Real-time hybrid testing of semiactive control strategies for vibration reduction in a structure with MR damper

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SUMMARY

Magnetorheological (MR) dampers have been widely studied and employed to solve the vibration problem in structures such as buildings and bridges. It is known that MR dampers can generate high damping forces with low-energy requirements and low-cost productions. However, the complex dynamics that characterize MR dampers make difficult the control design to achieve the vibration reduction goals in an efficient manner. In this paper, semiactive controllers based on the backstepping and quantitative feedback theory techniques are proposed and their performances are compared with each other on the problem of vibration control in a structure with an MR damper. They are applied to a large-scale three-story building with an MR damper at its first floor subject to seismic motions. The performance of the proposed controllers is experimentally evaluated by means of real-time hybrid testing scheme that accounts for time delays and actuator dynamics, allowing for the test of velocity-dependent devices. Copyright © 2009 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The protection of civil engineering structures has always been a major concern especially when these structures are built in places prone to hazardous weather conditions (e.g. hurricanes,

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tsunamis) or in zones of intense seismic activity or when the structure is subjected to heavy loadings (e.g. heavy traffic on a bridge). If a structure is not well protected against these phenomena, they can suffer severe damage and, as a consequence, produce personal injuries or death as could be seen during the earthquakes in Mexico City (1985), Kobe (1995), northwestern Turkey (1999), those that struck southern Asia in 2004 followed by the tsunamis or more recently in China (2008).

In order to make structures more resistant against these phenomena, passive and active dampers were initially proposed. Passive dampers alleviate the energy dissipation of the main structure by absorbing part of the input energy, without the need of external power sources. However, once installed, they are not adaptable to different loading conditions [1]. Active dampers, on the other hand, can respond to variations of the loading conditions and structural dynamics but require large power sources and additional hardware such as sensors and digital signal processors (DSP) to operate. Active dampers can also inject energy into the structure and may destabilize it in a bounded-input bounded-output sense [2].

Semiactive devices provide an effective solution to overcome the disadvantages of passive and active dampers [3]. They have shown to perform significantly better than passive devices as well as active devices without requiring large power sources, thus allowing for battery operation [4]. The main characteristics of semiactive devices are the rapid adaptability of their dynamic properties in real time but without injecting any energy into the system. Among diverse semiactive devices, magnetorheological (MR) fluid dampers are one of the most attractive and useful ones. MR dampers can generate high yield strength, have low costs of production, require low power and have fast response and small size. However, they are characterized by a complex nonlinear dynamics (typically hysteresis), which make mathematical treatment challenging, especially in the modeling and identification of the hysteretic dynamics and the development of control laws for its implementation for vibration mitigation purposes.

A number of control techniques have been developed for vibration control of structures equipped with MR dampers. The clipped optimal control [5] was one of the first controllers developed for this class of systems. An optimal controller is designed to estimate the force that mitigates the vibrations in the structure and the control signal takes only two values according to an algorithm, in which the MR damper dynamics are ignored. Control techniques based on Lyapunov’s stability theory have been proposed and successfully tested in structures such as buildings, bridges and car suspension systems [6–11]. The general control objective is achieved through the choice of control inputs that make the Lyapunov function derivative as negative as possible and consequently obtain the maximum energy dissipation. Other control methods have also been proposed such as bang-bang control [6,12,13], sliding mode control [10,14,15] and intelligent control such as fuzzy logic control [16] and neuro-fuzzy control [17].

Backstepping control in systems with MR dampers was first proposed by Villamizar et al. [18] and Luo et al. [19]. Backstepping control design consists of selecting appropriate functions of state variables as pseudocontrol inputs for lower-dimension subsystems of the overall system. Each backstepping stage is a new pseudocontrol design in terms of the preceding stages. Finally, a feedback design for the true control input results, which achieves the original design objective by virtue of a final Lyapunov function, which is formed by summing up the Lyapunov functions associated with each individual design stage. Numerical simulations and experiments on small-scale specimens showed the feasibility of backstepping control implementation in larger systems [20].

Since the behavior of controlled structures depends not only on the magnitude of the external excitation but also on its frequency modes, the modal frequency control is of great interest for
achieving the structural safety and human comfort. As one of the frequency structural control strategies, the quantitative feedback theory (QFT) control technique was firstly introduced by Luo et al. [21] for the vibration reduction in linear structures and was extended to structures equipped with MR dampers by Villamizar et al. [22]. Numerical simulations and experiments on small-scale specimens showed the feasibility of applying QFT control in larger systems. In these studies, however, an algorithm similar to the clipped optimal control was followed, i.e. the nonlinear dynamics of the MR damper were ignored. A step further was performed by Zapateiro et al. [23] by proposing the inclusion of the hysteretic dynamics of MR dampers in the QFT control design and its feasibility was proved by numerical simulations.

This paper presents our research work carried out on an experimental setup available at the Smart Structures Technology Laboratory, University of Illinois at Urbana-Champaign (SSTL), which represents the problem of vibration control of a large-scale three-story building with an MR damper at its first floor subject to seismic motions. This setup uses model-based methods to compensate for time delays and actuator dynamics and combines fast hardware and software (for high-speed computations and communication) with high-performance hydraulic components. Owing to the complex dynamics characterized by MR dampers, both backstepping and QFT control approaches were used for achieving the vibration reduction goals in an efficient manner. The structural performance of the proposed controllers was experimentally tested in the novel real-time hybrid testing (RTHT) facility [24] at the SSTL.

