Affective decision making: A theory of optimism bias

Anat Bracha\textsuperscript{a,}\textsuperscript{*}, Donald J. Brown\textsuperscript{b}

\textsuperscript{a} Research Department, The Federal Reserve Bank of Boston, 600 Atlantic Avenue, Boston, MA 02210, USA
\textsuperscript{b} Department of Economics, Yale University, Box 208268, New Haven, CT 06520-8268, USA

\begin{abstract}
Optimism bias is inconsistent with the independence of decision weights and payoffs found in models of choice under risk and uncertainty, such as expected utility theory, subjective expected utility, and prospect theory. We therefore propose an alternative model of risky and uncertain choice where decision weights—affective or perceived risk—are endogenous. Affective decision making (ADM) is a strategic model of choice under risk and uncertainty where we posit two cognitive processes—the “rational” and the “emotional” process. The two processes interact in a simultaneous-move intrapersonal potential game, and observed choice is the result of a pure strategy Nash equilibrium in this game. We show that regular ADM potential games have an odd number of locally unique pure strategy Nash equilibria, and demonstrate this finding for affective decision making in insurance markets. We prove that ADM potential games are refutable by axiomatizing the ADM potential maximizers.
\end{abstract}

\section{Introduction}

Many of our everyday decisions such as whether to work on a project, take a flu shot, or buy insurance, require an estimate of probabilities of future events: the probability of a project’s success, of getting sick, or of being involved in an accident. Nevertheless, it appears that in many such instances decision makers are repeatedly optimistically biased, where optimism bias is defined as the tendency to overestimate the likelihood of favorable future outcomes and underestimate the likelihood of unfavorable future outcomes (Irwin, 1953; Weinstein, 1980; Slovic et al., 1982; Slovic, 2000). A young woman drinking at a bar thinking it would be safe for her to drive home is an example; an entrepreneur who starts a new business, confident that she is going to succeed where others have failed, is another. Indeed, one can argue that although statistics for these events are well documented, these individuals have private information concerning their tolerance for alcohol and entrepreneurial ability, respectively. Hence, each woman may have good reasons to believe that overall empirical frequencies do not apply to her. The common feature in these examples is that decision makers have some freedom in choosing their probabilistic beliefs, and they are often optimistic—they appear to choose beliefs that are biased towards favorable outcomes.

Optimism bias is not merely a hypothetical bias; instead it translates into both microeconomic and macroeconomic activity. For example, optimism bias influences high-stakes decisions, such as startup investment, investment behavior, and merger decisions. It was found that 68 percent of startups’ entrepreneurs believe their company is more likely to succeed than similar companies, while in reality only 50 percent of startup companies survive beyond three years of activity (Baker et al., 2006 and references therein). Malmendier and Tate (2005) find that CEOs who are optimistic regarding their firm’s future performance have greater sensitivity to investment cash flow, leading to distortions in investment. In their 2008 paper, Malmendier and Tate find that the optimistic CEOs are 65 percent more likely to complete mergers, are more likely to overpay for those target companies, and are more likely to undertake value-destroying mergers. On the macroeconomic level,
Robert Shiller in his now classic book (2000) defines irrational exuberance as “wishful thinking on the part of investors that blinds us to the truth of our situation,” and makes the case that irrational exuberance contributes to generating bubbles in financial markets. Shiller points out several psychological and cultural factors that affect individuals’ beliefs and consequently investment behavior, leading to real macro-level effects. Many of these factors can be summarized as optimistically biased beliefs.

Optimism bias is inconsistent with the independence of decision weights and payoffs found in models of choice under risk and uncertainty, such as expected utility, subjective expected utility, and prospect theory. Hence, we propose a theory of optimism bias for decision making under risk and uncertainty. That is, decision making under risk where the objective probabilities over future outcomes are known to the decision maker, and decision making under uncertainty where objective probabilities of future outcomes are unknown to the decision maker. In our alternative model of risky and uncertain choice, decision weights—which we label affective or perceived risk—are endogenous. More specifically, we consider two systems of reasoning, which we label the rational process and the emotional process. The rational process decides on an action, while the emotional process forms perception of risk and in doing so is optimistically biased. The two processes interact to reach a decision. This interaction is modeled as a simultaneous-move, intrapersonal potential game, and consistency between the two processes, which represents choice, is the equilibrium outcome realized as a pure strategy Nash equilibrium of the game.

This novel formulation of optimism bias, using a simultaneous choice of action and beliefs and where tradeoff is done by a game, is the first contribution of our paper. This formulation may be viewed as a model of specialization and integration of brain activity considered in the recent neuroscience literature. That is, recent studies in neuroscience identify distinct brain modules that specialize in different activities. For instance, the amygdala is associated with emotions, while the prefrontal cortex is associated with higher-level, deliberate thinking (e.g., Reisberg, 2001). Our model is also consistent with the psychology literature that draws a distinction between analytical and intuitive, or deliberate and emotional, processing (Chaiken and Tropé, 1999). However, in both neuroscience and psychology, behavior is thought to be a result of the different systems interacting (for example, Camerer et al., 2004; Damasio, 1994; Epstein, 1994; Gray et al., 2002; LeDoux, 2000; Pessoa, 2008; Sacks, 1985). Gray et al. (2002) for example conclude that, “at some point of processing, functional specialization is lost, and emotion and cognition jointly and equally contribute to the control of thought and behavior,” and recently, Pessoa (2008) argues that, “emotions and cognition not only strongly interact in the brain, but [that] they are often integrated so that they jointly contribute to behavior,” a point also made in the specific context of expectation formation.

Although the evidence on modular brain and the dual-processes theory cannot typically be pinned down to the formation of beliefs, given that beliefs formation is partly affected by the beliefs we would like to have—that is, by affective considerations—decision making under risk and uncertainty maps naturally to the interplay between two cognitive processes, as proposed by Kahneman (2003). That is, decision making under risk and uncertainty can be modeled as a deliberate process that chooses an optimal action, and an emotional cognitive process that forms risk perception.

Formally, the rational process coincides with the expected utility model, where for a given risk perception (affective probability distribution), the rational process chooses an action to maximize expected utility. The emotional process forms risk perception by selecting an optimal risk perception that balances two contradictory impulses: (1) affective motivation and (2) a taste for accuracy. This is a definition of motivated reasoning, a psychological mechanism where emotional goals motivate agent’s beliefs (see Kunda, 1990), and is a source of psychological biases, such as optimism bias. Affective motivation is the desire to hold a favorable personal risk perception—optimism—and in the model it is captured by the expected utility term. The desire for accuracy is modeled as a mental cost incurred by the agent for holding beliefs in lieu of her base rate probabilities given her desire for favorable risk beliefs. The base rate probabilities are the beliefs that minimize the mental cost function of the emotional process, i.e., the risk perception that is easiest and least costly to justify; in many instances, one can think of the baseline probabilities as the empirical, relative frequencies of the states of nature.

Quite intuitively, a desire to hold optimistic beliefs is present when the issue at stake is personal future outcome: having future success, being healthy, being successfully employed, or winning the lottery. When such outcomes are at stake we would expect individuals to exhibit optimism bias when the mental cost function allows it. This will be more likely to happen when individuals are familiar or are competent with the subject matter pertaining to risky or uncertain outcomes.

