Extended State Observer Based Adaptive Control Scheme for PMSM System

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Abstract: In this paper, the speed regulation problem for permanent magnet synchronous motor (PMSM) systems under vector control framework is studied. A modified composite control strategy combining model reference adaptive control (MRAC) method and extended state observer (ESO) technology, called MRAC+ESO method, is proposed. Controller is designed for the speed loop of the permanent magnet synchronous motor (PMSM) to improve the performance of the system. The stability analysis, simulation and experimental results are presented to show the effectiveness of the proposed control method.

Key Words: Model reference adaptive control(MRAC), Permanent magnet synchronous motor(PMSM), Speed regulation system, Extended state observer(ESO), Composite controller

1 Introduction

The permanent magnet synchronous motor (PMSM) has been widely applied in robotic system, electric vehicle system and some other motion control systems for its excellent features such as simple structure, high efficiency, high power density and friendly maintenance. However, the PMSM is a nonlinear system with states coupling and parameters varying. Therefore, it is difficult for conventional linear control methods, including the proportional-integral (PI), to provide a high precision performance [1].

To enhance the control performance, many advanced control methods have been introduced to PMSM servo systems, e.g., adaptive control [2]-[5], disturbance estimation based control [6], slide mode control [7], predictive control [8]-[9], fuzzy control [10], neural network control [11], etc. These methods have improved the performance of PMSM servo systems in different ways.

Among these methods, the adaptive control method has been widely used in the situations where the system parameters are inaccurately known or the system is operated over a wide range of different operating conditions. In these situations the conventional controller with fixed gains may not provide a satisfactory performance. One adaptive concept is to identify the parameters of the unknown system and use these parameters to choose suitable control gains. This approach is easy to be understood and implemented, but it needs to identify the parameters online which may not be allowed in some situations. For example, in many application cases, the PMSM system may not be allowed to add a sufficiently exciting periodic input signal for the online inertia identification [2].

Another concept is the model reference adaptive control (MRAC) approach. It employs a reference model to generate a reference output. The adaptive laws derived by means of the Lyapunov stability theory modify the parameters of the controller without the necessity of a sufficiently exciting system input signal [12]. Many cheering research reports on the applications of the MRAC method in the motion control system have been done. In [13], MRAC technique is employed for velocity and currents loops of induction motor control systems. In [14], MRAC algorithm for robust control of PMSM is proposed and applied to the three control loops including a speed loop and two current loops. [15] and [16] combine MRAC method and fuzzy control method, MRAC method and variable structure control method in the speed loop to improve the performance of DC motor system and induction motor system, respectively.

In industrial situations, disturbances, e.g., friction force, load disturbances and unmodeled dynamics are unavoidable. Conventional control methods may not react directly and fast to reject these disturbances. In this case, many control methods based on feedforward compensation techniques for disturbances have been proposed and show an effective way to suppress the influences caused by the disturbances [17], [18]. Extended state observer (ESO) is one of the most useful observers in the disturbance observer field. To improve disturbance rejection ability, we propose a composite speed control method using MRAC method and disturbance estimation technique for the speed loop of a PMSM servo system. Simulation and experimental comparison results of the proposed method and PI method are also presented.

This paper is organized as follows. Section 2 introduces the PMSM model. The MRAC+ESO speed control scheme is proposed in Section 3. Section 4 shows the simulation and experimental results of the proposed composite control strategy. A conclusion is given in Section 5.

2 The mathematical model of PMSM

The model of the surface mounted permanent magnet synchronous motor is expressed as follows [19]:

\[
\begin{pmatrix}
i_d \\
i_q \\
\omega
\end{pmatrix} = \begin{pmatrix}
\frac{-R_p}{L_p} & -\frac{\pi_0}{L_p} & \frac{0}{L_p} \\
-\frac{\pi_0}{L_p} & \frac{R_p}{L_p} & -\frac{\pi_x}{L_p} \\
0 & \frac{\pi_x}{L_p} & \frac{-B}{L_p}
\end{pmatrix}
\begin{pmatrix}
i_d \\
i_q \\
\omega
\end{pmatrix} + \begin{pmatrix}
\frac{u_d}{L_p} \\
\frac{u_q}{L_p} \\
\frac{-\pi_y}{L_p}
\end{pmatrix}
\]

(1)

where \(R_p\) is the stator resistance, \(i_d\) and \(i_q\) are the \(d\) and \(q\) axes stator currents, \(u_d\) and \(u_q\) are the \(d\) and \(q\) axes stator...
voltages, \( n_p \) is the pole pairs number, \( L \) is the stator inductance, \( \omega \) is the rotor angular velocity, \( \psi_f \) is the flux linkage, \( K_t \) is the torque constant, \( J \) is the rotor inertia, \( B \) is the viscous friction coefficient, and \( T_L \) is the load torque.

