Centrifuge experimental study of thaw settlement characteristics of mucky clay after artificial ground freezing

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ABSTRACT

Artificial ground freezing (AGF) plays a significant role in the construction of subway tunnel cross passages, underground commercial streets and underground pumping stations, etc., especially under adverse hydro-geological conditions. Associated with the properties of high water content and large void ratio in saturated Shanghai mucky clay, large thaw settlement has arisen after AGF construction. The thawing response is much more complicated compared to freezing and less research work has been documented in the AGF method partly due to the long-period and variability in the redistribution of water released from melting ice crystals and the rewetting of the soil aggregates in clay. This paper presented a series of centrifuge model tests to derive the thawing function development and predict the thaw settlement. The results show that the thawing rate was distinct and larger than the conventional results of small sample tests without stress acting. The thawing front advanced much faster compared to the development in natural frozen soil. An improved gray prediction model emphasizes that a simple analytical result with new thaw front function is much consistent with the experimental and prediction results. All the discussions demonstrate that small scale specimen thaw test at 1 g is not capable to directly apply into full scale prediction of the thaw settlement in thick soft saturated clay after artificial ground freezing construction, in which self-weight dominates the process of consolidation. This could motivate, in the near future of computation model development, that the large strain thaw consolidation theory should be applied to prognosticate the long-term settlement of thick soft mucky clay after artificial ground freezing construction.

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1. Introduction

Artificial ground freezing (AGF) is used as both structural support system and a water barrier for tunnel construction. For practical purposes it eliminates the need for sheeting of the earth, site dewatering, soil stabilization, or concern for the movement of the adjacent ground. The mechanical properties of frozen ground are more dependent on time and temperature than on the geology of strata. Hence ground freezing may be used in any soil or rock formation but it has best advantages in soft clays, loose sand, and gravel soils with high ground water or high water pressure level (Andersland and Ladanyi, 2004). That is to say AGF is less sensitive to the geologic conditions than other alternative construction methods, such as dewatering and grouting while the cost of AGF is usually much higher if the project size is huge. Comprehensive compared, AGF technology is much more used in the short tunnel of the cross passage connecting subway tunnels under locally adverse hydro-geological condition of high groundwater level, high water content, large void ration and high compressibility etc., in saturated Shanghai soft clay (Dassargues et al., 1991; Xu et al., 2009), without other alternation. Frost heave and subsequent thaw-induced settlement are the two main soil responses, directly relevant to the AGF method. Clays with higher plasticity are generally less susceptible to frost heave (Andersland and Ladanyi, 2004) but more additional settlement will be resulted during thaw. On thawing, the ice will disappear, and for existing overburden pressures the soil skeleton must now adapt itself to a new equilibrium void ratio (generally smaller after thaw consolidation). In fine-grained soil (especially in clay), slow freezing permits local ice segregation and moisture migration, even for closed drainage conditions. The resulted overburden pressure beneath the freezing front has a negative effect for frost heave but this over-consolidation increases the thaw deformation. Moreover, freeze–thaw has a softening effect on soil structure with denser pore structure, which aggravates the long-term settlement of the soil after the completion of thawing during the subway operation. The resulting post-thaw settlement is a very significant factor that should be considered for the adjacent construction and environmental conditions after artificial ground freezing in soft soil area. More importantly, frost heave as a soil response to freezing is much more immediate and visible compared to thaw settlement in the AGF construction. During artificial ground freezing in the subway cross passage, the freezing process is more likely
under control compared to thawing, it stands to reason that all the excavation work can only be started after the frozen wall is completely achieved, which is also the core control technology and mainly concerned aspect in this method. While usually the onset of subways operation will not wait to resume until the surrounding soils are fully thawed (it even takes many years in complete natural thawing), i.e. subways advance during the thawing process, especially in those sites where naturally thawing can only be applied.

Frost heave in soils was paid attention over decades, such as sands, silts and clays from theoretical, experimental modeling to construction application (Colombo et al., 2008; Han and Goodings, 2006; Lackner et al., 2005, 2008; Pimentel et al., 2012; Viggiani and De Sanctis, 2009), in which the frost heave is greatest in silts with identical or somewhat less subsequent thaw settlement, however, the situation is different in clayey soils (Han, 2005). Even though Russo et al. (2012) was concerned in the thaw settlement in geotechnical aspects, the thaw duration was usually short in cohesionless granular deposits and it was easily controlled by underpinning. Casini et al. (2013, 2014) presented a coupled thermo-hydraulic-mechanical model from a theoretical aspect and then validated and calibrated by laboratory tests on natural samples of volcanic ash, but it was also in a granular material and mainly focused on the freezing process. Except recently some thaw-induced slope instability publications (Bommer et al., 2012; Harris et al., 2008), relevant research for thawing in clayey soil is still not so much. As for mucky clay with low permeability, local ice segregation formed by groundwater migration drawn by pore-water suction during freezing may be not as much as silts. Nonetheless, the prediction of thaw settlement after AGF construction in site is necessary and of great significance in practice. While cold region practical experience in many areas is still quite limited and field tests are often prohibitive if the time scale is very long in the study, centrifuge modeling is an attractive proposition with its scale effects. The fundamental scaling laws pertinent to cold regions, which may not be totally identical to ‘conventional’ geotechnical considerations due to heat transfer and moisture migration (Taylor, 2005), actually have been verified by several researchers, but just focused on the freezing problem (Clack and Philips, 2003; Han and Goodings, 2006; Yang, 1997). As for thawing problem, rare publications can be found in thaw settlement by centrifuge modeling.