We present an example of the demand for insurance in a world with a bad state and a good state as an application of affective decision making. The relevant probability distribution in insurance markets is personal risk; hence, the demand for insurance may depend on optimism bias. Affective choice in insurance markets is defined as the insurance level and risk perception that constitute a pure strategy Nash equilibrium of the ADM intrapersonal potential game. Surprisingly, we show that it is possible for an affective agent to be pessimistic relative to the base rate probabilities. The reason is a special property of the insurance problem, where a “bad” can be converted into a “good” state and vice versa depending on the level of insurance; hence, although an agent may appear to be pessimistic to an outside observer, she is always optimistic in her own mind.

The systematic departure of the ADM model from the expected utility model shows, consistent with consumer research (Keller and Block, 1996), that campaigns intended to educate consumers on the magnitude of the potential loss in the unfavorable state can have the unintended consequence that consumers purchase less, rather than more, insurance. Hence, the ADM model suggests that the failure of the expected utility model to explain some data sets may be due to systematic
affective biases. Furthermore, the ADM intrapersonal game is a potential game—where a (potential) function of a penalized subjective expected utility (SEU) form defines the best response dynamic of the game. This property has the natural interpretation of the utility function of the composite agent, or integration of the two systems, and allows for axiomatic derivation of the ADM potential maximizers.

The second contribution of our paper is in deriving axiomatization of the ADM potential maximizers, and showing that choice based on the potential maximizers is refutable. More specifically, deviations from the basic models of rational choice often raise the concern that the theory lacks the discipline imposed by a clear paradigm, and, as a result, "anything goes". This concern arises in the ADM model, since we allow agents to choose both actions and beliefs. To investigate this concern we present an axiomatic characterization of ADM potential maximizers which has a variational form à la Maccheroni et al. (2006), and which subsumes the class of ADM with unique equilibrium. That shows that choosing both actions and beliefs in the ADM model does not per se imply that anything goes, and it implies a relationship between optimism and pessimism and ambiguity attitudes. This relationship is discussed in Sections 4 and 5.

1.1. Related literature

We are not the first to suggest agents may choose their beliefs in a self-serving manner. Economic models such as Akerlof and Dickens (1982), Bénabou and Tirole (2002), Bodner and Prelec (2002), Brunnermeier and Parker (2005), Caplin and Leahy (2004), Köszegi (2006) and Yariv (2002) recognize that possibility. The dual processes hypothesis, as well, was recently recognized in economic modeling. Specifically, in models of self-control and addiction such as Benhabib and Bisin (2005), Bernheim and Rangel (2004), Brocas and Carrillo (2008), Fudenberg and Levine (2006), Loewenstein and O’Donoghue (2004), and Thaler and Shefrin (1981). Existing models are restricted in the sense that choice of beliefs and choice of action are not made in tandem and in that the models assume that an agent chooses beliefs in a strategic manner to resolve a tradeoff between a standard instrumental payoff and some notion of psychologically based belief utility,1 while the existing models of dual processes are restricted in that the two systems, or decision modes, are conceived as mutually exclusive.

Although there are cases where a descriptive model seems to require mutually exclusive systems, as in the case of self-control and addiction, there are other cases where a descriptive model seems to require several different processes that together determine observed choice. We provide such a formulation: one process chooses action while the other forms perceptions, and both are necessary for decision making.

As mentioned, the ADM intrapersonal game is a potential game with a potential function defined as a penalized SEU model. This characterization allows us to axiomatize the set of ADM potential maximizers, which includes the class of unique equilibrium ADM models. The axiomatic foundation suggests an alternative interpretation of the ADM model as a model of ambiguity seeking, an additional difference between this paper and the existing literature.

The model in the literature that comes closest to optimistic preferences as ADM intrapersonal game is the Optimal Expectations model of Brunnermeier and Parker (2005). Optimal expectations consider an agent who chooses both beliefs and actions in a dynamic setting, where beliefs are chosen at period one for all future periods, trading off greater anticipated utility against the cost of poor decisions due to optimistic beliefs. Hence, optimal expectations are optimistic beliefs not constrained by reality. ADM, in contrast, is a static model, where beliefs and actions mutually determine observed choice, and where beliefs trade off greater anticipated utility against the mental cost of holding distorted beliefs—costs that are a function of reality. This captures the fact people do not hold arbitrary beliefs even in the absence of an action they can take based on their beliefs, in contrast to the optimal expectations model’s prediction. Moreover, the simultaneous framework of ADM where cost is based solely on beliefs is a parsimonious model, consistent with cognitive dissonance, difference between report and choice tasks, and integration of processes in the brain, and which has a utility function of the composite agent that will allow future welfare analysis. This formulation further allows, as mentioned above, axiomatic characterization of the ADM potential maximizers which then leads to an alternative interpretation of the ADM model as a model of ambiguity seeking. Both the axiomatic characterization and the alternative interpretation as an ambiguity seeking model are not shared by Brunnermeier and Parker (2005) or the other dual process models.

1.2. Organization

The remainder of the paper is organized as follows: In Section 2 we present the demand for insurance in a world with two states of nature. Section 3 presents an analysis of ADM potential games in a world with K states of nature with a formal definition of optimistic preferences in the general case. In Section 4 we present an axiomatic foundation of the ADM potential maximizers, and discuss the relationship between the axiomatized class of optimistic preferences and ambiguity attitudes. In the final section of the paper, we discuss the conditions we expect to lead to optimistic beliefs or ambiguity seeking attitudes. All proofs are in Appendix A.

1 The axiomatic foundation for this is provided by Caplin and Leahy (2001) and Yariv (2001).
2. Motivating example: The ADM model of the demand for insurance

In this section, we present a model of affective choice in insurance markets, where probability perceptions are endogenous. We use a simple example of an agent facing two possible future states of the world. In Section 3 we show that analogous results to this two-states case hold for K states of the world.

Consider an agent facing two states of the world, Bad and Good with associated wealth levels \( w_B \) and \( w_G \), where \( w_B < w_G \). The agent has a strictly increasing, strictly concave, smooth utility function of wealth, \( u(w) \), with \( \lim_{w \to 0} u'(w) = \infty, \lim_{w \to \infty} u'(w) = 0 \). Risk perception is defined as the perceived probability \( p \in [0, 1] \) of the Bad state occurring. For simplicity we allow the agent to purchase or sell insurance \( I \in R \) at the fixed insurance premium rate. \( \gamma \in (0, 1) \). The intuition and results for the case where the agent can only buy insurance are easily derived from this analysis.

