\alpha$ should be taken into account when we examine the impact of tariff reduction on the output levels of the firms. To consider this as well as the effect of tariff on the optimal state share itself, we shall, in the remainder of this section, use the same demand and cost specifications as those in Section 4, namely:

\[ P = a - Q_1 - Q_2 - Q_3, \quad C_1 = \frac{k_1}{32} Q_1^2, \quad C_2 = \frac{1}{2} Q_2^2, \quad C_3 = \frac{1}{2} Q_3^2. \] (16)

As before, $k_1 (>0)$ captures the relative cost efficiency of the state firm, and $a (>0)$ captures the demand level. Also note that parameter $a$ must be sufficiently large so as to guarantee positive equilibrium quantities. Substituting the above specifications to the first-order conditions and solving for $E_1$ yields:

\[ E_1(\alpha, t) = \frac{2a(2 + \alpha + 2\alpha \beta) + t(2 - 3\alpha + 2\alpha \beta)}{12 - 6\alpha + 12\alpha \beta + \frac{1}{2} k_1}, \] (17)

\[ E_2(\alpha, t) = \frac{2a\left(1 - \alpha + \alpha \beta + \frac{1}{16} k_1\right) + t\left(1 + \alpha \beta + \frac{1}{16} k_1\right)}{12 - 6\alpha + 12\alpha \beta + \frac{1}{2} k_1}, \] (18)

\[ E_3(\alpha, t) = \frac{2a\left(1 - \alpha + \alpha \beta + \frac{1}{16} k_1\right) - t\left(5 - 3\alpha + 5\alpha \beta + \frac{3}{16} k_1\right)}{12 - 6\alpha + 12\alpha \beta + \frac{1}{2} k_1}, \] (19)

where $12 - 6\alpha + 12\alpha \beta + 1/2 k_1 > 0$, $1 + \alpha \beta + 1/16 k_1 > 0$, and $5 - 3\alpha + 5\alpha \beta + 3/16 k_1 > 0$. As a result, we have $(\partial E_2)/(\partial t)>0$ and $(\partial E_3)/(\partial t)<0$. However, because the sign of $2 - 3\alpha + 2\alpha \beta$ is
ambiguous, depending on the specific values of $\alpha$ and $\beta$, the sign of $(\partial E_1)/(\partial t)$ is undetermined. These results are consistent with Proposition 6.2.

We now solve for the optimal state shares, $x_G^*$ and $x_W^*$. To simplify our discussion, we let $\beta=1/2$. After some tedious manipulation we obtain the expression for $x_G^*$, the state share that maximizes the government’s payoff, as:

$$x_G^* = -\frac{\left(12a - \frac{1}{2}k_1\right)(24 + k_1) - a\left(200 + \frac{11}{2}k_1\right) - t\left(52 + \frac{3}{4}k_1\right)}{68a - 50t}. \quad (20)$$

Hence,

$$\frac{\partial x_G^*}{\partial t} = \frac{(864 + 274k_1)a - 25(24 + k_1)k_1}{(68a - 50t)^2}. \quad (21)$$

The sign of $(\partial x_G^*)/(\partial t)$ is ambiguous and depends on the relative cost efficiency of the SOE. Specifically, let

$$k_1^0 = \frac{\sqrt{(300 - 137a)^2 + 21,600 - (300 - 137a)}}{25}. \quad (22)$$

Then we can show that if $0<k_1<k_1^0$, then $(\partial x_G^*)/(\partial t)>0$; and if $k_1=k_1^0$, then $(\partial x_G^*)/(\partial t)<0$. To provide an example, we can prove that as long as $a \geq 5$ and $k_1 \leq 32$, we have $(\partial x_G^*)/(\partial t)>0$. That is, as long as the SOE is cost efficient relative to the two private firms, the domestic government would reduce its share in the SOE as import tariff falls.