Thus, to figure out the specific thaw response of Shanghai mucky clay during the artificial ground freezing construction, several centrifuge modeling experiments were conducted to visualize, discuss and derive the freeze–thaw front development. An improved gray model was applied to predict the final thaw settlement based on the centrifuge modeling results.

2. Materials and methods

2.1. Model soil

To simulate the cross passage construction site in AGF, the soil in this research program was all retrieved from the cross passage construction field at the depth of around 12m beneath the soil surface (due to the limitations of the model box size (900 mm × 550 mm × 700 mm) model scales were set around 1:50). The soil used in this research was gray mucky clayey soil in the fourth layer of Quaternary coastal-shallow sea deposition in Shanghai (Q4). It is obtained from the foundation pit in South Sungkiang Station of Line 9 of Shanghai Metro. The geotechnical and thermal properties for the gray mucky clay are provided in Table 1.

The model soil was prepared from the slurries with initial water contents of 68% and 75%. All these processed slurries were molded from the field soils and provided uniform and repeatable experimental model soil samples. They consolidated under practical self-weight condition by 50 g centrifuge acceleration. Since the slurries were placed in the model box layer by layer, the model soil should be kept in 1 g dead load for 1 to 2 days to ensure the homogeneity of moisture before being conducted in the centrifuge for consolidation. The completion of consolidation was estimated through the stable value of buried pore pressure transducer. Generally, the self-weight consolidation can result in at least 90% of self-weight consolidation settlement. Before freezing, the water contents of model soils were measured to be around 47%–50% with no detectable variation along the depth. It was similar to the field condition after complete consolidation.

2.2. Experimental system

The schematic diagram of experimental system is presented as Fig. 1. It involves the testing model box, freezing circulation system, and transducer measurement system. To diminish the boundary effect, the largest model container with an insider dimension of

<table>
<thead>
<tr>
<th>Table 1</th>
<th>The basic physical and mechanical properties of mucky clay.</th>
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<tr>
<td>Engineering index</td>
<td>Mucky clay</td>
</tr>
<tr>
<td>Water content, w (%)</td>
<td>49.2</td>
</tr>
<tr>
<td>Natural bulk density, γ (kN/m³)</td>
<td>17.1</td>
</tr>
<tr>
<td>Void ratio, ε</td>
<td>1.435</td>
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<tr>
<td>Specific gravity, Gs</td>
<td>2.74</td>
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<tr>
<td>Liquid limit, wL(%)</td>
<td>50.5</td>
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<tr>
<td>Plastic limit, wP(%)</td>
<td>23.8</td>
</tr>
<tr>
<td>Plasticity index, Ip</td>
<td>25.4</td>
</tr>
<tr>
<td>USCS symbol</td>
<td>CH</td>
</tr>
<tr>
<td>Coefficient of compressibility, a11 − a22(MPa−1)</td>
<td>1.08</td>
</tr>
<tr>
<td>Coefficient of volume compressibility, cu(MPa−1)</td>
<td>0.93</td>
</tr>
<tr>
<td>Vertical permeability, Kc(cm/s)</td>
<td>9.13E−08</td>
</tr>
<tr>
<td>Unfrozen specific heat capacity cu(kJ/(kgK))</td>
<td>1.747</td>
</tr>
<tr>
<td>Unfrozen thermal conductivity, ku(W/(mK))</td>
<td>1.149</td>
</tr>
<tr>
<td>Frozen thermal conductivity, kv(W/(mK))</td>
<td>1.178</td>
</tr>
</tbody>
</table>

a The values are weighted-average by the heat capacity of each composited component, such as organic content, mineral content and water content.

b The values are measured based on the data collected in our self-designed model tests (Tang et al., 2014).
900 mm × 550 mm × 700 mm was utilized in this testing. The TJ Geotechnical centrifuge is TLJ-150 G-Accelerator (Fig. 2a). The design capacity is 150 g-tons. Two swinging platforms are located at either end of the centrifuge arm, in which one platform holds the model container and the other fixes the counterweight for the static and dynamic balance. The effective rotation radius is 3.0 m and the largest acceleration is 200 g. The cryostat tank supplies cooled glycol liquid for the freezing circulation (Fig. 2b), where it pumps out from a valve through silicone hoses into the freezing pipe system (Fig. 2c) equipped within testing soil in the model.
box and after one-circle heat exchange within soil the fluid returns back (Fig. 2a). The 90% glycol was employed as circulation fluid to prevent frost during testing. And the temperatures can be controlled around -25 °C in the experimental patterns in all models (Table 2) at an accuracy of 0.1 °C. Detailed inlet temperature and outlet temperature scheme are added in Fig. 3. Transducers including temperature and pressure were all equipped in arrays at the prefixed locations when filling the model soil. In addition, displacement transducers (LVDTs) were positioned both in the center and four corners of the model soil to monitor the displacements in the surface including frost heave, thaw settlement, and long-term settlement (Fig. 2d).