The rational process chooses an optimal insurance \( I^* \) to maximize expected utility given a perceived risk \( p \). Specifically, the rational process maximizes the following objective function:

\[
\max_I \left\{ pu(w_B + (1-\gamma)I) + (1-p)u(w_G - \gamma I) \right\}.
\]

The emotional process chooses an optimal risk perception \( p^* \) given an insurance level \( I \), to balance affective motivation and a taste for accuracy. Specifically, the emotional process maximizes the following objective function:

\[
\max_p \left\{ pu(w_B + (1-\gamma)I) + (1-p)u(w_G - \gamma I) - J^*(p; p_0) \right\}.
\]

Affective motivation is captured by the expected utility term—the agent would like to assign the highest possible weight to her preferred state of the world. Taste for accuracy is modeled by introducing a mental cost function \( J^*(p; p_0) \) that is nonnegative, strictly convex and reaches a minimum at \( p = p_0 \), where \( p_0 \) is the baseline probability. \( J^*(p; p_0) \) is essentially smooth—smooth on \((0, 1)\) with \( \lim_{p \to 0} |DJ^*(p; p_0)| = \lim_{p \to 1} |DJ^*(p; p_0)| = +\infty \)—and it’s limit \( \lim_{p \to 0} J^*(p; p_0) = \lim_{p \to 1} J^*(p; p_0) = +\infty \). See Fig. 1.

![Fig. 1. The mental cost function.](image_url)

Why do we assume this shape of the mental cost function? To formally reflect insights from psychology. More specifically, the literature in psychology argues that individuals tend to use mental strategies, such as bias search through memory, to find justifications for their desired beliefs (Kunda, 1990). As the desired beliefs are further away from some baseline odds \( p_0 \)—the odds that immediately come to mind and are easiest to justify (could be the empirical relative frequency of state of nature or available statistics like mortality tables)—the search costs are likely to increase. That is, the more optimistic the views are, the more difficult it is to find anecdotes to support them. The shape of the mental cost function is a formal description of this insight. In addition, it seems that decision makers assign a special quality to certain situations: getting $100 for sure is qualitatively different from a 99 percent chance lottery to win $100 (Kahneman and Tversky, 1979).

In the current simple settings, certainty corresponds to the extreme beliefs \( p \in \{0, 1\} \), and the behavior at the extremes, i.e., infinite increase in marginal cost, captures this dramatic difference. Hence, the psychology literature is consistent with a mental cost \( J^*(p) \) that is strictly convex, and essentially smooth on the interior of the probability simplex \( \Delta \). Moreover, since people do not hold beliefs that reflect certainty in situations that are inherently risky, we demand that in the limit, as probability approaches zero or one, the cost function approaches infinity at a higher rate than the utility function such that the insurance desired by the rational process for probability approaching the boundary is not sufficient to support these beliefs; hence, one would never hold beliefs \( p \in \{0, 1\} \).

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2 All qualitative results remain the same for the case of \( \lim_{w \to 0} u'(w) = \infty, \lim_{w \to \infty} u'(w) = 0 \).

3 Alternatively, one can assume that the utility is bounded with the limit of its derivative approaching the lower bound is positive infinity and the limit of its derivative approaching the upper bound is zero. Then, this shape of the utility function together with the assumption on the cost function guarantees that extreme beliefs reflecting certainty will never be chosen.
The fact that the mental cost is a function solely of probability is appealing as well. It formally reflects the description of reasoning coming from psychology and gives rise to the special and important property of our model of interaction between the rational and emotional processes—the intrapersonal game—namely, the intrapersonal game is a potential game. As such, this type of mental cost is consistent with integration of the two processes, a property supported by psychology and recent research in neuroscience. Moreover, Section 3 shows that, surprisingly, the psychology description of the cost function is in fact necessary and sufficient for optimistic preferences, where optimistic preferences are such that the assigned probabilities are skewed towards the good state and away from the bad state.

We now consider the interaction of the two processes in decision making. We model this interaction as an intrapersonal simultaneous-move game. This choice reflects a recent view in cognitive neuroscience that both processes mutually determine the performance of the task at hand (Damasio, 1994).

**Definition 1.** An intrapersonal game is a simultaneous move game of two players, namely, the rational and the emotional processes. The strategy of the rational process is an insurance level, \( I \in \mathbb{R} \), and the strategy of the emotional process is a risk perception, \( p \in (0,1) \). The payoff function for the rational process \( g : (0,1) \times \mathbb{R} \rightarrow \mathbb{R} \) is \( g(p, I) \equiv pu(w_B + (1 - \gamma)I) + (1 - p)u(w_C - \gamma I) \). The payoff function for the emotional process \( \Theta : (0,1) \times \mathbb{R} \rightarrow \mathbb{R} \) is \( \Theta(p, I) = J^*(p) \), where \( J^*(\cdot) \) is the mental cost function of holding belief \( p \), which reaches a minimum at \( p_0 \).

Proposition 2 below indicates that the intrapersonal game defined above is a potential game. Potential games are a class of strategic games introduced by Monderer and Shapley (1996), where all players have a common goal and therefore the game can be represented with one global common payoff function. This global payoff function is called the potential function of the game, and is used by each player to determine her best response. In the case of individual choice, since the players are decision processes and the game is a model of decision making, the potential function has an intuitive interpretation as a utility function of the composite agent. Below is the formal proposition:

**Proposition 2.** The intrapersonal game is a potential game, in which the emotional process’s objective function is the potential function for the game. Because the potential function is strictly concave in each variable (risk perception and insurance), its critical points are the pure strategy Nash equilibria of the game.

It is straightforward to show that the emotional process’s objective function is the potential of the game, as its first order conditions with respect to \( I \) and \( p \) are the same as those of the rational process and emotional process, respectively. That is the potential function \( \Theta(I, p) = (U(I), p) - J^*(p) \) captures the best-response dynamics of the intrapersonal game, and this potential game is therefore a model of affective decision making.

The equilibrium notion in the potential game is pure strategy Nash equilibrium, which is a natural candidate for choice, as it reflects a mutually determined choice and consistency between the rational and emotional processes. Excluding the case of tangency between the best responses of the two processes and given that the potential is bi-concave we have the following existence theorem (see Fig. 2 for illustration).

**Proposition 3.** The ADM intrapersonal game has an odd number of pure strategy Nash equilibria.

![Fig. 2. Pure strategy Nash equilibria in the intrapersonal game.](image-url)
How exactly does affective choice in insurance markets differ from the demand for insurance in the expected utility model? Proposition 4 below shows that the expected utility outcome in the case of an actuarially fair insurance market (full insurance) falls within the choice set of the ADM agent. However, if the insurance market is not actuarially fair, then this is no longer the case.

**Proposition 4.** If \( \gamma = p_0 \), there exists at least one Nash equilibrium \((p^*, I^*)\) with \( p^* = p_0 = \gamma \), and \( I^* = \text{full insurance} \).

If \( \gamma > p_0 \), there exists at least one Nash equilibrium \((p^*, I^*)\) with \( p^* < p_0 \) and \( I^* < I^*(p_0) \).

If \( \gamma < p_0 \), there exists at least one Nash equilibrium \((p^*, I^*)\) with \( p_0 < p^* \) and \( I^* > I^*(p_0) \).