Similarly, we can calculate $x_W^*$, the state share that maximizes social welfare, as:

$$x_W^* = \frac{a\left(12 + \frac{3}{4}k_1\right) + \frac{1}{8}k_1 t - k_1\left(3 + \frac{1}{8}k_1\right)}{3(2a - t)}, \quad (23)$$

where $a$ is assumed large enough such that $2a - t$ and hence $x_W^*$ are positive. It follows that

$$\frac{\partial x_W^*}{\partial t} = \frac{\frac{1}{8}k_1(2a - 1) + 12a + 3k_1(\frac{a}{4} - 1)}{3(2a - t)^2} > 0. \quad (24)$$

We summarize the above discussion as:

**Proposition 7.** Given the demand and cost specifications in Eq.(16) and $\beta=1/2$, 

(1) when state share $x$ is chosen to maximize the government payoff, then $(\partial x_G^*)/(\partial t)>0$, and $<0$ if $k_1<k_1^0$, = $k_1^0$, and > $k_1^0$ respectively, where $k_1^0$ is given by Eq.(22). That is, as the tariff rate falls, it is in the government’s interest to reduce state shares only when the state firm is relatively cost efficient;

(2) when $x$ is chosen to maximize social welfare, then $(\partial x_W^*)/(\partial t)>0$, that is, as the tariff rate falls, reducing the state share is socially efficient.
Proposition 7.2 shows that as the tariff rate falls, reducing state shares is socially efficient irrespective of relative cost efficiency of the SOE. This is because, as tariff falls, import competition becomes more significant. As a consequence, the procompetitive effect of state ownership becomes less important in ensuring domestic welfare maximization. On the other hand, the welfare distortion of state ownership can be alleviated with a reduction in \(a\), thereby giving rise to the above result.

In contrast, Proposition 7.1 indicates that as the tariff rate falls, it is in the government’s interest to reduce state shares only when the state firm is relatively efficient. If not, then there is an incentive for the government to raise the state share in Firm 1. This suggests that in the likely case where the SOE is relatively inefficient, the WTO entry might actually result in a slowdown in the privatization of SOEs, although such slowdown is against social efficiency. This is a somewhat surprising result, and it may be explained as follows. In this case, as foreign competition intensifies, the market price and the SOE’s output would fall, thereby adversely affecting SOE employment. With the given employment pressure \(b\), the government needs to increase the output of the SOE to counteract the output reduction as well as to compensate for the lowered price, which can only be done by increasing the state share.

This difference has an important implication for how the SOE’s output would change as the tariff rate changes. From Eq. (17), we have \(E_1^G = E_1(z_0^*(t), t)\). Differentiating \(E_1^G\) with respect to \(t\) gives:

\[
\frac{\partial E_1^G}{\partial t} = \frac{\partial E_1(z_0^*, t)}{\partial t} = \frac{\partial E_1}{\partial z} \frac{\partial z_0^*}{\partial t} + \frac{\partial E_1}{\partial t}.
\]

Notice, from Eq. (17), that when \(\beta = 1/2\), the sign of \((\partial E_1) / (\partial t)\) is the same as the sign of \(2 - 3z + 2 \beta - 2(1 - z) \geq 0\), so \((\partial E_1) / (\partial t) \geq 0\), which is the “normal” result. Furthermore, because \((\partial E_1) / (\partial z)\) is positive by Proposition 6.1, the first term on the right-hand side of Eq. (25) is positive if \(k_1 < k_1^0\) (by Proposition 7.1); consequently, \((\partial E_1^G) / (\partial t) > 0\) if \(k_1 < k_1^0\). On the other hand, if \(k_1 \geq k_1^0\), then the first term is negative so the sign of \((\partial E_1^G) / (\partial t)\) is undetermined. In effect, we can show examples where the (negative) first term outweighs the second term, leading to \((\partial E_1^G) / (\partial t)\) being negative.

**Proposition 8.** When the state firm is sufficiently inefficient relative to the domestic private firm and the foreign firm, its output may rise following the WTO entry, owing to an increase in state ownership. This output increase is, however, against social efficiency.

**Proof.** The first part has been proved in the text. For the second part, if \(z\) is chosen to maximize social welfare, then \(E_1^W = E_1(z_1^*, t)\). Differentiating \(E_1^W\) with respect to \(t\) then yields:

\[
\frac{\partial E_1^W}{\partial t} = \frac{\partial E_1(z_1^*, t)}{\partial t} = \frac{\partial E_1}{\partial z} \frac{\partial z_1^*}{\partial t} + \frac{\partial E_1}{\partial t} > 0,
\]

for any \(k_1\). The positive sign follows from Proposition 6.1, Proposition 7.1, and the fact that \((\partial E_1) / (\partial t) > 0\) at \(\beta = 1/2\). \(\square\)
Similarly, we can show that the domestic private firm’s output decreases following the WTO entry, whilst from social perspective, its output should decrease or increase. As for foreign import, if \( a \) is chosen to maximize social welfare, then import will rise after the WTO entry. From the government-payoff maximization, however, foreign import might actually fall if the state firm is sufficiently inefficient and the government’s employment burden is heavy.\(^{16}\)