This paper is organized as follows: Section 2 describes the RTHT principle and the experimental setup. Section 3 gives an overview about backstepping and QFT control and explains the details of the controller formulation. Section 4 describes the experiments performed to evaluate the performance of the controllers and, finally, Section 5 summarizes the conclusions.

2. EXPERIMENTAL SETUP

This section describes the experimental environment where the semiactive controllers will be tested. Experiments are executed in an RTHT configuration available at the SSTL, U.S.A.

2.1. Hybrid testing background

The experimental testing of the control performance in civil engineering structures is an important issue in structural control. It is well known that testing vibration reduction systems at large scale in structures such as buildings or bridges is rather prohibitive because of the dimensions, the power required to do so and the costs that such tests imply. This is why experiments are usually run at small or midscale laboratory specimens. Experiments can be performed in one of three ways: shaking table tests, quasi-static tests and pseudodynamic or hybrid tests [25].

Shaking tables are moved by hydraulic actuators to recreate the motion of the base of the structure in events such as earthquakes at a correct rate. But these are not suitable for structures that are not well represented at small scale (reinforced concrete, for instance). At smaller scales, it is difficult to investigate structural details such as bond, shear and anchorage. Furthermore, shaking tables do not allow the representation of other types of motion such as that of strong winds [25,26].
Quasi-static testing, on the other hand, is a much simpler testing method that can be used to test structural members at large scales, but these tests require a predefined displacement history, which can be later difficult to relate to the seismic demands on the structure [27]. The predefined inputs (displacements or forces) are applied to the structural component on an extended timescale (i.e. slow rates); therefore, the interaction with the structure to which it is to be attached and the dynamic and rate-dependent behavior of the structure are not considered. Typically, this type of test is used to investigate the hysteretic or cyclic behavior of structural materials or components under earthquake loading [24].

The limitations on the shaking tables and quasi-static tests led to the development of pseudodynamic or hybrid tests. This was initially proposed by Hakuno et al. [28]. In these tests, systems are divided into two substructures: a numerical subsystem and a physical subsystem. The numerical subsystem usually corresponds to that of a structure whose dynamics are well known and in general are assumed to exhibit a linear behavior. The physical subsystem is, on the other hand, the critical component of the system and is usually a nonlinear device such as an MR damper. In this way, large- and full-scale experiments can be performed because the main structure is reduced to a numerical model solved in a computer or DSP and the critical components can be physically realized at a large or full scale provided reasonable space, energy and costs. Generally, hydraulic actuators are used to drive the physical specimens in the experiment.

One significant advantage of hybrid simulation is that it removes a large source of epistemic uncertainty compared with pure numerical simulations by replacing structural element models that are not well understood with physical specimens on the laboratory test floor [27]. There are two main drawbacks with the hybrid test method. First, the method relies on the assumption that the mass of the structure is concentrated at discrete points. Second, the loading is applied over a greatly expanded timescale so that time-dependent nonlinear behavior is not correctly reproduced in the physical component. In hybrid testing, the displacements are imposed on an extended timescale that typically ranges from 100 to 1000 times the actual earthquake duration to allow for the use of larger actuators without high hydraulic flow requirements, careful observation of the response of the structure during the test and the ability to pause and resume the experiment. In particular, the method cannot be applied to the testing of highly rate-sensitive components such as visco-elastic dampers [26,29].

RTHT is a variation of the pseudodynamic test method in which the imposed displacements and response analysis are executed in real time, thus allowing testing of systems with rate-dependent components. RTHT makes possible the testing of a large class of structural components associated with vibration control, including passive, semiactive and active control devices (e.g. base isolators and dampers), which are typically nonlinear and rate-dependent [24,30]. RTHT is challenging because it is necessary to perform all of the calculations, apply the displacements and measure and feedback the forces within a single time step (typically less than 10 ms). Because the test is conducted in real time, the dynamics of the testing system and specimen become important. For example, when hydraulic actuators are used to apply forces to the test specimen, they inevitably include a response delay, which is equivalent to negative damping in a real-time hybrid experiment [31].

The response delay is one of the main research focuses in RTHT. In general, it is frequency-dependent; therefore, its approximation as a pure time delay is valid only in a limited frequency range (e.g. low frequency). The effect of the dynamic response of hydraulic actuators on real-time hybrid experiments was initially considered by Horiuchi et al. [32], whose method based on
polynomial extrapolation is still widely used. Other solutions to this problem have been proposed by Horiuchi et al. [31], Nakashima and Masaoka [30], Darby et al. [26] and Jung and Shing [29], to name a few. Carrion and Spencer [24] proposed a method for RTHT that incorporates model-based compensation techniques for time delays and actuator dynamics, and combines fast hardware and software (for high-speed computations and communication) with high-performance hydraulic equipment, allowing accurate testing of systems with larger natural frequencies (e.g. stiff structures or multi-degree-of-freedom systems) and handling larger delays/lags, typically associated with actuators with high force capacity.