To understand the intuition behind these results, consider a standard myopic adjustment process where the processes alternate moves. If \( \gamma > p_0 \), at \( p_0 \) the rational process, similar to the expected utility model, prescribes buying less than full insurance. The emotional process, in turn, leads the decision maker to believe “this is not going to happen to me” and determines that she is at a lower risk. This effect causes a further reduction in the insurance purchase, with a result of less than full insurance, even less than what the expected utility model would predict. This proposition gives both an intuitive understanding of the effect of the emotional process in the ADM model, and intuitively shows existence of pure-strategy Nash equilibrium, affective choice. In the case of a unique ADM models, Proposition 4 fully characterizes affective choice; in the case of multiple equilibria it points out only one equilibrium out of many possible ones. However, the effect of the two processes in enhancing each other’s initial tendency holds whether one considers unique ADM models or the entire class of ADM models.\(^4\)

Note that Proposition 4 also implies that, from the viewpoint of an outside observer, both optimism and pessimism (relative to \( p_0 \)) are possible. The possibility of agents holding optimistic beliefs is not surprising, as ADM is a model of optimism bias. However, the possibility of pessimistic beliefs seems counterintuitive. How can it be that a model of optimism yields pessimistic views? Consider the case in Proposition 4 where insurance premium is relatively cheap (\( \gamma < p_0 \)). In that case, at \( p_0 \) the rational process prescribes buying more than full insurance, as would the expected utility model. This, in turn, leads the mental process to believe “this is going to happen to me” and to hold pessimistic beliefs. Together the result is choice (an equilibrium) where the agent purchases more than full insurance, even more than prescribed by the expected utility model, and holds pessimistic beliefs \( p^* > p_0 \). What makes the mental process choose pessimistic beliefs? The key is the characteristics of the insurance problem: more than full insurance in effect converts the “bad” state into a “good” state, and vice versa. Hence, although the agent seems to be pessimistic to an outside observer, she is always optimistic in her own mind. The implication of this insight is that when there is an effective action, i.e., an action that can change a bad state into a good state and vice versa, we could observe pessimism (relative to \( p_0 \)) and more-than-optimal insurance. However in the absence of such effective action, as is often the case, we should expect optimism and less-than-optimal insurance.

Here is another example of the difference between affective choice and the demand for insurance in the expected utility model. Consider an educational campaign—a campaign of the type that is often run by a non-profit organization or a government using media such as radio and television messages, posters, house visits, etc. in order to increase the public awareness of a certain issue. Very often these issues are related to public health, public safety, and potential natural disasters. Some contemporary examples include campaigns against smoking, against excessive alcohol consumption, for preventive measures against skin cancer, awareness of the importance of safe driving, safe sex, protecting one’s property against a natural disaster, and alike. Many of these educational campaigns focus on raising awareness of the possible catastrophe—cigarette warning labels warn that “Smoking Kills”, campaigns against speeding show vivid pictures of people severely injured or killed in car accidents, and flood warnings say “Like Never Before”. Formally, these are attempts to raise people’s awareness of the higher potential loss compared to what they imagined.\(^5\) The effect of such campaigns according to the standard expected utility model would be to increase purchase of insurance. However, in the ADM model, if an agent realizes she faces higher possible loss she might purchase less insurance. The increased loss size affects both the emotional and the rational processes in different directions; the rational process prescribes more insurance, the emotional process prescribes lower risk belief to every insurance level (due to greater incentives to live in denial). If the emotional effect is stronger the agent will buy less insurance than previously. That is, if the loss is great, agents might prefer to remain in denial and ignore the possible catastrophes altogether, leading them to take fewer precautions such as buying insurance. This is consistent with consumer research showing that high fear arousal in educating people on the health hazards of smoking leads to a discounting of

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\(^4\) Considering such an adjustment process, the ADM model is consistent with two widely discussed phenomena: cognitive dissonance and attention effects. Cognitive dissonance is when one holds two contradicting beliefs at the same time. Hence if one thinks of the adjustment process as a process of reaching a decision, in this process the agent suffers cognitive dissonance and choice represents a resolution of it. As for attention effects—if one’s attention is manipulated to first think of an action, or first think of risk beliefs, generally he or she will end up with different choices. In particular, according to our general ADM model, thinking first of probabilities of adverse events leads to greater optimism and lower insurance purchased than if the agent’s attention is given to thinking of insurance first.

\(^5\) One can think of educational campaigns as efforts to change the base-line probability \( p_0 \), or to change the mental costs directly. We take a different approach, that is, we think of educational campaigns as efforts to change the perceived loss if the bad state occurs. The reason for our approach is that we found, as described in the text, that campaigns against smoking, alcohol consumption, speeding, and natural disasters stress the outcome and not the probability of getting into a bad state. That is, many educational ads do not add information on the probability of suffering from the adverse consequence, but rather draw the attention of the public to the possible dramatic consequences.
the threat (Keller and Block, 1996; see also Ringold, 2002 and references therein). Proposition 5 and Fig. 3 summarize the conditions for educational campaigns to produce the counter-intuitive affective result.

**Proposition 5.** An educational campaign results in more optimistic beliefs if

\[
\frac{u'(w_B + (1 - \gamma)I)}{r(w_B + (1 - \gamma)I)} > \frac{u'(w_G - \gamma I)}{r(w_G - \gamma I)},
\]

and this will result in less insurance if

\[
\frac{u'(w_B + (1 - \gamma)I)}{J^{**}(p; p_0)} > r(w_B + (1 - \gamma)I)p(1 - p),
\]

where \( r(\cdot) \) is the absolute risk aversion property of the utility function \( u(\cdot) \).

![Fig. 3. Possible effect of educational campaigns.](image)

The two conditions above are statements on the relative movement of the two processes' best responses. We focus on the second condition as it gives the condition under which educational campaigns would backfire, i.e., lead to the unintended result of less insurance. In this condition, the term on the left is the mental process's increased motivation to hold favorable beliefs, i.e., the change in the bad-state utility, over the increase in cost if one decides to do so. In other words, this is the mental value to cost of further distorting beliefs. The higher this value is, the stronger the shift in the mental process's best response, and the higher the insurance level needed for the mental process to maintain the original beliefs. This value to cost is compared with the relative risk aversion property of the utility function estimated at the bad state. The relative risk aversion property determines the change prescribed by the rational process due to an increase in the bad-state loss size. If the insurance needed by the mental process to maintain the same beliefs is greater than the prescribed change by the rational process (i.e., large value per cost of further distorting beliefs), then the result is a backfire effect of lower insurance as it is in the figure above.

To illustrate the two conditions, suppose the utility function \( u(\cdot) \) exhibits constant or increasing absolute risk aversion. In this case, educational campaigns will lead to a more optimistic view if and only if the agent initially buys less than full insurance. This will translate into lower insurance purchase depending on the curvature of the utility function at the bad state relative to the cost function. Taking the cost function discussed in the next section, \( \sum_{j=1}^{K} p_j \log(p_j/p_0) \), a backfire effect will occur if the marginal utility at the bad state is greater than the coefficient of absolute risk aversion at that point. Hence, for such utility functions, educational campaigns can divide the insurance market into a set of agents who purchase more insurance—the intended consequence—and a set of agents who purchase less insurance—the unintended consequence. These conditions are true for any equilibrium, even in the case of multiplicity of equilibria.