Finally, we consider the effect of the WTO entry on the government payoff and social welfare. Using \( G^* = G(E_1^G, E_2^G, E_3^G; \alpha_0, t) \) and \( W^* = W(E_1^W, E_2^W, E_3^W; \alpha_0, t) \) to denote the equilibrium government payoff and social welfare, respectively, we have the following result:

**Proposition 9.** Given the demand and cost specifications in Eq.(16) and \( \beta = 1/2 \),

1. when state share \( \alpha \) is chosen to maximize the government’s payoff, then there exists a tariff rate \( t^0 \), such that \( (\partial G^*)/(\partial t) > 0, = 0, \) and \( < 0 \), if \( 0 < t < t^0 \), \( t = t^0 \), and \( t > t^0 \), respectively. Thus, as the tariff rate falls, the government’s payoff will rise initially, but will fall as the tariff rate approaches zero;

2. when \( \alpha \) is chosen to maximize social welfare, then \( (\partial W^*)/(\partial t) < 0 \) for any tariff rate. Social welfare will therefore rise after the WTO entry in the case of social-welfare maximization.

Thus, the WTO entry would improve the society’s well-being in the benchmark case of welfare maximization. It would also improve the government payoff initially, but can hurt the government when trade liberalization deepens. Our result therefore suggests that given the government’s employment burden, whilst it welcomes initial, small trade liberalization, it may resist complete free trade once the initial liberalization is achieved.

### 6. Concluding remarks and future research

Our main objectives in writing this article are to investigate the effects of employment burden on the choice of optimal state shares within an SOE, and to consider the impact of increasing foreign competition following China’s entry to the WTO. In our mix-oligopoly model, we found that given sufficient cost asymmetries between public and private firms, complete state ownership or complete privatization is not optimal for the government or from the social perspective. We also found that for cases where state firms are less efficient than their private counterparts, the effects of employment burden on the optimal state shares are different between the government and welfare-maximizing solutions. Whilst the government’s optimal response to increasing employment pressure is to raise state shares, the socially efficient solution implies a reduction in the state share, suggesting that the government’s objective and social efficiency may not be fulfilled simultaneously. In effect, the state share that maximizes the government’s payoff is too high from the social perspective.

\(^{16}\) Essentially, we can show \( \frac{\partial E_1^G}{\partial t} = \frac{\partial E_2^G}{\partial t} = \frac{\partial E_3^G}{\partial t} = \frac{\partial E_1^W}{\partial t} = \frac{\partial E_2^W}{\partial t} = \frac{\partial E_3^W}{\partial t} \). But the signs for \( \frac{\partial E_1^G}{\partial t} \) and \( \frac{\partial E_3^G}{\partial t} \) are undetermined.
Whilst this comparison of the two perspectives, the government’s and the social planner’s, is useful, perhaps the more interesting results lie in those about foreign competition. Extending our basic model to the case of foreign competition, we found that as the tariff rate falls and foreign competition intensifies, the social planner always wants to reduce the state share but the government does not want to do that if the SOE is sufficiently cost inefficient relative to the private firms. Thus, for the likely case where the SOE is relatively inefficient, the WTO entry may actually result in a slowdown in the privatization of SOEs, although such slowdown is against social efficiency. Somewhat surprisingly, when the SOE is sufficiently inefficient relative to its private counterparts, it is possible that its output increases post-WTO entry. The reason is that as competition intensifies, the government needs to increase the output of the SOE to compensate for the lowered price as well as the reduced output of the SOE, which can only be done by increasing the state share. This output increase is, however, against social efficiency. Our analysis also suggested that given the employment burden, the government of a transition economy might resist complete free trade once the initial liberalization is achieved.