Pseudodynamic tests have also been extended for geographically distributed applications. This allows to run experiments with different substructures located in different places. Because the time required for network communication is relatively large, response prediction–correction methods are required to generate the actuator command signals in continuous or fast-rate tests. See, for example, the works in [27,33–35].

2.2. RTHT system

RTHT is particularly suitable for testing structures with rate-dependent devices. Recently, special attention has been directed toward the application of this technique to evaluate the response of structures with MR dampers. Real-time hybrid experiments of structures with MR dampers have already been proposed and/or conducted by Emmons and Christenson [36], Wu et al. [37] and Carrion and Spencer [24], among others. The system designed by Carrion and Spencer [24] is used in this study to implement and evaluate the performance of different semiactive controllers for vibration mitigation in a structure with an MR damper.

The RTHT system schematic used in this study is shown in Figure 1. A fully detailed description of this implementation can be found in [24].

The RTHT system of Figure 1 consists of a computer that simulates the structure to be controlled and generates the commanding signals (displacements and control signals); a small-scale MR damper that is driven by a hydraulic actuator, which in turn is controlled by a

Figure 1. RTHT system schematic.
servo-hydraulic controller; and DSP, A/D and D/A hardware for signal processing. Sensors available include a linear variable displacement transformer (LVDT) for displacement measurements and a load cell to measure the MR damper force. In Figure 1, \( x_{\text{cmd}} \) is the commanded displacement, \( f_{\text{mr}} \) is the MR damper force measured by the load cell, \( x_{\text{meas}} \) is the displacement measured by the LVDT and \( i \) is the control current sent to the hydraulic actuator.

2.3. Structure model

The structure to be controlled is a low-damping three-story building and its schematic is shown in Figure 2. The dynamic behavior of the structure can be modeled with the second-order motion equation:

\[
M_s \ddot{x} + C_s \dot{x} + K_s x = G_s f - M_s L_s \ddot{x}_g
\]  

(1)

where the matrices and vectors \( M_s, C_s, K_s, G_s \) and \( L_s \) are given by

\[
M_s = \begin{bmatrix}
m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{bmatrix} = \begin{bmatrix}
20,253 & 0 & 0 \\
0 & 20,253 & 0 \\
0 & 0 & 20,253
\end{bmatrix} \text{ kg}
\]

(2)

\[
C_s = \begin{bmatrix}
c_{11} & c_{12} & 0 \\
c_{21} & c_{22} & c_{23} \\
0 & c_{32} & c_{33}
\end{bmatrix} = \begin{bmatrix}
7,243.2 & -2,070 & 0 \\
-2,070 & 4,138.2 & -2,070 \\
0 & -2,070 & 2,070
\end{bmatrix} \text{ Ns/m}
\]

(3)
\[ K_s = \begin{bmatrix} k_{11} & k_{12} & 0 \\ k_{21} & k_{22} & k_{23} \\ 0 & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} 9932 & -5661 & 0 \\ -5661 & 11338 & -5661 \\ 0 & -5661 & 5661 \end{bmatrix} \text{N/m} \]  

(4)

\[ G_s = [-1, 0, 0]^T, \quad L_s = [1, 1, 1]^T \]  

(5)

\[ x \] is the vector of relative displacements, i.e. with respect to the ground, \( f \) is the MR damper force and \( \ddot{x}_g \) is the incoming earthquake acceleration. \( \ddot{x}_i \) is the absolute acceleration of the \( i \)th floor. Absolute displacement is measured with respect to an inertial frame; therefore, the relationship between relative and absolute displacements is \( x = x_a - x_g \). The natural frequencies and the damping ratios of the structure corresponding to the first, second and third modes are 1.09 Hz (0.31%), 3.17 Hz (0.62%) and 4.74 Hz (0.63%), respectively.

### 2.4. MR damper

The MR damper used in the experiments is the RD-1005 manufactured by the Lord Corporation (www.lord.com), which is shown in Figure 3. The damper is 216 mm long in its extended position, is 38.1 mm in diameter and has a stroke of 25.4 mm. It contains 50 ml of MR fluid and can generate forces up to 3000 N approximately. The magnetic field is generated by the current from a pulse width modulator (PWM) amplifier (the RD-1002 Wonder Box from Lord Corporation).

The dynamics of the damper can be modeled by the Bouc–Wen model [38] as shown in the following equations:

\[ f_{mr} = (c_0a + c_0b)(x_1^r) + (k_{0a} + k_{0b})(x_1^r) + (z_a + z_b)z \]  

(6)

\[ \dot{z} = -\gamma (\dot{x}_1^r) |z|^\eta - \beta \dot{x}_1^r |z|^\alpha + A \dot{x}_1^r \]  

(7)

where \( z \) is an evolutionary variable that describes the hysteretic behavior of the damper and \( u \) is the output of the first-order filter introduced to describe the time delay existing in the electronic circuit of the MR damper with a constant parameter \( \eta \):

\[ \dot{u} = -\eta (u - v) \]  

(8)
The parameters of the MR damper specimen are: \( a = 33.27 \text{ N/m}, \) \( b = 182.65 \text{ N/m V}, \) \( c_0 = 754.41 \text{ N s/m}, \) \( c_0b = 712.73 \text{ N s/m V}, \) \( k_0 = 1137.57 \text{ N/m}, \) \( k_0b = 1443.50 \text{ N/m V}, \) \( x_0 = 0 \text{ m}, \) \( \gamma = 4209.8 \text{ m}^{-2}, \) \( \beta = 4205.2 \text{ m}^{-2}, \) \( A = 10246, \) \( n = 2 \) and \( \eta = 57 \text{ s}^{-1}. \) The following scaling factors are used to integrate the physical small-scale MR damper to the numerical large-scale structure: the first-floor relative displacement is reduced by a factor \( S_L = 7.25 \) to obtain the damper piston displacement and the MR damper force is increased by a factor \( S_F = 60 \) to obtain the input force to the structure.