In this paper, we presented the results of several group centrifuge model experiments, in which all the models were frozen at 1 g under the same heat flux per surface area in accordance with the field AGF construction, but thawed at different scales respectively; the long-term settlement was also monitored at different scales. Table 2 summarizes the model design parameters in each model of this experimental program. The comparison of 1 g and 50 g is aimed to check the self-weight effect on thawing problem. The paralleling models of 40 g, 50 g and 60 g are to check the effectiveness of the scaling laws when the testing values are scaled up to prediction without an applicable full scale prototype data.

2.3. Transducer arrangement

Temperature sensors (TSs) were deployed in arrays at depths within the model soil above the freezing-pipe surface. Each TS array had four TSs paralleling distributed at the same depth with equal spacing of 0 mm shown as Fig. 4. One TSA was placed right above the freezing-pipe surface with the distances of 0 mm (250 mm in depth, TSA1:T5, T6, T7, T8); another TSA was just beside the freezing-pipe boundary with the lateral distance of 20 mm (250 mm in depth, TSA2: T9, T10, T11, T12) for the boundary temperature distribution. For the vertical temperature distribution, two additional TSAs were respectively paralleling to TSA1 in different depths: 230 mm (TSA3: T13, T14, T15, T16, frozen wall) and 200 mm (TSA4: T17, T18, T19, T20, temperature monitoring-boundary), accompanying with the pressure transducers (P1–P7), were distributed along the vertical direction. Shown as Fig. 4b, P1 was just beside TSA1 at the depth of 250 mm (freezing-pipe surface); P2, P3 and P4 were paralleling at the depth of 230 mm just beside TSA3; and similar to P5, P6 and P7, they were at the depth of 200 mm just beside TSA4. All the temperature, pressure and displacement data were respectively acquired and recorded through two data acquisition systems.

3. Results and discussion

3.1. Temperature distribution and self-weight effect

The temperature data indicated the similar process on temperature variation at each specific monitoring surface parallel to the freeze pipes during the freezing stage (Fig. 5). There are also roughly differences in existence between them. The temperatures among the freeze pipes are almost the same in each TS as shown in Fig. 5a. Compared to the temperature distribution in the freezing area, around the lateral boundary side, the temperature ramps more slowly and decreases along the lateral distance (Fig. 5b). While both in the 2 cm and 5 cm top surfaces of the freeze pipe, the temperature is nearly symmetrically distributed between two sides of each freeze pipe (in Fig. 5c and d, TS No.13/14, 15/16, 17/18 and 19/20). In this experimental program, the freezing wall is set to be 10 cm in total, i.e. 5 cm in each side of the freezing pipe surface (the freeze pipe was prefixed in the central surface of the model soil). Thus the TS array on the 5-cm top surfaces is also located to monitoring the temperature for the termination of each stage, such as the freezing stage and thawing stage. Fig. 6 shows the temperature during the whole process including freezing stage and thawing stage, in which TS 17 and 19 on the 5-cm top surface are good indicators for the testing time. Especially in TS 19, when the temperature approached below 0 °C (for the safe consideration almost —2 °C), the freezing stage was ceased and fluid circulation was stopped for the beginning of the thawing stage. Until the temperature recovered to above 0 °C at freezing wall (as shown in T 17 and T19), the time then can be marked as the termination of the thawing stage. Then the long-term settlement can start recording. When the displacements in LVDTs are stable in value (the criterion that is the variation monitored is no more than 0.5 mm in field scale).

In addition, model soil size, preparation, and freezing mode in Models A and B were identical in all respects except one: thawing of Model A was conducted at 1 g, whereas Model B was thawed at 50 g. These two tests were designed to check the self-weight effects in the scaled model soil on thawing behavior after subjected to artificial freezing. Fig. 6 shows the temperature variations of T5 and T7 located around the freezing pipes both from Model A (1 g) and Model B (50 g); and T17 and T19 in freezing wall boundary are also presented (it should be illustrated that actually two TSAs of freezing pipe surface and freezing wall boundary are chosen. Based on the temperature analysis above, T5/T6, T7/T8 and T17/T18, T19/T20 are all symmetrically distributed in each pair. Thus Fig. 5 just presents T5, T7, T17 and T19 for a clear exhibition).

Note that even though they were thawed at 1 g and 50 g respectively, Models A and B had roughly the same temperature development. But apparently in Model B thawing advanced faster and this is much conspicuous in T5. The similar temperature distributions during thawing in Models A and B provide comparison basic for the following inference of thaw settlement in time domain by means of centrifuge modeling.

Fig. 7 depicts the surface displacement curves during the thawing stage respectively in Models A and B. Thaw settlement in Model B (6.38 mm) was strikingly larger than that in Model A (5.23 mm). Clearly and also visibly, self-weight stress condition has a significant impact on the development of thaw settlement. Note here that the settlement in Model B ramped initially due to the change from 1 g freezing to 50 g