The design procedure is based on the framework of vector control, where a structure of cascade control loop including a speed tracking loop and two current tracking loops is employed. Here we adopt two PI controllers in two current control, where a structure of cascade control loop included by clarke and park transformation. The reference current \( i^*_d \) can be calculated from \( \omega \) and \( i^*_q \) by clarke and park transformation. The reference current \( i^*_q \) is determined by the speed loop controller. In this paper, we concentrate on the design of the speed loop controller.

\[
\dot{\omega} = -\frac{K_t}{J} i_q - \frac{B \omega}{J} \frac{T_L}{J} - K_t \left( i^*_q - i_q \right) = -\frac{B \omega}{J} + \frac{K_t}{J} i^*_q + d(t) - a \omega + b i^*_q + d(t)
\]

(2)

where \( a = \frac{B}{J}, \ b = \frac{K_t}{J} \), \( d(t) = -\frac{T_L}{J} - \frac{K_t}{J} \left( i^*_q - i_q \right) \) can be considered as the lumped disturbances including the load torque disturbance and the tracking error of \( q \) axis current loop.

3 Controller strategy

3.1 MRAC controller for PMSM

The MRAC structure is shown in Fig. 2, where \( G_m(s) \) is the reference model, and \( \omega^* \) is the reference speed. In Fig. 2, the “Generalized PMSM” represents the two current loops that include the PMSM and other components [2].

The reference model is chosen as:

\[
\dot{\omega}_m = -a_m \omega_m + b_m \omega^*
\]

(3)

where \( a_m > 0, b_m > 0 \) are the parameters of the reference model.

Suppose the control law is described as:

\[
i^*_q = h(t) \omega + k(t) \omega^*
\]

(4)

where \( k(t) \) is the variable feedforward gain and \( h(t) \) is the variable feedback gain.

Fig. 2: Block diagram of MRAC method for PMSM system

Substituting (4) into (2), yields

\[
\dot{\omega} = -(a - bh(t)) \omega + bk(t) \omega^*
\]

(5)

Here, define the speed tracking error and parameter error as follows:

\[
e = \omega_m - \omega
\]

(6)

\[
\phi = \begin{bmatrix} k^* - k(t) \\ h^* - h(t) \end{bmatrix}
\]

(7)

where \( k^* = \frac{b_m}{b} \), \( h^* = \frac{a_m - a_m}{b_m} \).

Then, differentiating (6) along system (3) and (5) yields

\[
\dot{e} = -a_m \omega_m + b_m \omega + a \omega - bh(t) \omega - bk(t) \omega = -a_m e + b \phi^T [\omega^* \omega] \phi
\]

(8)

Considering the Lyapunov function:

\[
V = \frac{1}{2} e^2 + \frac{b}{2} \phi^T \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix} \phi
\]

(9)

where \( \gamma_1 > 0, \gamma_2 > 0 \).

The differentiation of (9) along the trajectory of (8) yields

\[
\dot{V} = e \dot{e} + b \phi^T \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix} \phi
\]

\[
= -a_m e^2 + b \phi^T \begin{bmatrix} \omega^* & \omega \end{bmatrix} \dot{\phi}
\]

\[
= -a_m e^2 + b \phi^T \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix} \dot{h}(t)
\]

(10)

Choose

\[
\begin{bmatrix} k(t) \\ h(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{\gamma_1} & \frac{1}{\gamma_2} \end{bmatrix} \begin{bmatrix} e \omega^* + k^* - k(t) \\ h^* - h(t) \end{bmatrix}
\]

(11)

Then it can be obtained that

\[
\dot{V} = -a_m e^2 - b |\phi|^2 \leq 0
\]

(12)

Therefore, according to the Lyapunov stability theorem, it can be concluded that the close loop system is asymptotically stable.
3.2 MRAC+ESO composite controller design for PMSM

In this paper, an ESO is added to the MRAC controller. It regards the disturbances \( d(t) \) as a new state of system and estimates both the states and the disturbances. The detail principle of ESO can be found in [20] and [21].

According to (2), define \( x_1 = \omega, \ x_2 = d(t) \), then (2) can be written as

\[
\begin{cases}
\dot{x}_1 = x_2 - ax_1 + bi_q^* \\
\dot{x}_2 = c(t)
\end{cases}
\]

(13)

where \( c(t) \) is the derivative of \( d(t) \).