2.5. Hydraulic actuator dynamics

The MR damper is driven by a hydraulic actuator that receives a commanding signal from the computer where the simulation runs to impose a displacement to it. A block diagram that shows the interaction between the numerical model and the dynamic system is illustrated in Figure 4.

The entire physical system can be modeled by a transfer function \( G_{xu}(s) \) whose input \( u_c \) is the commanded displacement and the output \( x \) is the piston displacement. Modeling the system dynamics is useful for simulating the RTHT experiments. The transfer function \( G_{xu}(s) \) varies according to the MR damper input voltage. Two cases are identified corresponding to the damper operating at \( V_0 = 0 \text{ V} \) \( (G_{xu,0}(s)) \) and \( V_{\text{max}} = 5 \text{ V} \) \( (G_{xu,\text{max}}(s)) \), respectively. These transfer functions are given by

\[
G_{xu,0}(s) = \frac{1}{(0.0062s + 1)(2.639 \times 10^{-5}s^2 + 0.059s + 1)} \tag{9}
\]

\[
G_{xu,\text{max}}(s) = \frac{1}{(0.0094s + 1)(2.618 \times 10^{-5}s^2 + 0.058s + 1)} \tag{10}
\]

An algorithm was designed by Carrion and Spencer [24] to provide a smooth transition from \( G_{xu,0}(s) \) to \( G_{xu,\text{max}}(s) \) and vice versa when the damper voltage varies during the experiments. A block diagram of this algorithm is shown in Figure 5. The Laplace transform of the model is described by

\[
X(s) = X_a(s) + X_b(s)W(s) \tag{11}
\]

\[
X_a(s) = G_a(s)U_c(s) = G_{xu,0}(s)U_c(s) \tag{12}
\]

\[
X_b(s) = G_b(s)U_c(s) = (G_{xu,\text{max}}(s) - G_{xu,0}(s))U_c(s) \tag{13}
\]

\[
W(s) = G_t(s)V(s) \tag{14}
\]
where $G_t(s)$ is used to model the dynamics of the actuator associated with the change in the voltage of the MR damper, providing a smooth transition between $G_a(s)$ (Equation (12)) and $G_b(s)$ (Equation (13)), and is given by

$$G_t(s) = \frac{0.2}{\tau_t s + 1}$$

where $\tau_t = 0.0048\, s$ is the transition filter time constant. As the time constant becomes small, the transition becomes faster, approaching a simple switching algorithm, while for large values of the time constant the transition is slower and smoother. Owing to the inherent dynamics of the physical system (e.g. time delays), a pre-compensator $G_{ff}(s)$ is added to the system for compensation purposes. In this way, the commanded displacement ($u_c$, input to the physical system) is calculated based on the desired displacement ($d$, output from the simulations) and the inverse dynamics of the physical system. As a result, $x \approx d$. A schematic of the compensated system is shown in Figure 6.

Once again, two compensators are designed: one for the MR damper operating at $V_0 = 0\, V$ ($G_{ff,V_0}(s)$) and the other for the damper operating at $V_{\text{max}} = 5\, V$ ($G_{ff,V_{\text{max}}}(s)$). The transfer functions are given by

$$G_{ff,V_0}(s) = \frac{(0.062s + 1)(2.639 \times 10^{-5}s^2 + 0.059s + 1)}{(4.129 \times 10^{-4}s + 1)(1.173 \times 10^{-7}s^2 + 3.909 \times 10^{-4}s + 1)}$$

$$G_{ff,V_{\text{max}}}(s) = \frac{(0.0094s + 1)(2.618 \times 10^{-5}s^2 + 0.058s + 1)}{(6.289 \times 10^{-4}s + 1)(1.164 \times 10^{-7}s^2 + 3.857 \times 10^{-4}s + 1)}$$