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**Table 2**

<table>
<thead>
<tr>
<th>Experimental patterns in all models.</th>
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<tbody>
<tr>
<td><strong>Thawing</strong></td>
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<tr>
<td><strong>Scale</strong></td>
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<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
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<td>C</td>
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<tr>
<td>D</td>
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**Fig. 3.** Temperature schemes during the freezing period.
scaled thawing; hence the final thaw settlement was the value set aside this part. The soil deformation during thawing stage embraces several aspects, including volumetric change due to water phase conversion, simultaneous process of water squeezing out soil by self-weight of the soils, etc. Chamberlain (1981) reported the consolidation of soil during thaw and proved that over-consolidation stress condition had possible great effects on thaw settlement. Particularly the model soil used herein is typical Shanghai mucky clay with a high water content of around 50% and a large initial void ratio up to 1.3 (Table 1). The thaw settlement results did confirm that marginally large deformation occurred. Mikasa (1984) and Gibson (Gibson et al., 1981) underscored a lot that self-weight played an important role on the finite strain deformation. Patrick et al. (2005) performed centrifuge modeling tests to prove the theory. In addition, a 50 g increase in self-weight of water ramps the water pressure gradient, which brings similarity with the full scale water dissipation conditions. This comparison indicates that small scale thawing test at 1 g is not capable to directly apply into full scale prediction of the thaw settlement in thick soft saturated clay during artificial ground freezing method, in which self-weight dominates the process of consolidation.

3.2. Freeze–thaw front development

In artificial ground freezing, the freeze–thaw interface forms the lower boundary of the study area in the frost-heave and thaw-settlement problem. The movement rate of the freeze–thaw interface, usually taken as 0 °C isotherm location is determined accordingly. In this modeling experiments, Models C, D and E were conducted in a slightly slower cooled freezing mode for a longer period than A and B to get more specific temperature development data. Temperatures along depths are drawn in different times to speculate the approximate 0 °C locations at each time in T5, 13 17 and three additional single thermocouples 1, 2 and 3 respectively in frozen thicknesses of 1 cm, 3 cm and 4 cm for complementary points. Thus there are six sensors along different vertical distances (0, 1, 2, 3, 4, 5 cm) in the central line starting from the freezing-pipe surface. Moreover the temperature data from all the previous TSs did indicate that the thaw plane was planar and advanced with a certain relationship with time. From our point of view, temperature data in these six locations are able to make clear sense to the freeze–thaw front movement. Fig. 8a reveals the temperature development along different distances to the freeze-pipe surface. Based on the time of temperature approaching around 0 °C in each position, the freezing front can be deduced in Fig. 8b. The freezing front development over time and the temperature variations at the locations from the freeze pipe surface are mutually relevant. In the previous freezing stage, the temperature decreased very quickly and the freezing front advanced relatively fast. As the surface moved up gradually, the temperature varied slowly and the frost heave deformation became stable, then the temperature distribution was almost in a stable state. Note that all the freezing stages in these models are in 1 g condition, in which the model test is more similar to a small scale model test regardless of field stress condition. A good correlation fitting was obtained as Eq.(1). This is much consistent with the well-known one dimensional consolidation of thawing soils derived by Morgenstern and Nixon (Morgenstern and Nixon, 1971) in the small sample laboratory experiments.

\[ H = 1.120\sqrt{t} \]  

where \( H \) (m) is the freeze distance, \( t \) (min) the time, and 1.120 is the fitting constant \( \sqrt{m/\text{min}} \). Nixon and McRoberts (1973) noted that the relationship between \( H \) and the square root of time \( \sqrt{t} \) is

![Fig. 4. Layout of temperature and pressure transducers: (a) plan view; (b) central profile.](image-url)
predominated by several variables (latent heat $L$, thermal conductivity of unfrozen soil $k_u$, thermal conductivity of frozen soil $k_f$, specific heat capacity of unfrozen soil $c_u$, specific heat capacity of frozen soil $c_f$, etc. when the soil temperature and cooled circulation fluid are both constant.)

Presently, the linear relationship with square root of time in the movement of thaw surface is used mostly by researchers (Bommer et al., 2012; Harris et al., 2000, 2008; Morgenstern and Smith, 1973; Nixon and McRoberts, 1973; Nixon and Morgenstern, 1974), even in many complicated three dimensional thawing settlement numerical simulations (Foriero and Ladanyi, 1995; Yao et al., 2012). In this modeling experiment, Models C, D and E were thawed at centrifugal acceleration conditions. During the thawing stage, re-consolidation by self-weight secondary compression, especially in this high-water content soft mucky clay soil, self-weight played an important role on the finite strain deformation. Moreover, an increase in self-weight of water ramps the water pressure gradient, which brings similarity with the full scale water dissipation conditions. Three models all present the thaw front advanced far swifter than predicted in the square root of time. Fig. 9 demonstrates the fitting results in the field scale. Eqs. (2)–(4) indicate the fitting curves under accelerations of 40 $g$, 50 $g$ and 60 $g$ respectively with very high correlation coefficient (R), in which $X$ here is labeled as distance to the freezing-pipe surface from the thaw front plane and $t$ is relevant time. The fitting curves, expressed in Eqs.(1)–(3) also all address the development of the thaw front plane in this modeling of artificial ground freezing method can hardly be consistent with the most well-known movement of the thaw plane in the conventional thaw consolidation, in which the rate of thaw is proportional to the square root of time. They are shown to be strikingly similar in pattern and very close in values (the full scale equivalent value according to the hypothesized model scaling relationship as $t_{\text{full scale}} = t_{\text{model}} \cdot (\text{model scale} N)^{2}$, $X_{\text{full scale}} = X_{\text{model}} \cdot (\text{model scale} N)$, note that the freezing pipes were arranged on the mid-surface in all models; also complementary pore pressure variation curves with time can make more clear sense to the thaw front movement (Fig. 10)).