The present paper models the government’s preference for employment as a preference for greater SOE’s revenue. We have also used the output of the SOE (rather than revenue) as a proxy for employment. Our basic results survive this modification. In particular, when the SOE is sufficiently inefficient relative to the private firms, reducing the tariff rate can still result in an increase in the output produced by the SOE and a decrease in import. Whilst in the revenue case, this surprising result arises because the government needs to increase the output of the SOE (via an increase in the state share) to compensate for both the lowered price and the reduced SOE’s output as foreign competition intensifies; in the output case, the government needs to increase the output of the SOE to counteract the SOE’s shrinking output due to foreign competition. Nonetheless, this somewhat surprising result is less likely to emerge in the output case than in the revenue case.

Whilst our analysis allows for an exogenous cost difference across state and private firms, we did not model the degree of relative inefficiency as a function of state ownership \( \alpha \). One way to incorporate this is to let the cost function of the SOE be \( C_1(Q_1, \alpha) \). Empirical evidence appears to suggest \( (\partial C_1(\partial \alpha)) > 0 \), that is, there is a total-cost efficiency improvement effect associated with less state ownership. It is less clear, however, regarding the effect of \( \alpha \) on the SOE’s marginal cost. If we assume that \( (\partial^2 C_1(\partial \alpha \partial Q_1)) \neq 0 \), then the effect of changes in \( \alpha \) on the equilibrium output of the SOE would be ambiguous, so would all the other results that are based on Proposition 1. Our basic results extend, nevertheless, to the case where \( (\partial^2 C_1(\partial \alpha \partial Q_1)) = 0 \); hence, incorporating \( \alpha \) into the cost function may not be necessary for this case. A related approach linking the degree of state ownership and efficiency may be to specify a production function with labor and capital as inputs. The government’s objective could then be specified as a function of labor (rather than revenue or output). Under this specification and assuming the output price is exogenously given, the SOE would use “too much” labor. The reason is that employment burden makes the SOE to produce under a seemingly “lower” cost, which in effect violates the profit maximization principle. In this framework, therefore, the cost difference between the SOE and the private firm will arise endogenously. Combining a competitive framework (recall the output price is exogenously given) with oligopolistic rivalry in the output market is, however, quite complex and messy. It is hard to derive useful results...
even for the special case of linear demand and constant marginal costs. The extensions along this long are nevertheless seen as an interesting and important research area.

Another interesting line of research concerns the nature of oligopolistic rivalry in the product market. This paper derives results under Cournot-quantity competition. Alternatively, the firms may compete in a Bertrand fashion. It would be interesting to compare our results with ones under the price competition with differentiated products. For example, we have found that the government’s preference for employment might yield higher equilibrium profits for the SOE. This result is similar to the well-known result obtained by Fershtman and Judd (1987), Ross (1987), and Sklivas (1987), that owners of a firm can increase profits by writing management contracts that reward revenue, not profits. These authors also found that, in equilibrium, each owner will give his manager a positive incentive for sales revenue in a Cournot-quantity game, but a negative incentive for sales (or equivalently, a positive incentive for keeping sales low) in a Bertrand-price game. In terms of the optimal state shares \( a \) within SOEs, because the reaction functions are upward sloping in a price game (rather than downward sloping in a quantity game), an increase in \( a \) could result in an increase in equilibrium prices. Thus, the procompetitive effect of state ownership may not be as strong as that in the quantity competition, suggesting that the optimal degree of state ownership tends to be lower in the price competition.

Finally, in the analysis of Section 5, the state share \( a \) is chosen optimally but the tariff is treated as an exogenous parameter and, therefore, is not necessarily the optimal tariff. Our analysis focuses primarily on the effect of China’s WTO entry on the more recent debate concerning about the sale of state shares in SOEs to private investors. As discussed earlier, China has as part of its WTO accession made substantial market access commitments including lowering import tariffs. Our treatment of tariff has thus implicitly assumed that the decision regarding whether to enter the WTO and the extent of market access commitments was made under the influence of other factors, which might reasonably be regarded as given when we discuss the issue of optimal state shares within SOEs. Nonetheless, not allowing the tariff to be chosen optimally appears inconsistent with the assumption that the government rationally pursues its objective. One way to allow the tariff to be chosen optimally is to consider a three-stage game with the tariff rate \( t \) being chosen at the first stage. We see full development of such model as a natural extension of the analysis presented here, although beyond the scope of the present article.