A similar approach to that of Figure 5 is followed to provide a smooth transition between both compensators. The block diagram is shown in Figure 7 and the model is described by

$$U_{ff}(s) = U_a(s) + U_b(s)W(s)$$

$$U_a(s) = G_{ff,a}(s)D(s) = G_{ff,V_0}(s)D(s)$$

$$U_b(s) = G_{ff,b}(s)D(s) = (G_{ff,V_{\text{max}}}(s) - G_{ff,V_0}(s))D(s)$$

$$W(s) = G_f(s)V(s)$$

where $G_f(s)$ is used to provide a smooth transition between both compensators:

$$G_f(s) = \frac{0.2}{0.0048s + 1}$$
3. CONTROLLER FORMULATION

To begin with the design of the controllers, the structure model of Equation (1) is divided into two subsystems accounting for the first-floor dynamics, where the MR damper is attached, and the rest of the structure that accounts for the dynamics of all the floors above the base. Thus, the building can be modeled by the following set of equations:

\[ S_m : M_{s2} \ddot{x}_{23a} + C_{s2} \dot{x}_{23a} + K_{s2} x_{23a} = B_{s2} x_{1a} + F_g \]  

\[ S_b : m_{1} \ddot{x}_{1a} + c_{11} \dot{x}_{1a} + k_{11} x_{1a} = -f_{mr} - f_c + F_g \]

where \( S_m \) stands for the uncontrolled structure subsystem (the second and third floors) and \( S_b \) represents the controlled structure system (with the MR damper at the first floor). The \( a \) sub-index means absolute coordinates; \( x_{23a} = [x_{2a}, x_{3a}]^T \) is the absolute displacement vector of the two upper floors and \( x_{1a} = [x_{1a}, \dot{x}_{1a}]^T \) is a vector composed of the absolute displacement and the velocity of the first floor. \( F_g = [(c_1 + c_2 + c_3) \ddot{x}_g + (k_1 + k_2 + k_3)x_g, 0]^T \approx [0, 0]^T \). The matrices \( M_{s2}, C_{s2}, K_{s2} \) and \( B_{s2} \) are given by

\[
M_{s2} = \begin{bmatrix} m_2 & 0 \\ 0 & m_3 \end{bmatrix}, \quad C_{s2} = \begin{bmatrix} c_{22} & c_{23} \\ c_{32} & c_{33} \end{bmatrix}, \\
K_{s2} = \begin{bmatrix} k_{22} & k_{23} \\ k_{32} & k_{33} \end{bmatrix}, \quad B_{s2} = \begin{bmatrix} -k_{21} & -c_{21} \\ 0 & 0 \end{bmatrix}
\]
\( f_{mr} \) is the MR damper force as given in Equation (6), \( f_c \) is the coupling force between the base and the main structure and \( f_g \) is the force due to the seismic motion:

\[
f_c = c_{12} \dot{x}_{2a} + k_{12} x_a
\]

\[
f_g = (c_{11} + c_{12}) \ddot{x}_g + (k_{11} + k_{12}) x_g
\]

The following propositions about the intrinsic stability of the structure will be used in formulating some control laws [39]:

**Proposition 1**

The unforced main structure subsystem \( S_m \) is globally exponentially stable for any bounded initial conditions.

**Proposition 2**

If the coordinates \((x_{1a}, \dot{x}_{1a})\) of the base and the coupling term \(B_{12}x_{1a}\) are uniformly bounded, then the main structure subsystem is stable and the coordinates \((x_{2a}, \dot{x}_{2a})\) of the main structure are uniformly bounded for all \(t \geq 0\) and any bounded initial conditions.

In this way, the controllers designed for the first-floor subsystem will ensure the stability of the overall system.

### 3.1. Backstepping controller formulation

As the first step of the backstepping technique, Equation (24) is expressed in state space forms as follows:

\[
\dot{y}_1 = y_2
\]

\[
\dot{y}_2 = \left[ -\frac{k_{11}}{m_1} y_1 - \frac{c_{11}}{m_1} y_2 - \frac{1}{m_1} (f_{mr} + f_c - f_g) \right]
\]

where \(y_1 = x_{1a}\) is the first-floor absolute displacement and \(y_2 = \dot{x}_{1a}\) is the first-floor absolute velocity.

Next, the following standard backstepping variables are defined:

\[
e_1 = y_1, \quad e_2 = y_2 - z_1, \quad z_1 = -h_1 e_1
\]

where \(h_1\) is a positive constant. Now consider the following Lyapunov function candidate and its derivative, respectively:

\[
V = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2
\]

\[
\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2
\]

Substitution of Equations (28)–(30) into Equation (32) yields

\[
\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2
\]

\[
= e_1 y_1 + e_2 (\dot{y}_2 - \dot{z}_1) = e_1 y_1 + e_2 \dot{y}_2 + h_1 \dot{e}_1
\]

\[
= e_1 y_1 + e_2 \left[ -\frac{k_{11}}{m_1} y_1 - \frac{c_{11}}{m_1} y_2 - \frac{1}{m_1} (f_{mr} + f_c - f_g) \right] + h_1 y_2
\]

The objective is to make \(\dot{V}\) as negative as possible. In order to achieve this goal, the following control law is proposed for generating the MR damper force \(f_{mr}\):

\[
f_{mr} = (m_1 - k_{11} + h_1 h_2 m_1) y_1 + (h_1 m_1 - c_{11} + h_2 m_1) y_2 - f_c + f_g
\]

where \(h_2\) is another positive constant.
Substitution of Equation (34) into Equation (33) yields

\[ \dot{V} = -h_1 e_1^2 - h_2 e_2^2 < 0 \]  

(35)

According to Lyapunov’s stability theory, \( e_1 \to 0 \) and \( e_2 \to 0 \). Consequently, \( y = y_1 = e_1 \to 0 \) and \( \dot{y} = y_2 = e_2 + h_1 e_1 \to 0 \). According to Propositions 1 and 2, the vibration of the base is asymptotically attenuated and the asymptotic stability of the main structure is guaranteed.