Fig. 10 presents the pore-water pressure variations during thawing at 50 $g$. The locations of pore-water pressure transducers P1, P2 and P4 are at the thaw front thickness of 50 mm, 25 mm and 20 mm at 50 g (2.5 m, 1.5 m and 1.0 m in full scale equivalent meters) along the central
0.70
6.00
5.00
4.00
3.00
2.00
1.00
0.00

0
60
120
180
240
300
360
420
480
540
600
660
720

Model time (min)

Fig. 7. Thaw settlement during thawing of mucky clay at 1 g and 50 g (displacements are plotted at model scale).

It stands to reason that pore water pressure value reflected by pore-water pressure transducer acts to ramp or resume when the surrounding soil starts to thaw. From Fig. 10, we can clearly see that the ramped points presented by arrows pointed to different times; and corresponding to Fig. 10, TSA1, TSA3 and TSA4 are the exact X value of arrow ends, and the corresponding times revealed by arrows are implied great consistence with those in Fig. 9. And vice versa, all these could indicate a consistent position of the thaw front at a known certain time during the course of thawing. This could motivate, in the near future, a more precise large scale or field experiments can be conducted for the proving. Some computation simulation may be able to predict the proving. Some computation simulation may be able to predict the proving. Some computation simulation may be able to predict the proving. Some computation simulation may be able to predict.

3.3. Gray model prediction to experimental results

The gray prediction theory is one of the most important application theories in many research areas proposed by Deng (1982), such as agriculture, geology engineering, meteorology, hydraulic engineering and process control. Compared with conventional statistical prediction models, one of the major advantages of the gray model (GM) is that it requires only a limited amount of data to predict the system behavior (Kayacan et al., 2010; Liu et al., 2010). Thaw process usually exists during a long period. Settlement even happens in many years during the natural thawing. Engineering in cold regions is a very expensive undertaking. Practical experience in many areas is still quite limited. Field tests are often prohibitive due to the high costs. Thus, it is too difficult to get a large amount of data by continuously monitoring in a long period of time. The gray prediction model can just compensate for the lack of monitoring data. In gray systems theory, GM (n, m) denotes a gray model, where n is the order of the differential equation and m is the number of variables. GM (1, 1) is the most widely used because of its simplicity and satisfactory accuracy. Several other gray models were developed based on GM (1, 1), such as GM (1, n) or discrete gray model DGM (1, 1). All of the above models require the 1-AGO (accumulated generating operation) data to obey the law of exponential growth with time sequence. This is the essence of the prediction model, by use of accumulated generating operation (AGO), smooth discrete data can be transformed into a sequence having the approximate exponential regularity. Coincidently, one of the most used thaw interfaces is depicted as the power law expression $X(t) = Ct^n$, using exponential function to fit and predict is the algorithm nature of gray models. However, the above results of the thaw front movement (Fig. 9) and thaw settlement (Fig. 11) both show that the data regularity follows a non-homogeneous exponential distribution. Therefore, 1-AGO data also follow the non-homogeneous exponential distribution, but not exponential distribution. The above-mentioned gray models are not good enough to directly use to predict the thaw front development and the long-term thaw settlement. In addition, the general gray prediction model requires that the data used for modeling is equal interval. The obtained data are not always in an equal interval.

Thus, in this paper the non-equidistant non-homogeneous gray model (NNGM) (Ren et al., 2012; Tang et al., 2008; Xie et al., 2005; Zhang et al., 1999) is applied for the thaw front fitting and settlement prediction. This NNGM (1, 1), is improved based on the traditional gray model GM (1, 1). It includes several steps to accomplish this new model: Firstly, transforming the non-equal interval $X_0^{(0)} = \{X_0^{(0)}(1), X_0^{(0)}(2), ..., X_0^{(0)}(k), ..., X_0^{(0)}(M)\}$ into the equal

$$X(t) = 2.93 - 0.065t - 0.000925t^2$$

$$X(t) = 2.94 - 0.061t - 0.0000105t^2$$

$$X(t) = 2.95 - 0.070t - 0.000123t^2$$
interval $X_1^{(0)} = \{X_1^{(0)}(1), X_1^{(0)}(2), \ldots, X_1^{(0)}(k), \ldots, X_1^{(0)}(N)\}$ by use of the cubic spline interpolation method; secondly establishing non-homogenous gray model NNGM (1, 1) by 1-AGO manipulation (the results are called the first order AGO series based on $X_1^{(0)}$), $X_1^{(1)} = \{X_1^{(1)}(1), X_1^{(1)}(2), \ldots, X_1^{(1)}(k), \ldots, X_1^{(1)}(N)\}$, where $x_1^{(1)}(k) = \sum_{i=1}^{k} x_1^{(0)}(i)$; then defining the one variable, and first order differential equation in each data $\frac{dx_1^{(1)}}{dt} + ax_1^{(1)} = bt + c$ and calculating the parameters $a$, $b$, and $c$ by the least square method to get the final solution $x_1^{(1)}(t) = Ce^{-\alpha t} + \frac{b}{\alpha} t - \frac{b}{\alpha^2} + \frac{c}{\alpha}$; finally returning the solution $X_1^{(1)}(k), x_1^{(0)}(k')$ back to $x_1^{(0)}(k), x_1^{(0)}(k')$, where $k$ and $k'$ represent fitting and prediction points respectively.