**Appendix A**

**Proof of Proposition 1.** By differentiating the first-order conditions (4) and (5) and collecting the terms, we obtain:

\[
\frac{dE_1}{dz} = \frac{\left[(1 - \beta)P' E_1 - \beta P\right]\left[P'' E_2 + 2P' - C_2''\right]}{\Delta},
\]

\[
\frac{dE_2}{dz} = -\frac{\left[(1 - \beta)P' E_1 - \beta P\right]\left[P'' E_2 + P'\right]}{\Delta}.
\]

Because \((1 - \beta)P Q_1 - \beta P < 0\), it follows, using the second-order conditions and the stability condition, that \((dE_1)/(dz) > 0\) and \((dE_2)/(dz) < 0\). Furthermore, using Eq. (7), we have \((dE_1)/\)
Proof of Proposition 6. The first-order conditions for Firms 1, 2, and 3 are, respectively:

\[ P'(1 - \alpha + \alpha\beta)Q_1 + (1 + \alpha\beta)P - C_1' - \alpha P' Q_3 = 0, \]  
\[ P'Q_2 + P - C_2' = 0, \]  
\[ P'Q_3 + P - (C_3' + t) = 0. \]

Differentiating Eqs. (A3)–(A5) with respect to \( x \) at equilibrium outputs \( E_1, E_2 \) and \( E_3 \), we obtain:

\[
\frac{\partial E_1}{\partial x} K_1 + \frac{\partial E_2}{\partial x} H_1 + \frac{\partial E_3}{\partial x} L_1 = (1 - \beta)P' E_1 - \beta P + P' E_3, \tag{A6}
\]

\[
\frac{\partial E_1}{\partial x} [P'' E_2 + P'] + \frac{\partial E_2}{\partial x} [P'' E_2 + 2P' - C_2''] + \frac{\partial E_3}{\partial x} [P'' E_2 + P'] = 0, \tag{A7}
\]

\[
\frac{\partial E_1}{\partial x} [P'' E_3 + P'] + \frac{\partial E_2}{\partial x} [P'' E_3 + P'] + \frac{\partial E_3}{\partial x} [P'' E_3 + 2P' - C_3''] = 0. \tag{A8}
\]

Manipulating Eqs. (A6)–(A8), we have:

\[
\frac{\partial E_1}{\partial x} = \frac{\Delta_{23}}{\Delta_1} [(1 - \beta)P' E_1 - \beta P + P' E_3], \tag{A9}
\]

\[
\frac{\partial E_2}{\partial x} = - \frac{1}{\Delta_1} [(1 - \beta)P' E_1 - \beta P + P' E_3] (P'' E_2 + P') (P'' - C_3''), \tag{A10}
\]

\[
\frac{\partial E_3}{\partial x} = - \frac{1}{\Delta_1} [(1 - \beta)P' E_1 - \beta P + P' E_3] (P'' E_3 + P') (P' - C_2''), \tag{A11}
\]

where

\[
\Delta_1 = \begin{vmatrix} K_1 & H_1 & L_1 \\ P'' E_2 + P' & P'' E_2 + 2P' - C_2'' & P'' E_2 + P' \\ P'' E_3 + P' & P'' E_3 + P' & P'' E_3 + 2P' - C_3'' \end{vmatrix},
\]

\[ K_1 = P'' [(1 - \alpha + \alpha\beta)E_1 - \alpha E_3] + P' (2 - \alpha + 2\alpha\beta) - C_1'', \]

\[ H_1 = P'' [(1 - \alpha + \alpha\beta)E_1 - \alpha E_3] + P' (1 + \alpha\beta), \]

\[ L_1 = P'' [(1 - \alpha + \alpha\beta)E_1 - \alpha E_3] + P' (1 - \alpha + \alpha\beta), \]
By Eq. (A12), the sign of \( \Delta_2 \equiv \begin{vmatrix} P'' E_2 + 2P' - C_2'' & P'' E_2 + P' \\ P'' E_3 + P' & P'' E_3 + 2P' - C_3'' \end{vmatrix} \).