The control law of Equation (34) cannot be implemented directly because the force to the MR damper cannot be commanded. Instead, a voltage signal must be sent to the damper to approximately generate the desired force. This can be achieved by substituting \( f_{mr} \) in the Bouc–Wen model of Equation (6) and solving for \( u \). Thus, the following control law is obtained for generating an implementable voltage signal to the MR damper:

\[
u = \frac{(m_1 - k_{11} + h_1 h_2 m_1) y_1 + (h_1 m_1 - c_{11} + h_2 m_1) y_2}{S_F[c_{0b} \dot{x}_{1r} / S_L + k_{0b} x_{1r} / S_L + z_b z]} - \frac{+f_c - f_g + S_F[c_{0a} \dot{x}_{1r} / S_L + k_{0a} x_{1r} / S_L + z_a z]}{S_F[c_{0b} \dot{x}_{1r} / S_L + k_{0b} x_{1r} / S_L + z_b z]} \]  

(36)

provided that \( S_F[c_{0b} \dot{x}_{1r} / S_L + k_{0b} x_{1r} / S_L + z_b z] \neq 0 \); otherwise, \( u = 0 \).

### 3.2. QFT controller formulation

QFT is a frequency control methodology based on the notion that feedback is only necessary when there are uncertainty and nonmeasurable disturbances actuating on the plant. Despite the fact that QFT was initially thought for linear time-invariant (LTI) systems, this methodology can be extended to nonlinear systems. As it has been stated throughout this paper, the MR damper is a nonlinear device. However, a representation of the damper as a linear plant with uncertain parameters that approximates the nonlinear dynamics can be made to apply the methodology.

The basic developments with QFT are focused on the control design problem for uncertain LTI systems as shown in Figure 8. In this figure, \( R \) represents the command input set and \( P \), the plant set. \( H(s) \) is the sensor transfer function and \( N(s) \) is the measurement noise. Let \( T(s) \) be the transfer function from \( Y(s) \) to \( R(s) \) and \( T \) the transfer function set. For each \( R(s) \in R, P(s) \in P \), the closed-loop output will be \( Y(s) = T(S)R(s) \) for some \( T(s) \in T \). For a large class of such problems, QFT is executable, i.e. a pair of controllers \( F(s) \) and \( G(s) \) can be found to guarantee that \( Y(s) = T(S)R(s) \). Suppose that the plant \( P(s) \) is an uncertain but known member of the set \( P \). The designer is free to choose the prefilter \( F(s) \) and the compensator \( G(s) \) in order to ensure that the system transfer function \( T(s) = F(s)P(s)G(s)/(1 + P(s)G(s)) \) satisfies that assigned specifications.

The uncertain plant model \( P(s) \) and its frequency and time domain specifications are represented in the Nichols chart through the use of the Horowitz–Sidi bounds. These bounds determine the regions where the nominal open-loop transfer function \( L_0(s) = G(s)P_0(s) \) may lie so that all the design specifications can be achieved.

The QFT methodology design can be summarized as follows [40]:

1. **Plant model, template generation and nominal plant selection:** The plant is represented in the Laplace domain; each uncertain parameter is assigned a range of variation and the
frequencies of interest are chosen within the expected operation range. At each frequency
of interest and for each possible value of the uncertain parameters, the plant model \( P(j\omega) \)
becomes a complex number that can be represented in the Nichols chart (dB, \( \Phi \)). This set
of complex numbers is called the templates.

2. **Design specifications**: The inputs to the system of Figure 8 are \( R(s) \) (the reference), \( W(s) \),
\( D_1(s) \) and \( D_2(s) \) (the disturbances) and \( N(s) \) (the noise). \( Y(s) \) is the variable to be
controlled, \( E(s) \) is the error and \( U(s) \) is the control signal. The following transfer functions
can be obtained:

3. 

\[
Y = \frac{1}{1 + PGH} D_2 + \frac{P}{1 + PGH} D_1 + \frac{PG}{1 + PGH} (W + FR) - \frac{PGH}{1 + PGH} N \tag{37}
\]

4. 

\[
U = \frac{G}{1 + PGH} (W + FR) - \frac{GH}{1 + PGH} (N + D_2 + D_1) \tag{38}
\]

5. 

\[
E = - \frac{H}{1 + PGH} D_2 + \frac{PH}{1 + PGH} D_1 + \frac{PGH}{1 + PGH} W + \frac{1}{1 + PGH} FR - \frac{H}{1 + PGH} N \tag{39}
\]

6. By limiting the transfer function magnitudes of Equations (37)–(39), it is possible to set
the stability and robustness specifications such as disturbance rejection, tracking and
noise rejection.

7. **Bound generation**: Once the nominal plant is chosen, the next step is to transform the
closed-loop specifications of uncertainty plants in a set of restriction curves or bounds
known as Horowitz–Sidi bounds for each frequency of interest on the Nichols chart. This
information synthesis allows the design of the controller using only the nominal plant.
For each frequency and for each design specification there is one bound. When all these
bounds are calculated, then the most restrictive bound per frequency is kept.