It is worth noting here, in general GM (1, 1), the first order differential equation is defined as $\frac{dx_1^{(1)}}{dt} + ax_1^{(1)} = c$. All the specific equations and calculating procedures are presented in Appendix A.

Moreover, an analytical model is calculated as well for comparison. Thaw settlement includes two parts corresponding to the two synchronized processes: one thawing term $A_0 = \frac{e_f - e_{th}}{1 + e_f}$ (thaw settlement parameter) and one thaw consolidation compression terms $m_c$ due to self-weight or external loading. Thus the total thaw settlement can be analytically calculated by the following equation (Andersland and Ladanyi, 2004).

$$S = A_0 X + m_c \int_0^X \gamma' X \, dX$$  \hspace{1cm} (5)

Fig. 12 presents the gray model NNGM (1, 1) prediction of thaw front locations (Fig. 12a) and thaw displacements (Fig. 12b) compared to the experimental results. $\alpha$, $C$ and $p$ are the three parameters for the model accuracy checking (details can be seen in Appendix B). Apparently, the NNGM (1, 1) gray model prediction results match much better in thaw function than the thaw displacement. It is understandable that thaw function is known as power law relationship with time, which happens to the essence of NNGM (1, 1) gray model. While the thaw displacement movement is much more unknown and the data regularity is very crucial in prediction. For better understanding, the thaw process completion time was calculated through the thaw function, i.e., the time when the $X$ value approaches 0. The final thaw settlements were calculated by the simplified analytical model as well based on the fitting $X$ function. The power constant $n$ was performed in two values, respectively 1.0833 (derived by the fitting and gray prediction) and conventional 0.5 derived by small sample tests. All the results are presented in Table 3. It can be seen that the final thaw settlement value with thaw function in power constant of 0.5 is much smaller than both the experimental and the gray model prediction results. While the results at the power constant around 1 match well it should be realized that small scale thawing test at 1 g is not capable of directly applying into full scale prediction of the thaw settlement in thick soft saturated clay under artificial ground freezing construction, in which self-weight dominates the process of thaw consolidation.

4. Conclusions

Our research project is primarily intended to establish a prediction model of thaw settlement of muddy clay during the artificial ground freezing (AGF) construction, which should facilitate the application and much more precise than the small laboratory sample test results. Based on our previous AGF projects under the conventional thaw problem calculation, this paper is trying to figure out the thawing response of the whole artificial ground freezing model system by means of
centrifuge modeling. Some comments, more convincible and suitable in field circumstance, can be advanced as follows:

Even with the existence of acceleration condition in centrifuge, roughly the same temperature distribution and development were observed in different testing models. It provides comparison basic for the following inference by means of centrifuge modeling. Similarities are revealed in thaw settlement scale and time scale, respectively compared with the conventional scaling effects. Freezing front in 1 g freeze model reveals good proportional relation with square root of time, which is much consistent with the well-known theoretical deduced results. While the thawing front under centrifugal acceleration condition, which simulates the field full-scale circumstance, advances much faster and higher power law index exists rather than square root. The predicted results of improved gray model NNGM (1, 1) with the new thaw front function presents good match with the experimental results. All results reveal that small scale thawing test at 1 g is not capable to directly apply into full scale prediction of the thaw settlement in thick soft saturated clay under artificial ground freezing construction.

Centrifuge modeling is an attractive and effective proposition especially when long-period freezing or thawing behavior is of interest and practical experiences or field tests are still quite limited for reference due to the expensive undertaking with high costs in cold region engineering. More cases are needed to check the practical ability of centrifuge modeling to compress time scales, size scales on thawing problem induced by artificial ground freezing construction. The field thaw-consolidation response in high-thickness muddy clay is different to the small laboratory specimen tests. It has only been reported once by Foriero and Ladanyi (1995) in the form of FEM assessment. The present study provides specific thawing case in centrifuge; demonstrates the possibility of conducting thaw problems by means of centrifuge modeling; and confirms the early findings of large strain thaw consolidation in high-thickness soft clay. Less effective, higher cost and less timely on thaw settlement management could not be improved without more specific and precise thaw settlement predicting model.

Acknowledgments

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Appendix A. Modeling method for NNGM (1, 1) gray prediction model

First, transforming a non-equal interval sequence into a equal interval sequence. Define a raw non-equal interval data series as:

$$X_0(0) = \{x_0(0)(0), x_0(0)(2), ..., x_0(0)(k), ..., x_0(0)(N)\}, k \geq 1$$

where $N$ is the data number to establish gray model.

Table 3 Comparison of predicted results based on experimental data.