### 3. ADM potential games

In this section we present a state-preference model of affective decision making where an agent maximizes her utility subject to a budget constraint and is facing \( K \) possible states of nature; this section therefore extends the analysis in Section 2. More specifically, consider an agent who faces \( K \) possible states of nature and has a utility function \( u \) over outcomes. The rational process chooses an action \( z \) from the interior of the budget hyperplane \( Z \) to maximize expected utility, while the emotional process chooses beliefs \( p \) from the interior of the probability simplex \( \Delta \) to maximize its objective function. The emotional process's objective function is the potential function for the ADM intrapersonal game and is given...
by $\Theta(z, p) = (U(z), p) - J^*(p)$, where $U(z)$ is the state-utility vector and $J^*(p)$ is a function of Legendre-type—i.e., a strictly convex, essentially smooth function on the interior of the probability simplex $\Delta$.\footnote{The notion of a strictly convex, essentially smooth function is due to Rockafellar (1970)—see Chapter 26. If $\Omega$ is the interior of the effective domain of a proper, extended real-valued convex function $f : R^k \rightarrow [+-\infty]$, then $f$ is essentially smooth if: (i) $\Omega$ is not empty, (ii) $f$ is differentiable throughout $\Omega$, (iii) $\lim_{t \rightarrow \infty} \|\nabla f(x)\| = +\infty$ whenever $x_1, x_2, \ldots$ is a sequence in $\Omega$ converging to a boundary point $x$ of $\Omega$. He defines the class of strictly convex and essentially smooth functions to be of Legendre-type.}

In the general $K$-state case we define optimism-bias to be:

**Definition 6 (Optimism bias).** We say an agent is optimistically biased if for all state-utility vectors $U(z)$ and $U(y)$:

$$\left[\nabla U(z) J(U(z)) - \nabla U(y) J(U(y))\right] \cdot [U(z) - U(y)] > 0,$$

where:

$$J(U(z)) = \max_{p \in \Delta} \{[U(z), p] - J^*(p)\}.$$

This condition says that affective probabilities are a strictly increasing monotone map of the state-utility vector $U(z)$. To see this, note that the properties of the mental cost function $J^*(p)$ allow us to represent the emotional process problem in the domain of state-utility vectors by using the Legendre-Fenchel conjugate of a smooth, convex function. More specifically:

$$J(U(z)) = \max_{p \in \Delta} \{[U(z), p] - J^*(p)\},$$

where the gradient $\nabla J^*(p)$ maps probability distributions into state-utility vectors. In other words the function $J(U(z))$ is an alternative representation of the emotional process problem, and it is a function of Legendre type as well. This alternative representation will play an important role in our analysis, as will become clear in Section 4. By using the biconjugate theorem, the following holds true as well:

$$J^*(p) \equiv \max_{U(z) \in R^n} \{[U(z), p] - J(U(z))\},$$

i.e., the double conjugate of $(J^*)^\ast$ is $J^*$ itself, and both functions ($J^*$ and $J$) are convex functions of Legendre type. Note that this equivalence is similar to the well-known dual relationship between the cost and profit functions of a price-taking, profit-maximizing firm producing a single good.

This dual relationship gives rise to our definition of optimism-bias in the general $K$-state case: it follows from this relationship that the affective probabilities chosen by the emotional process for the act $z$ are

$$\nabla U(z) J(U(z)) = \max_{p \in \Delta} \{[U(z), p] - J^*(p)\}.$$

Hence, our definition of optimism bias says, in fact, that affective probabilities $\nabla U(z) J(U(z))$ are a strictly increasing monotone map of the state-utility vector $U(z)$. This means that the assigned probability is skewed towards the more favorable outcomes of an act $z$. In the two states case this relationship implies, for instance, that comparing no insurance to full insurance, the assigned probability to the bad state occurring will be lower under no insurance. Likewise, comparing any two positive insurance levels, the assigned probability to the bad state occurring will be lower under no insurance. Therefore this definition subsumes the intuitive definition of optimism bias in the two state case and it is our definition of optimism bias in the $K$-state world.

As noted $J(U(z))$ is a convex function of Legendre type (since $J^*(p)$ is), and for this reason its differential mapping is a one-to-one monotone map from the interior of the budget plane to the interior of the probability simplex (see Corollary 26.3.1 in Rockafellar, 1970); in other words, our definition of optimism bias holds. Remarkably, this shows that the assumed shape of the mental cost function in the demand for insurance example (strictly convex and essentially smooth), reflecting the psychological characterization of affective costs, is both necessary and sufficient for optimism-bias. It also means that although beliefs are unobservable, we do have conditions on the state-utility vector which reflect optimism in beliefs.

Using this framework we now extend the existence and local uniqueness of pure strategy Nash equilibria results in the two-state ADM potential game to ADM potential games with $K$ states of the world. To do this, we define regular potential games. Regular potential games are potential games where the potential function $\Theta(z, p)$ is a Morse function or equivalently $0$ is a regular value of $\nabla_{z, p}[\Theta(z, p)]$, where $z$ is an action and $p$ is a belief. That is, the Hessian of the potential function $\Theta(z, p)$ evaluated at a critical point of the potential is non-singular. See Chapter 1 in Guillemin and Pollack (1974) for a discussion of Morse functions, where they show that “most” smooth functions are Morse functions.

We show that essentially smooth and strictly bi-concave potential games that are regular have an odd number of locally unique pure strategy Nash equilibria. In the proof we use the homotopy principle, which implies an algorithmic interpreta-
tion and allows for the computation of a pure-strategy Nash equilibrium. Moreover, we show that if the potential function \( \Theta(z, p) \) is essentially smooth and strictly bi-concave, then the set of pure strategy Nash equilibria is non-empty, where 0 need not be a regular value of \( V(z, p)[\Theta(z, p)] \).

**Proposition 7.** If \( \Theta(z, p) \) is the potential function for a regular potential game \( G \), where the strategy-sets \( Z \) and \( \Pi \) are the interiors of non-empty, convex, compact subsets of \( R^k \), is essentially smooth and strictly bi-concave, then \( G \) has an odd number of locally unique, pure strategy Nash equilibria.

In the context of ADM, \( \Theta(z, p) = (U(z, p) − J^*(p)) \) is the potential function for a regular ADM potential game \( G \), where \( Z \) is the interior of the budget hyperplane and \( \Pi \) is the interior of the probability simplex. By Proposition 7, if \( \Theta(z, p) \) is essentially smooth and strictly bi-concave, then \( G \) has an odd number of locally unique, pure strategy Nash equilibria.

**Proposition 8.** If \( \Theta(z, p) \) is the potential function for a potential game \( G \), where the strategy-sets \( Z \) and \( \Pi \) are the interiors of non-empty, convex, compact subsets of \( R^k \), is essentially smooth and strictly bi-concave, then the set of pure strategy Nash equilibria is non-empty.

Taking \( \Theta(z, p) = (U(z, p) − J^*(p)) \) to be the potential function for an ADM potential game \( G \), \( Z \) to be the interior of the budget hyperplane and \( \Pi \) the interior of the probability simplex, Proposition 8 implies that if the potential function \( \Theta(z, p) \) is essentially smooth and strictly bi-concave, then the set of pure strategy Nash equilibria for the ADM potential game is non-empty.

The conditions for uniqueness are given below, where they extend Neyman (1997) theorem on the uniqueness of pure strategy Nash equilibrium for potential games with compact strategy sets to potential functions with bounded, open strategy sets.