Then, a second-order linear ESO for system (13) is designed as follows:

\[
\begin{cases}
\dot{z}_1 = z_2 - 2p (z_1 - x_1) + bi_q^* \\
\dot{z}_2 = -p^2 (z_1 - x_1)
\end{cases}
\]

(14)

where \( z_1 \) is an estimation of speed \( x_1 \), and \( z_2 \) is an estimation of \( x_2 \), \(-p^2\) is the desired double pole of ESO with \( p > 0 \).

The block diagram of the composite control method is shown in Fig. 3. It can be observed that a MRAC controller and an ESO are employed to construct the composite MRAC+ESO structure. Under this control method, the composite control form is

\[
u = i_q^* - \hat{d}(t)
\]

(15)

![Fig. 3: Block diagram of MRAC+ESO method for PMSM system](image)

3.3 Stability analysis

**Assumption 1.** The disturbance \( d(t) \) is bounded, and it satisfies \( \lim_{t \to \infty} d(t) = 0 \).

**Lemma 1.** Let \( V: [0, \infty) \times \mathbb{R}^n \to \mathbb{R} \) be a continuously differentiable function such that

\[
\begin{aligned}
\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}(t, x, u) &\leq -W_3(x), \forall \ |x| \geq \rho(\|u\|) > 0 \\
\forall (t, x, u) &\in [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m, \text{ where } \alpha_1, \alpha_2 \text{ are class } K_\infty \text{ functions, } \rho \text{ is a class } K \text{ function, and } W_3(x) \text{ is a continuous positive definite function on } \mathbb{R}^n. \text{ Then, the system } \dot{x} = f(t, x, u), x \in \mathbb{R}^n, u \in \mathbb{R}^m \text{ is input-to-state stable with } \gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho.
\end{aligned}
\]

**Lemma 2.** If the following system

\[
\dot{x} = f(t, x, u)
\]

(16)
satisfies: 1. system (16) is globally input-state stable; 2. \( \lim_{t \to \infty} u = 0 \), then the states of system (16) are asymptotically convergent to zero.

By substituting (15) into (2) and combining with (3), the speed tracking error is given by

\[
\dot{\hat{\omega}} = -a_0 \omega m + b_0 \omega^* + a \omega - bh(t) \omega - bk(t) \omega - b \left[ d(t) - \hat{d}(t) \right]
\]

(17)

For system (17), choose Lyapunov function as

\[
V = \frac{1}{2} e^2 + \frac{b}{2} \phi^T \left[ \gamma_1 \ \gamma_2 \right] \phi
\]

(18)

where \( \gamma_1 > 0, \gamma_2 > 0 \).

The differentiation of (18) along the trajectory of (17) yields

\[
\dot{V} \leq -a_0 \omega m^2 - be \left[ d(t) - \hat{d}(t) \right] - b \| \phi \|^2
\]

(19)

(1) If \( b \geq a_0, \) then (19) can be written as

\[
\dot{V} \leq -a_0 m \left( |e|^2 + \| \phi \|^2 \right) - a_0 m \left( |e|^2 + \| \phi \|^2 \right) - (b - a_0) m \| \phi \|^2 + b |e|^2 + \| \phi \|^2 \| d(t) - \hat{d}(t) \| \leq 0
\]

(20)

Supposing that

\[
-a_0 \left( |e|^2 + \| \phi \|^2 \right) + b |e|^2 + \| \phi \|^2 \| d(t) - \hat{d}(t) \| \leq 0
\]

(21)

That is to say

\[
|e|^2 + \| \phi \|^2 \| d(t) - \hat{d}(t) \| \leq 0
\]

(22)

In this case, it can be seen that

\[
\dot{V} \leq -a_0 m \left( |e|^2 + \| \phi \|^2 \right) - (b - a_0) \| \phi \|^2
\]

(23)

(2) If \( b < a_0, \) then (19) can also be written as

\[
\dot{V} \leq -b \left( |e|^2 + \| \phi \|^2 \right) - b \left( |e|^2 + \| \phi \|^2 \right) - (a_0 - b) |e|^2 + b |e|^2 + \| \phi \|^2 \| d(t) - \hat{d}(t) \| \leq 0
\]

(24)

Supposing that

\[
-b \left( |e|^2 + \| \phi \|^2 \right) + b |e|^2 + \| \phi \|^2 \| d(t) - \hat{d}(t) \| \leq 0
\]

(25)

That is to say

\[
|e|^2 + \| \phi \|^2 \| d(t) - \hat{d}(t) \| \leq 0
\]

(26)

Then it can be seen that

\[
\dot{V} \leq -b \left( |e|^2 + \| \phi \|^2 \right) - (a_0 - b) |e|^2
\]

(27)
From the two cases analysing above, by Lemma 1 it can be obtain that system (17) is input-state stable.