<table>
<thead>
<tr>
<th>Time (d)</th>
<th>Thaw settlement (cm)</th>
<th>NNGM(1,1) predicted</th>
<th>Analytical calculated base on fitting X</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time (d)</td>
<td>Thaw completion (d)</td>
<td>Thaw settlement</td>
</tr>
<tr>
<td>40 g</td>
<td>2189</td>
<td>33.68</td>
<td>34.92</td>
</tr>
<tr>
<td>50 g</td>
<td>2153</td>
<td>32.25</td>
<td>33.74</td>
</tr>
<tr>
<td>60 g</td>
<td>2225</td>
<td>31.62</td>
<td>31.90</td>
</tr>
</tbody>
</table>

Fig. 12. The gray model prediction results: (a) thaw front (at 40 g); (b) thaw displacement (at 40 g).
The time interval of data is
\[ \Delta t_k = t_{k+1} - t_k, \quad k = 1, 2, \ldots, N-1. \] (2)

Its average time interval can be
\[ \Delta T = \frac{1}{N-1} \sum_{k=1}^{N-1} \Delta t_k = \frac{1}{N-1} (t_N - t_1). \] (3)

The cubic spline interpolation method can be used to transform the non-equal interval data sequence (1) into a new equidistant data series of time interval \( \Delta T \).

\[ X_1^{(0)}(k) = \{x_1^{(0)}(1), x_1^{(0)}(2), \ldots, x_1^{(0)}(k), \ldots, x_1^{(0)}(N) \} \] (4)

where, \( x_1^{(0)}(k) = \frac{p_k}{\Delta t_k} x_0^{(0)}(t_k) + \frac{\Delta t_k - p_k}{\Delta t_k} x_0^{(0)}(t_k), \quad k = 1, \ldots, N, \) \( p_k \) is the difference between the average time point and observed time point as \( p_k = (k-1)\Delta T - (t_k - t_1) \).

Second, establishing non-homogeneous gray model NGM (1, 1).

To smooth the randomness, the generated new equidistant sequence (4) is manipulated by AGO, in which each data is replaced by the summation of previous data as \( x_1^{(1)}(k) = \sum_{i=1}^{k} x_1^{(0)}(i), \quad k = 1, 2, \ldots, N. \)

Thus after the first order AGO generation based on \( X_1^{(0)} \), a 1-AGO sequence \( X_1^{(1)} \), generally called whitening data, is

\[ X_1^{(1)} = \{x_1^{(1)}(1), x_1^{(1)}(2), \ldots, x_1^{(1)}(k), \ldots, x_1^{(1)}(N) \}. \] (5)

Then, then defining the one variable, and first order differential equation in each data calculating the parameters by least square method to get the final solution.

The gray prediction model can be expressed by one variable and first order differential equation, termed as whitening equation.

\[ \frac{dx_1^{(1)}}{dt} + ax_1^{(1)} = bt + c \] (6)

The derivative for the first order gray differential equation with 1-AGO is conventionally estimated by

\[ \frac{dx_1^{(1)}}{dt} = \lim_{\Delta t \to 0} \frac{x_1^{(1)}(t + \Delta t) - x_1^{(1)}(t)}{\Delta t}. \] (7)

Let \( \Delta t \to 1 \) and obtain

\[ \frac{dx_1^{(1)}}{dt} = x_1^{(1)}(t + 1) - x_1^{(1)}(t) = x_1^{(0)}(t + 1). \] (8)

Its discrete form is

\[ x_1^{(0)}(k) + ax_1^{(1)}(k) = bk + c \] (9)

where, \( z_1^{(1)}(k) \) is the mean value of adjacent data of 1-AGO sequence, called the background value of the gray first derivative \( \frac{dx_1^{(1)}}{dt} \) and obtained by \( z_1^{(1)}(k) = \frac{1}{2} \{x_1^{(1)}(k) + x_1^{(1)}(k + 1) \} \). This generated mean sequence \( Z_1^{(1)} = \{z_1^{(1)}(1), z_1^{(1)}(2), \ldots, z_1^{(1)}(k), \ldots, z_1^{(1)}(N-1) \} \) of \( X_1^{(1)} \) is called the background data. Simply to say, the first derivative \( \frac{dx_1^{(1)}}{dt} \) is calculated by difference of whitening data, and background value \( z_1^{(1)}(k) \) is calculated by mean value of whitening data in each interval \([k, k + 1]\), for \( k = 1, 2, \ldots, N - 1 \).

By the least square method for \((N - 1)\) order equation group, parameters \( a, b \) and \( c \) can be determined by coefficient matrix as follows:

\[ [a, b, c]^T = \left( B^T B \right)^{-1} B^T Y \] (10)

\[ B = \begin{bmatrix} z_1^{(1)}(1) & 3/2 & 1 \\ z_1^{(1)}(2) & 5/2 & 1 \\ \vdots & \vdots & \vdots \\ z_1^{(1)}(N-1) & (2N + 1)/2 & 1 \end{bmatrix} \quad Y = \begin{bmatrix} x_1^{(0)}(2) \\ x_1^{(0)}(3) \\ \vdots \\ x_1^{(0)}(N) \end{bmatrix}. \] (11)

It is worth remarking here that in the conventional gray model GM (1, 1) the differential equation is defined as \( x_1^{(0)}(k) + ax_1^{(0)}(k) = c \).

The solution of (8) is

\[ x_1^{(1)}(t) = Ce^{-at} + \frac{b}{a} - \frac{c}{a} \] (12)

where \( C \) is a constant parameter, which can be determined by the initial condition \( t = 1 \), \( x_1^{(1)}(1) = x_1^{(0)}(1) \). It results in \( C = \left( x_1^{(0)}(1) - \frac{b}{a} + \frac{c}{a} \right) \).