**Proposition 9.** If \( \Theta(z, p) = (U(z, p) − J^*(p)) \) is the potential function for a regular ADM intrapersonal game \( G \), where the strategy-sets \( Z \) is the interior of the budget hyperplane and \( \Pi \) is the interior of the probability simplex, is essentially smooth and strictly concave, then \( G \) has a unique pure strategy Nash equilibria.

In the insurance example, the affective cost function depends on baseline probabilities \( p_0 \). In the general \( K \)-states framework the analogous concept is Bregman divergence—a generalization of relative entropy, used in information theory to measure the “directed distance” from a fixed probability distribution \( q \) to other probability distributions \( p \) (see Banerjee et al., 2005 for a general discussion of Bregman divergences.) More specifically, the Bregman divergence associated with a convex function \( F \) on the interior of the simplex \( \Delta \) and a “prior” probability distribution \( q \in \Delta \) is:

\[
D(p \parallel q) = F(p) - F(q) - \nabla F(q) \cdot (p - q).
\]

Notice that for all \( q \) and \( p \) in the interior of \( \Delta \), the Bregman divergence is nonnegative, and reaches a minimum at \( q \). In other words, if \( q \) is the baseline probability, \( D(p \parallel q) \) is the “directed distance” of \( p \in \Delta \) from \( q \). Importantly, every convex function of Legendre type on the interior of \( \Delta \) and a “prior” probability distribution \( p_0 \) in the interior of \( \Delta \) defines a Bregman divergence \( D(p \parallel p_0) \) of Legendre type. Bergman divergences of Legendre type satisfy the conditions we impose on the mental cost function and are therefore of a particular interest for our purposes. In that class of Bregman divergences, the relative entropy \( J^*(p) = \sum_{j=1}^{K} [p_j \log(p_j/p_{0j})] \) is of a particular interest, as it is the measure of model misspecification used in the multiplier preferences by Hansen and Sargent (2000) and therefore shows that the structure of the mental cost function we suggest is consistent with a well-known cost function used in the literature.

Given this, here is an example of an essentially smooth, strictly bi-concave ADM intrapersonal potential game:

Let \( u(w) = w^\delta \), where \( \delta \in (0, 1) \)

and

\[
J^*(p) = \sum_{j=1}^{K} [p_j \log(p_j/p_{0j})],
\]

then

\[
\Theta(z, p) = \sum_{j=1}^{K} z_j^p p_j - \sum_{j=1}^{K} [p_j \log(p_j/p_{0j})].
\]

The general setup of ADM potential games can be applied to insurance markets in a straightforward manner: instead of two states as considered in Section 2 now the agent faces \( K \) possible future states of the world with an associated wealth.
level vector $w$ (where $w_i$ is the wealth level in state $i \in K$). The different acts available to the agent are equivalent to different insurance vectors $I$, and the rational process choose a utility vector $U(I)$, to maximize expected utility given a perceived risk $p$. Specifically, the rational process maximizes $U(I(p))$, while the emotional process chooses an optimal risk perception $p^{*}$ given an insurance $I$, to maximizes $\Theta(I, p) = (U(I), p - J^{*}(p))$. In the general case, we require that the mental cost function $J^{*}(p)$ be a strictly convex and essentially smooth function on the interior of the probability simplex $\Delta$, i.e., a convex function of the Legendre-type. Given this set up, all results of the general ADM potential games immediately follow.

### 4. Axioms for optimistic preferences

In Section 3 we show that attitudes towards optimism reduce to the convexity of the utility representation of preferences over acts. $J(\cdot)^{2}$: in this section we use this property to derive axioms for optimistic preferences. That is, we show that preferences over acts, $z$, are optimistic if and only if there exists a continuous utility function $u$ over outcomes and a continuous convex function $J$ over state-utility vectors $U(z)$, i.e., the vector of utilities of outcomes for the act $z$. The ADM model we present below is an example of optimistic preferences over acts.

The axiomatic characterization of optimistic preferences is an amendment of the axiomatic characterization of variational preferences in Maccheroni, Marinacci and Rustichini (2006) [MMR], where: $S$ is the set of states of the world; $\Sigma$ is an algebra of subsets of $S$, the set of events; and $X$, the set of consequences, is a convex subset of some vector space. $F$ is the set of (simple) acts, i.e., finite-valued $\Sigma$-measurable functions $f : S \rightarrow X$. $B(\Sigma)$ is the set of all bounded $\Sigma$-measurable functions, and endowed with the sup-norm it is an AM-space with unit, the constant function 1. $B_{b}(\Sigma)$ the set of $\Sigma$-measurable simple functions is norm dense in $B(\Sigma)$. The norm dual of $B(\Sigma)$ is $B^{\ast}(\Sigma)$, finitely additive signed measures of bounded variation on $\Sigma$ (see Aliprantis and Border, 1999 for further discussion). Below we present the axioms:

A.1 (Weak order): If $f$, $g$, $h \in F$, (a) either $g \succeq f$ or $f \succeq g$, and (b) $f \succeq g$ and $g \succeq h \Rightarrow f \succeq h$.

A.2 (Weak certainty independence): If $f$, $g$, $h \in F$, $x$, $y \in X$ and $\alpha \in (0, 1)$, then $\alpha f + (1 - \alpha)x \succeq \alpha g + (1 - \alpha)x \Rightarrow \alpha f + (1 - \alpha)y \succeq \alpha g + (1 - \alpha)y$.

A.3 (Continuity): If $f$, $g$, $h \in F$, the sets $\{ \alpha \in [0, 1] : \alpha f + (1 - \alpha)g \succeq h \}$ and $\{ \alpha \in [0, 1] : h \succeq \alpha f + (1 - \alpha)g \}$ are closed.

A.4 (Monotonicity): If $f$, $g \in F$ and $f(s) \succeq g(s)$ for all $s \in S$, the set of states, then $f \succeq g$.

A.5 (Quasi-convexity): If $f$, $g \in F$ and $\alpha \in (0, 1)$, then $f - \sim g \Rightarrow \alpha f + (1 - \alpha)g \succeq \alpha f - \sim \alpha g$.

A.6 (Nondegeneracy): $f \not\succeq g$ for some $f$, $g \in F$.

These axioms where $A.5$ is replaced by $A.5$ (quasi-concavity) are due to MMR (2006).

**Theorem 10.** Let $\succeq$ be a binary order on $F$. The following conditions are equivalent:

2. There exists a nonconstant function $u : X \rightarrow R$, unique up to a positive affine transformation, and a continuous convex function $J^{*} : \Delta \rightarrow [0, \infty]$ where for all $f$, $g \in F$, $f \succeq g \iff W(f) \succeq W(g)$, where $W(h) = J(U(h)) = \max_{p \in \Delta} \{ U(h), p - J^{*}(p) \}$ is a convex function of $U(h)$ by the biconjugate theorem.

Theorem 10 shows that a preference relation satisfying axioms A.1–A.6 is optimistic in the sense that it has a convex representation $J(U(h)) = \max_{p \in \Delta} \{ U(h), p - J^{*}(p) \}$, and choice can be represented as a pure strategy Nash equilibrium maximizing a potential ADM intrapersonal game $\arg \max_{z \in Z} J(U(z)) = \max_{p \in \Delta} \{ U(h), p - J^{*}(p) \}$.