8. **Loop shaping**: When the most restrictive bounds are found, the controller is synthesized
by adding a gain, poles and zeros such that the loop function \( L_0(j\omega) \) lies in the Nichols
chart in the regions where the design specifications can be achieved. The optimal
controller is the one that has the minimum gain and lies on the bounds at each frequency
of interest. In this case, it is possible to affirm that the controller accomplishes all the
design specifications.
9. **Prefilter**: When tracking specifications are required, the prefilter $F(s)$ must be designed. The prefilter synthesis is similar to that of the controller.

10. **Design validation**: This step involves the performance evaluation of the controller and its adjustment until all the design specifications are satisfied within acceptable limits.

Most of the design process can be performed with the help of software packages such as the QFT toolbox for Matlab. The loop shaping process is left to the ability and experience of the designer.

To begin with the QFT controller design, Equation (24) is rewritten in relative coordinates:

$$m_1 \ddot{x}_1 + c_{11} \dot{x}_1 - c_{12} \dot{x}_2 + k_{11} x_1 - k_{12} x_2 = -f_{\text{mr}} - m_1 \ddot{x}_g$$

(40)

Taking the Laplace transform of Equation (40) yields

$$m_1 s^2 X_1(s) + c_{11} s X_1(s) - c_{12} s X_2(s) + k_{11} X_1(s) - k_{12} X_2(s) = -F_{\text{mr}}(s) - m_1 \ddot{X}_g(s)$$

(41)

Rearranging terms from Equation (41) yields

$$X_1(s) = \frac{1}{m_1 s^2 + c_{11} s + k_{11}} [-F_{\text{mr}}(s) + c_{12} s X_2(s) + k_{12} X_2(s) - m_1 \ddot{X}_g(s)]$$

(42)

In order to apply the QFT methodology in this problem with the MR damper, an approximation to an uncertain linear plant is proposed. Consider again the Bouc–Wen model of the MR damper of Equation (6). It can be decomposed into two parts: one linear and another nonlinear.

Thus,

$$f_{\text{lin}} = (c_{0a} + c_{0b} u_0) \dot{x} + (k_{0a} + k_{0b} u_0) x = a_1 \dot{x} + a_2 x$$

(43)

$$f_{\text{nonlin}} = (z_a + z_b u) z_0 = u_d z_0$$

(44)

$$f_{\text{mr}} = f_{\text{lin}} + f_{\text{nonlin}}$$

(45)

$$u_d = (z_a + z_b u)$$

(46)

From Equation (43), it is observed that the parameters $a_1$ and $a_2$ vary only with the input voltage. The third parameter, $z_0$, in Equation (44) is a bounded parameter. See Figure 9.

![Figure 9. Example of a hysteresis loop.](image-url)
velocities, \( z \) is approximately constant and, thus, \( z_0 \) could take either the maximum or the minimum value depending on the signs of velocity. In this way, Equations (43) and (44) can be seen as a plant with three uncertain parameters, namely, \( a_1, a_2 \) and \( z_0 \), which describe the dynamics of the damper. In this way, the damper dynamics appear to follow the Bingham model [41]. Figure 10 illustrates this approach with a sinusoidal displacement excitation at three levels of voltage.

The representation of the MR damper as an uncertain linear plant can now be incorporated into Equation (41). The Laplace transform of Equations (43)–(46) yields

\[
F_{mr}(s) = \frac{a_1 S_F}{S_L} s X_{1r}(s) + \frac{a_2 S_F}{S_L} X_{1r}(s) + S_F z_0 U_D(s)
\]

(47)

Substitution of Equation (47) into Equation (41) yields

\[
X_{1r}(s) = \frac{-S_F z_0 U_D(s)}{m_1 s^2 + (c_{11} + a_1 S_F/S_L) s + (k_{11} + a_2 S_F/S_L)} + \frac{c_{12} s X_{2r}(s) + k_{12} X_{2s}(s) - m_1 \ddot{x}_g(s)}{m_1 s^2 + (c_{11} + a_1 S_F/S_L) s + (k_{11} + a_2 S_F/S_L)}
\]

(48)

Therefore, the plant \( P_1(s) \) \((P(s) \text{ in Figure 8})\) is given by

\[
P_1(s) = \frac{S_F z_0}{m_1 s^2 + (c_{11} + a_1 S_F/S_L) s + (k_{11} + a_2 S_F/S_L)}
\]

(49)

and the voltage can be estimated by manipulating the following equation:

\[
U_i(s) = -U_D(s) + \frac{1}{S_F z_0} (c_{12} s X_{2r}(s) + k_{12} X_{2s}(s))
\]

(50)

The uncertain parameters and QFT controller specifications are: \( a_1 = [754.41, 4318.06] \text{ Ns/m, } \)
\( a_2 = [1137.57, 6855.07] \text{ N/m and } z_0 \text{ has the values } -1.11 \text{ or } 1.11 \text{ m, respectively. The frequencies of interest are the natural frequencies of the system, i.e. 1.09, 3.17 and 4.74 Hz. The controller performance should accomplish the following bounds: robust performance } W_{s1} = 2 \text{ and disturbance rejection } W_{s3} = 3 \times 10^{-2}. \)
Figures 11–13 depict different stages of the controller design: templates and initial approach ($G_1(s) = 0$) and final loop. Figure 14 shows the analysis of the closed-loop response for the robust performance and disturbance rejection problems within the range of frequencies studied. The final controller $G_1(s)$ with a displacement input measured in meters and output $U_i(s)$ (measured in Newton meter) is given by

$$G_1(s) = \frac{298(0.016s^2 + 0.073s + 1)(7.3 \times -4s^2 + 0.051s + 1)(4.7 \times -3s^2 + 2.15 \times -3s + 1)}{(0.017s + 1)(0.033s + 1)(0.015s^2 + 0.095s + 1)(4.65 \times -3s^2 + 0.060s + 1)}$$