Thus, the solution of \( x_1^{(1)}(k) \) is

\[ x_1^{(1)}(k) = \left( x_1^{(0)}(1) - \frac{b}{a} + \frac{c}{a} \right) e^{-ak} + \frac{b}{a} - \frac{c}{a}, \quad k = 1, 2, \ldots, N. \] (13)

The solution of \( x_1^{(0)}(k) \) is easily recovered from \( x_1^{(1)}(k) \)

\[ x_1^{(0)}(k) = \begin{cases} (1 - e^a) \left( x_1^{(0)}(1) - \frac{b}{a} + \frac{c}{a} \right) e^{-ak} + \frac{b}{a}, & k = 1, 2, \ldots, N \\ x_1^{(0)}(k') & k' = N, N + 1, N + 2, \ldots \end{cases} \] (14)

Consequently, the prediction value \( x_1^{(0)}(k') \) can be computed by

\[ x_1^{(0)}(k' + 1) = (1 - e^a) \left( x_1^{(0)}(1) - \frac{b}{a} + \frac{c}{a} \right) e^{-ak} + \frac{b}{a}. \] (15)

Still by the cubic spline interpolation method, \( x_1^{(0)}(k) \) and \( x_1^{(0)}(k') \) can be transformed back into \( \hat{x}_1^{(0)}(k) \) and \( \hat{x}_1^{(0)}(k') \).

Appendix B. Checking for the model accuracy

There are three main checking methods on data residuals: mean relative error \( \alpha \), ratio of variances \( C \), and small error probability \( p \) (Liu and Lin, 2011).

B.1. Mean relative error

\[ \alpha = \frac{1}{N} \sum_{k=1}^{N} \frac{\varepsilon(k)}{|x_0^{(0)}(k)|} \] (16)

where \( \varepsilon(k) \) stands for the absolute degree of incidence between the raw data \( x_0^{(0)}(k) \) and the simulated value \( \hat{x}_0^{(0)}(k) \). \( \varepsilon_0^{(0)}(k) = x_0^{(0)}(k) - \hat{x}_0^{(0)}(k) \). Thus the error sequence is \( \varepsilon_0^{(0)} = \{\varepsilon_0^{(0)}(1), \varepsilon_0^{(0)}(2), \ldots, \varepsilon_0^{(0)}(k), \ldots, \varepsilon_0^{(0)}(N)\} = \{x_0^{(0)}(1) - \hat{x}_0^{(0)}(1), x_0^{(0)}(2) - \hat{x}_0^{(0)}(2), \ldots, x_0^{(0)}(k) - \hat{x}_0^{(0)}(k), \ldots, x_0^{(0)}(N) - \hat{x}_0^{(0)}(N)\} \).
Table B1
Criteria of model accuracy.

<table>
<thead>
<tr>
<th>Accuracy scale</th>
<th>Mean relative error $\alpha$</th>
<th>Degree of incidence $c$</th>
<th>Variance ratio $C$</th>
<th>Small error probability $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st level (excellent)</td>
<td>$0.00 \leq \alpha &lt; 0.01$</td>
<td>$1.00 \leq c &lt; 0.90$</td>
<td>$0.00 \leq C &lt; 0.25$</td>
<td>$1.00 \geq p &gt; 0.95$</td>
</tr>
<tr>
<td>2nd level (good)</td>
<td>$0.01 \leq \alpha &lt; 0.05$</td>
<td>$0.90 \leq c &lt; 0.80$</td>
<td>$0.35 \leq C &lt; 0.50$</td>
<td>$0.95 \geq p &gt; 0.80$</td>
</tr>
<tr>
<td>3rd level (satisfactory)</td>
<td>$0.05 \leq \alpha &lt; 0.10$</td>
<td>$0.80 \leq c &lt; 0.70$</td>
<td>$0.50 \leq C &lt; 0.65$</td>
<td>$0.80 \geq p &gt; 0.70$</td>
</tr>
<tr>
<td>4th level (poor)</td>
<td>$0.10 \leq \alpha &lt; 0.20$</td>
<td>$0.70 \leq c &lt; 0.60$</td>
<td>$0.65 \leq C &lt; 0.80$</td>
<td>$0.70 \geq p &gt; 0.60$</td>
</tr>
</tbody>
</table>


\[ \text{B.2. Variance ratio} \]

\[ C = \frac{S_2}{S_1} \quad (17) \]

in which, $S_1$ and $S_2$ are the variances of raw data sequence $X^{(0)}_k$ and the error sequence $e^{(0)}_k$. They can be calculated as follows:

\[ S_1 = \frac{1}{N} \sum_{k=1}^{N} (X^{(0)}_k - \bar{X})^2, \quad \bar{X} = \frac{1}{N} \sum_{k=1}^{N} X^{(0)}_k, \quad S_2 = \frac{1}{N} \sum_{k=1}^{N} (e^{(0)}_k - \bar{X})^2 = \frac{1}{N} \sum_{k=1}^{N} e^{(0)}_k^2. \]

\[ \text{B.3. Small error probability} \]

\[ p = P(\alpha \leq \bar{X} - 0.6745S_1) \quad (18) \]

For the mean relative error $\alpha$ and the simulation error, the smaller they are, the better. For the degree of incidence $c$, the greater it is the better. For the variance ratio $C$, the smaller the better, it is because a small $C$ indicates that $S_2$ is relatively small while $S_1$ is relatively large, meaning that the error variance is small while the variance of simulation results, the smaller $S_2$ is when compared to $S_1$, the better.

The most commonly used scales of accuracy for testing models are listed in Table B1.

References