In the standard approach, as in the expected utility theory and prospect theory, the agent maximizes over actions, not over both action and beliefs. In our set up, $\max_{z \in \Delta} J(U(z)) = \max_{p \in \Delta} \Theta(z, p)$ are the potential maximizers, a subset of the set of pure-strategy Nash equilibria of the ADM potential game. If the ADM model has a unique pure strategy Nash equilibrium then maximizing the composite utility function $J(U(z))$ over actions and maximizing the potential $\Theta(z, p)$ over actions and beliefs rationalize the same observed choices. Hence, these models are refutable. That is, not every data set can be rationalized with an ADM potential game.

The set of axioms, and therefore the ADM model, is closely related to existing models in the literature: if axiom $A.5$—ambiguity seeking—is replaced by axiom 5 of MMR—ambiguity aversion—then we get the variational preferences model of MMR. If we replace axiom $A.5$—ambiguity seeking—with axiom 5 of ambiguity-neutrality we get the subjective expected utility model (both results are proved in MMR, 2006). This relationship suggests an equivalence between optimism bias and

---

7 Strict convexity and essentially smoothness are generic properties of convex functions on $R^k$. This is an immediate consequence of Howe’s (1982) theorem where he proved that the generic convex function on $R^k$ is in fact strictly convex and smooth. To show that the generic convex function on $R^k$ is also essentially smooth, we use the gradient map. The gradient map of a strictly convex and differentiable function is strictly monotone, which implies the gradient map is one-to-one. By Corollary 26.3.1. in Rockafellar these functions must be strictly convex and essentially smooth. That is, strict convexity and essentially smoothness are generic properties of convex functions on $R^k$. 
pessimism bias to ambiguity seeking and aversion, respectively. That is, the ADM model has an alternative interpretation of ambiguity-seeking behavior and variational preference has an alternative interpretation of pessimism bias.

Under the ADM formulation ambiguity is endogenous, generated by the individual in a skewed manner. If the individual is optimistic then the generated endogenous ambiguity will be favorable to her, therefore being optimistic is being ambiguity seeking in this case. That is, the individual prefers the situation where she does not know the exact probability distribution. If the individual is pessimistic, then the generated endogenous ambiguity will be unfavorable to her, therefore being pessimistic is being ambiguity averse. In that sense, ambiguity attitudes are the same behavior as optimistic or pessimistic attitudes. In contrast, the variation preference formulation conceives ambiguity as exogenous, imposed on the individual.

This distinction between different types of ambiguity (self-generated or imposed) helps clarify the relationship between the game against nature in MMR's variational preferences model and the game against the emotional process in the ADM model: in both variational preferences and ADM, nature and the emotional process, respectively, are formal descriptions of uncertainty. In MMR this is a formal description of an imposed uncertainty, while in the ADM model it is a formal description of generated uncertainty. Even if one considers the unique equilibrium ADM case, the emotional self still generates ambiguity in the sense that it allows the agent to consider several possible probability distributions.

5. Discussion

In light of the ambiguity averse attitudes often observed in the Ellsberg experiment and the popularity of models based on ambiguity aversion in financial markets, the alternative interpretation of ADM as ambiguity seeking leads to a natural question: when should we expect agents to exhibit ambiguity aversion or pessimism bias and when should we expect agents to exhibit ambiguity seeking or optimism bias? The two motivating examples in the introduction as well as the evidence on CEOs who are optimistic regarding the performance of their own companies, illustrate optimism bias in situations where one's own ability or skill is at stake. Indeed, there is, quite intuitively, high motivation to hold favorable beliefs when one's own ability is at stake. At the same time, information on personal ability is abundant and accessible to the self, making a biased search relatively inexpensive and the mental cost low.

Interestingly, one of the main factors recognized as contributing to optimism bias even in chance events is the illusion of control—the tendency to believe that, or act as if, one can skillfully influence and control the outcomes of chance events. Rolling a dice softly for low numbers or hard for high numbers is an example (Henslin, 1967). Placing a higher value on a lottery ticket when it was personally selected is another (Langer, 1975). Hence, skill, even if unrelated as in the case of winning a lottery, may still lead to optimism bias. Familiarity, as other factors that mimic skill situations, is an important factor contributing to the illusion of control (e.g., Langer, 1975; Harris, 1996). In the context of motivated reasoning, familiarity is likely to reduce the mental cost since, as in the case of personal abilities, the more familiar one is with a topic the easier it is to search the memory to find support for optimistic views.

Skill has also been found to contribute to ambiguity seeking attitudes. Using compound gambles involving both skill and chance components, Cohen and Hansel (1959) and Howell (1971) find that most choices reflect a preference for skill over chance, holding the overall probability of winning constant. This is in spite of the fact the skill component was more ambiguous. These studies contributed to the “competence hypothesis” suggested by Heath and Tversky (1991), where the willingness to bet on an uncertain event depends on one's general knowledge or understanding of the relevant context. That is, holding judged probability constant, people prefer to bet in a context where they consider themselves knowledgeable or competent than in a context where they feel ignorant or uninformed. Heath and Tversky argue that familiarity encourages the feeling of competence, and demonstrated this competence effect in several studies.

Considering motivated reasoning we expect a desire to hold optimistic beliefs to be present when the issue at stake is a personal future outcome: future success, health, employment, or financial gain. When such outcomes are at stake we would expect optimism bias when the mental cost function allows it. In light of the above factors, we expect this to be more likely to happen when individuals are familiar with the subject matter pertaining to the risky or uncertain outcomes. If one is familiar or competent in a given subject, then it is easier to rationalize optimistic views, i.e., the mental cost of supporting optimistic beliefs is relatively low.

We therefore conjecture that individuals are more likely to be optimistically biased or ambiguity seeking in markets such as the insurance market and the labor market, while ambiguity averse to investment in unfamiliar markets, such as bond markets in emerging economies.

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Appendix A. Proofs

Proof of Proposition 2. Denote the rational process's payoff function as $R$ and the emotional process's payoff function as $E$. A necessary and sufficient condition for the intrapersonal game to have a potential function (Monderer and Shapley, 1996) is $\frac{\partial^2 \Pi}{\partial p \partial z} = \frac{\partial^2 \Pi}{\partial q \partial z}$. This condition clearly is satisfied in the ADM model. The potential function $\theta(p, I)$ is a function such that (Monderer and Shapley, 1996): $\frac{\partial^2 \theta}{\partial p \partial z} = \frac{\partial^2 \theta}{\partial q \partial z} = \frac{\partial \theta}{\partial z}$ and $\frac{\partial \theta}{\partial q} = \frac{\partial \theta}{\partial z}$. Because $\frac{\partial \theta}{\partial z} = \frac{\partial E}{\partial z}$, $E$ can serve as a potential function. The critical points of the potential function are $\frac{\partial \theta}{\partial p} = \frac{\partial \theta}{\partial q} = 0$, $\frac{\partial \theta}{\partial z} = 0$. The potential function is strictly concave in each variable, so at each critical point, each process is maximizing its objective function given the strategy of the other process. Therefore, the critical points of the potential function are the pure strategy Nash equilibria of the intrapersonal game, and all pure strategy Nash equilibria are critical points of the potential function. \qed