Consider the situations where the lumped disturbances \( d(t) \) satisfy Assumption 1. According to the analysis in [22]-[23], the ESO states \( z_1(t) \rightarrow x_1(t) \) and \( z_2(t) \rightarrow x_2(t) \). Then the estimation of the disturbance \( \hat{d}(t) = z_2(t) \) satisfies:

\[
\lim_{t \to \infty} \left[ d(t) - \hat{d}(t) \right] = 0 \quad (28)
\]

Treat \( d(t) - \hat{d}(t) \) as the system input, then together with (28) and Lemma 2, and it can be concluded that the close loop system is asymptotically stable.

4 Simulation and experimental results

To demonstrate the efficiency of the MRAC+ESO method, some simulation and experiments on the PMSM servo system have been done. Two methods, i.e., MRAC+ESO and PI, are both tested on the PMSM system.

The parameters of the PMSM model used in the simulation and experiments are given as: stator resistance \( R_s = 1.74 \Omega \), stator inductances \( L_d = L_q = L = 0.004 H \), number of pole pairs \( n_p = 4 \), moment of inertia \( J = 1.78 \times 10^{-4} Kg \cdot m^2 \), rotor flux \( \phi_f = 0.402 Wb \), viscous coefficient \( B = 7.4 \times 10^{-5} N \cdot m \cdot s/\text{rad} \). The PI parameters of both current loops are: \( K_p = 42, K_i = 2600 \). The saturation limit of \( q \)-axis reference current is \( \pm 9.42 A \).

4.1 Simulation result

In the simulation, the parameters for PI speed controller are: \( K_p = 0.2, K_i = 40 \), for MRAC+ESO speed controller are: \( a_m = b_{m} = 100, \gamma_1 = \gamma_2 = 0.015, p = 450 \). Fig. 4 (a) and (b) show the response curves of MRAC+ESO based controller and PI controller in the case of 1000 rpm reference speed. A load torque \( T_L = 2N \cdot m \) is applied at \( t = 0.6s \), the speed and \( i_q^* \) response curves are also given in Fig. 4 (c) and (d). Compared with PI controller, MRAC+ESO based controller has a smaller overshoot and less speed decrease amplitude.

4.2 Experimental result

In the experiments, the parameters for PI speed controller are selected as: \( K_p = 340, K_i = 22 \). The parameters of MRAC+ESO speed controller are selected as: \( a_m = b_{m} = 100, \gamma_1 = \gamma_2 = 1, p = 250 \). The speed responses under MRAC+ESO and PI method when the reference speed is 1000 rpm and 2000 rpm are shown in Fig. 5 and Fig. 6, respectively. Tests also have been done to evaluate the disturbance rejection ability of MRAC+ESO method when a sudden load torque is added suddenly and removed after some duration. The results are shown in Fig. 7 and Fig. 8. Compared with the PI method, the proposed MRAC+ESO method has smaller speed decrease and shorter recovery time while maintaining a good dynamic performance at the same time.

Then the inertia \( J \) of the PMSM system is increased to \( 11J \). Experiments are performed to test the adaptability of the MRAC+ESO method. The results are shown in Fig. 9 and Fig. 10.

From all the experimental results above, it can be observed that the MRAC+ESO control strategy can obtain a better performance of speed tracking, disturbance rejection and adaptation ability compared with the conventional PI method.

5 Conclusions

In this paper, a composite speed controller based on MRAC method and ESO method has been proposed. Simulation and experimental results have validated that the composite method can improve the abilities of speed tracking, disturbance rejection and adaptation of the PMSM servo system.
Fig. 6: Experimental response curves when the reference speed is 2000 rpm: (a) speed and (b) $i^*_q$

Fig. 7: Experimental response curves in the presence of sudden load torque disturbance in the case of 1000 rpm: (a) speed (PI). (b) $i^*_q$ (PI). (c) speed (MRAC+ESO). (d) $i^*_q$ (MRAC+ESO)

Fig. 8: Experimental response curves in the presence of sudden load torque disturbance in the case of 2000 rpm: (a) speed (PI). (b) $i^*_q$ (PI). (c) speed (MRAC+ESO). (d) $i^*_q$ (MRAC+ESO)

Fig. 9: Experimental response curves when $J$ is increased to $11J$ and reference speed is 1000 rpm: (a) speed and (b) $i^*_q$

References


Fig. 10: Experimental response curves when $J$ is increased to 11 and reference speed is 2000 rpm: (a) speed and (b) $i_q^*$.