(51)

It can be seen from Figure 13 that the closed-loop response (in black solid curve) lies above the bounds at the frequencies of interest and does not cross the ray $(0, -180^\circ)$. Thus, according
to the QFT, the closed-loop system using the designed controller is stable even in the presence of uncertainties. Furthermore, Figure 14 shows that the closed-loop system performs well under the disturbance rejection and robust performance constraints.

Figure 13. QFT controller final loop.

Figure 14. QFT controller closed-loop analysis: upper—robust performance and lower—disturbance rejection.
4. EXPERIMENTAL RESULTS

The numerical models corresponding to the three-story building, the compensator, the MR damper, the hydraulic actuator and the controller were implemented in Matlab/Simulink. The ordinary differential equation solver used is the fourth-order Runge–Kutta method with a time step $T_s = 5 \times 10^{-4}$ s. The experiments began by evaluating the RTHT system performance. Figure 15 shows the performance of the compensator. This figure illustrates a comparison between the desired, the commanded and the measured piston displacements during the execution of an experiment. The lower curve is a close-up of the upper curve.

On the other hand, Figures 16 and 17 show a comparison between the experimental dynamics of the MR damper and those predicted by the Bouc–Wen model. In the first case, the damper is subject to a sinusoidal displacement at 5 Hz and 0.254 cm amplitude. The voltage periodically switches from 0 to 5 V. In the second case, the damper is subject to a random displacement and a random voltage excitation.

Figure 18 compares the MR damper piston displacement as measured during an experiment and that obtained by the model of the overall system. That is, the system of Figure 6 was implemented in Simulink and simulated and the results compared with the experimental response. To make this comparison, the El Centro seismic motion records and the MR damper voltage were taken as inputs to the RTHT system. The results show good accuracy of the system model.

To evaluate the controller performance, the structure is subject to three different earthquake records, namely, El Centro, Loma Prieta and Northridge as shown in Figure 19; the scale amplitude used is 0.4. Table I gives the performance indices used to evaluate the control performance.

The backstepping controller is implemented with $h_1 = 1 \times 10^{-3}$ and $h_2 = 1 \times 10^{-6}$. The performance indices of both controllers for the different seismic excitations are given in

![Figure 15](image-url)
Figure 16. Time behavior of the MR damper characteristics: force response to sinusoidal displacement and switching voltage inputs.

Figure 17. Time behavior of the MR damper characteristics: force response to random displacement and voltage inputs.
Figure 18. Comparison between the experimental and the model displacement responses of the RTHT system.

Figure 19. Records of El Centro, Loma Prieta and Northridge earthquakes.
Table II. Figures 20–23 show the structure response and the MR damper performance when subject to some seismic excitations. Figures 21 and 23 show the performance of the MR damper (the actual damper, i.e. not scaled) and, particularly, a comparison of the dynamics predicted by the Bouc–Wen model and that obtained experimentally.

In the RTHT experiments, the control program was run first and the incoming earthquake motion was added to show when it actually began during the experiments. It is observed from the response figures that the controllers can achieve the rapid (although not immediate) reduction of the seismically excited response mainly due to the effects of asymptotic stabilization.

Analysis of indices $J_1$–$J_4$ shows that both controllers are able to reduce the structure response when subject to a seismic motion. It can also be observed that the backstepping controller performs better than the QFT controller in most cases. However, by analyzing indices $J_5$ and $J_6$, it can be seen that the QFT controller makes use of a larger control effort to achieve the response reduction. It may be caused by the conservativeness around the controller design when assigning the uncertainty range of the parameters.
dynamic compensator, which had been designed in previous studies to account for the time lags and delays of the system components. The overall configuration had also been modeled in order to design the compensator. This model is also useful to simulate the experiments with high accuracy as was shown in this paper.

Figure 20. Model-based backstepping: structure response under El Centro earthquake.

Figure 21. Model-based backstepping: MR damper response under El Centro earthquake.
The backstepping controller design has been performed in the time domain with the main objective of reducing the structural time response, while the QFT controller design has been carried out in the frequency domain through the loop shaping of the system response at the

Figure 22. Model-based QFT: structure response under El Centro earthquake.

Figure 23. Model-based QFT: MR damper response under El Centro earthquake.

The backstepping controller design has been performed in the time domain with the main objective of reducing the structural time response, while the QFT controller design has been carried out in the frequency domain through the loop shaping of the system response at the
natural frequencies by taking into account the bound information on uncertain parameters and external disturbance. The proposed controllers were experimentally tested in a novel RTHT setup at the SSTL. Both controllers successfully achieved the goal of reducing the structure response when subject to a seismic motion.

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