The corresponding second-order conditions and the stability conditions imply:

\[
\begin{align*}
\Delta_1 &< 0 \\
\Delta_{12} &> 0 \\
\Delta_{23} &> 0 \\
K_1 &< 0 \\
P'' E_2 + 2P' - C_2'' &< 0 \\
P'' E_3 + 2P' - C_3'' &< 0
\end{align*}
\]  
(A12)

By Eq. (A12), the sign of \((\partial E_1)/(\partial x)\) and that of \((1-\beta)P'E_1-\beta P+E_3\) are opposite to each other. Because the sign of \((1-\beta)P'E_1-\beta P+E_3\) is negative, we have \((\partial E_1)/(\partial x)>0\). Furthermore, because \(P'-C_3''<0\), it follows, using Condition (6), that \((\partial E_2)/(\partial x)<0\). Similarly, we have \((\partial E_3)/(\partial x)<0\). Finally, \(\partial E/\partial x=(\partial E_1)/(\partial x)+(\partial E_2)/(\partial x)+(\partial E_3)/(\partial x)=1/\Delta_1[(1-\beta)P'E_1-\beta P+E_3](P'-C_3'')(P'-C_3'')>0\), and hence, \(\partial P/\partial x=P'(\partial E)/\partial x<0\).

Differentiating Eqs. (A3)–(A5) with respect to \(t\) at equilibrium outputs and manipulating the terms, we obtain:

\[
\begin{align*}
\frac{\partial E_1}{\partial t} &= \frac{1}{\Delta_1} \left[ H_1 \begin{vmatrix} P'' E_2 + 2P' & P'' E_2 + P' \\ P'' E_3 + P' & P'' E_3 + 2P' \end{vmatrix} \right] + \frac{L_1}{\Delta_1} , \\
\frac{\partial E_2}{\partial t} &= -\frac{1}{\Delta_1} (P'' E_2 + P') [P' (1 + \alpha \beta) - C_2''], \\
\frac{\partial E_3}{\partial t} &= \frac{\Delta_{12}}{\Delta_1} ,
\end{align*}
\]  
(A13)
(A14)
(A15)

where \(\Delta_{12} \equiv \begin{vmatrix} K_1 & H_1 \\ P'' E_2 + P' & P'' E_2 + 2P' - C_3'' \end{vmatrix} \). By Eq. (A12), the sign of Eq. (A14) is positive, whilst the sign of Eq. (A15) is negative. However, the sign of Eq. (A13) is undetermined.

**Proof of Proposition 9.**

(1) By differentiating the government’s payoff with respect to tariff \(t\) and collecting the terms, we obtain:

\[
\frac{\partial G^*}{\partial t} = \frac{a[56 - 56x + 3k_1 - \frac{1}{2} k_1^2 + 50x^2 - 3k_1 x] + (1 - 164 + 68x - \frac{1}{4} k_1 - 41x^2 - \frac{1}{4} k_1^2 + \frac{1}{2} k_1 x - \frac{1}{4} k_1 (24 + x_1) (1 - x)]}{4(12 + \frac{1}{2} k_1)^3} .
\]

Because \(-164 + 68x - \frac{25}{2} k_1 - 41x^2 - 17/64 k_1^2 + 3/4 k_1 x < 0\),
\[
\frac{\partial G^*}{\partial t} > 0 \iff t < \frac{a(56 - 56x + 3k_1 - \frac{1}{52}k_1^2 + 50x^2 - 3k_1x) - \frac{1}{2}k_1 \times (24 + k_1)(1 - x)}{164 - 68x + \frac{25}{2}k_1 + 41x^2 + \frac{17}{64}k_1^2 - \frac{3}{2}k_1x} = t^0.
\]

We can show that \( t^0 > 0 \). Hence, if \( 0 < t < t^0 \), then \( (\partial G^*)/(\partial t) > 0 \); if \( t = t^0 \), \( (\partial G^*)/(\partial t) = 0 \); and if \( t > t^0 \), \( (\partial G^*)/(\partial t) < 0 \).

(2) Now differentiating \( W^* \) with respect to \( t \) yields:

\[
\frac{\partial W^*}{\partial t} = \frac{a \left[ 4x(2 - x) + \frac{1}{2}k_1 + \frac{1}{2}k_1^2 + \frac{1}{2}k_1x \right] + t \left[ 30 + x + \frac{1}{2}k_1 + 2x^2 + \frac{1}{128}k_1^2 \right] + \frac{1}{2}k_1 \times (12 + \frac{1}{2}k_1)(1 - x)}{(12 + \frac{1}{2}k_1)^2} < 0.
\]

References