Proof of Proposition 3. Given the assumption on the mental cost and the utility functions, we know the relationship between the emotional process and the rational process best response at the extreme beliefs $[0, 1]$. More specifically, as $p \to 0$, the rational process best response is “higher” and this relationship exactly flips when $p \to 1$. Hence, there exist a pure strategy Nash equilibria; since the best responses are monotonically increasing, it follows that there exists odd number of Nash equilibria. \qed

Proof of Proposition 4. Consider the case in which $\gamma = p_0$. At full insurance, there is no mental gain for holding beliefs $p \neq p_0$ but there exists mental cost. Therefore, at full insurance, the emotional process’s best response is $p = p_0$. Given that $\gamma = p_0 = p$, the rational process’s best response is full insurance. Consequently, full insurance and $p = p_0$ is a Nash equilibrium of this case. Next, consider the case $\gamma > p_0$; because the insurance premium is higher than $p_0$, $I^* = p_0 < z$, where $z$ is the loss size. Also, $p^* = p_0$ only at full insurance, where $I = z$. Therefore, at $p = p_0$ the emotional process’s best response falls above the rational process’s best response. This relationship is reversed at the limit $p \to 0$, and both the emotional and the rational best responses increase; therefore, there exists a Nash equilibrium with $p < p_0$ and less insurance than predicted by the expected utility model. A similar argument can be used to prove the result when $\gamma < p_0$. \qed

Proof of Proposition 5. Define $\tilde I(p; p_0)$ as the inverse function $p^{* -1}$. Define $\Pi(p; p_0) = I^*(p) - \tilde I(p; p_0)$; a solution $\Pi(p; p_0) = 0$ is a NE, and $\frac{\partial \Pi}{\partial z} < 0$, where $z$ is the loss size, represents the unintended consequence of greater optimism due to such campaigns.

$$
\frac{\partial \Pi}{\partial z} < 0 \iff \frac{\partial \tilde I}{\partial z} > 1,
$$

$$
\frac{\partial I^*}{\partial z} = \frac{u''(w_G - z + (1 - \gamma)I^*)|u'(w_G - \gamma I^*)|^2}{u'(w_G - z + (1 - \gamma)I^*)|u''(w_G - \gamma I^*)|},
$$

$$
\frac{\partial \tilde I}{\partial z} = \frac{u'(w_G - z + (1 - \gamma)\tilde I)}{u'(w_G + (1 - \gamma)\tilde I)}
$$

$$
\frac{\partial \Pi}{\partial z} < 0 \iff \frac{r(w_G - \gamma I)}{u'(w_G - \gamma I)} > \frac{r(w_B + (1 - \gamma)I)}{u'(w_B + (1 - \gamma)I)},
$$

where $r(x) = -\frac{u''(x)}{u'(x)}$.

Define $\hat p(I)$ as the inverse function $p^{*-1}$. Define $\hat \Pi(I) = p^*(I) - \hat p(I)$; a solution $\hat \Pi(I) = 0$ is a NE, and $\frac{\partial \hat \Pi}{\partial z} < 0$, where $z$ is the loss size, represents the unintended consequence of less insurance due to such campaigns.

$$
\frac{\partial \hat \Pi}{\partial z} < 0 \iff \frac{\partial \hat p}{\partial z} > 1,
$$

$$
\frac{\partial p^*}{\partial z} = \frac{u'(w_B + (1 - \gamma)I)}{\Pi''},
$$

$$
\frac{\partial \hat p}{\partial z} = \frac{pu''(w_B + (1 - \gamma)I)(1 - \gamma)(1 - \gamma)}{u'(w_B + (1 - \gamma)I)}
$$

$$
\frac{\partial \hat \Pi}{\partial z} < 0 \iff \frac{u'(w_B + (1 - \gamma)I)}{r(w_B + (1 - \gamma)I)} > p(1 - p)\Pi''(p),
$$

where $r(x) = -\frac{u''(x)}{u'(x)}$.

Note that when $\frac{\partial \Pi}{\partial z} < 0$ or $\frac{\partial \hat \Pi}{\partial z} < 0$ at an unstable equilibrium (under the usual Tatoumout adjustment process), the result would be still greater optimism and less insurance respectively. This is because as a result of the change, a person whose choice was according to an unstable NE would converge now to a stable, lower, NE. \qed
The proofs of Propositions 7 and 8 use the homotopy principle—see Chapters 1, 3 and 22 in Garcia and Zangwill (1981). The homotopy principle admits an algorithmic interpretation that can be used to compute a pure strategy Nash equilibrium of the potential game—see Chapter 2 in Garcia and Zangwill (1981).

**Proof of Proposition 7.** Consider the following homotopy: $H(t, \theta, \theta_0) = (1 - t)(\theta - \theta_0) + t\nabla[\Theta(\theta)]$, where $\theta = (z, \pi)$, $\theta_0 = (z_0, \pi_0)$ and $t \in [0, 1]$. $0$ is a regular value of $H(0, \theta, \theta_0)$, since $[\partial H(0, \theta, \theta_0)/\partial \theta] = I_{2K}$, the identity matrix on $R^K \times R^K$. $0$ is also a regular value of $H(1, \theta, \theta_0)$, since $[\partial H(1, \theta, \theta_0)/\partial \theta] = \text{Hess}[\Theta(\theta)]$ and $\Theta(\theta)$ is a Morse function. For $t \in (0, 1)$, $[\partial H(t, \theta, \theta_0)/\partial \theta] = -I_{2K}$. Hence $0$ is a regular value of $H(t, \theta, \theta_0)$ for all $t \in (0, 1)$ by the transversality theorem (parametric Sard’s theorem). That is, $0$ is a regular value of $H(t, \theta_0)$ for almost all $\theta_0 \in \Phi$, where $\Phi = K \times \Pi$—see Chapter 2 in Guillemin and Pollack (1974) for a proof of the transversality theorem. The assumption that $\Theta(\theta)$ is essentially smooth, i.e., $\|\nabla_\theta[\Theta(\theta)]\| \to \infty$, as $\theta \to bdry(\Phi)$ implies that the homotopy is boundary-free. Hence by the homotopy principle, $\nabla_\theta[\Theta(\theta)]$ has an odd number of regular points—see the proof of Theorem 3.2.3 in Garcia and Zangwill (1981). Since $\Theta(\theta)$ is strictly bi-concave, it follows that $\Theta(\theta)$ has an odd number of locally unique, pure strategy Nash equilibria. □

**Proof of Proposition 8.** If the set of pure strategy Nash equilibria is empty, then the set of critical points is empty and $0$ is a regular value, contradicting Proposition 7. Hence there exists at least one singular critical point. That is, there exists at least one pure strategy Nash equilibrium. □

**Proof of Theorem 10.** Axioms 1–4 are used in MMR to derive a nonconstant utility function, $u$, unique up to a positive affine transformation, over the space of consequences, $X$. $u$ is extended to the space of simple acts, $F$, using certainty equivalents. That is, $U(f) = u(x_f) \in B_0(\Sigma)$ for each $f \in F$, where $x_f$ is the certainty equivalent of $f$. This is Lemma 28 in MMR, where $I(f) = U(f)$ is a niveloid on $\Phi = \{\varphi: \varphi = u(f) \text{ for some } f \in F\}$. Niveloids are functionals on function spaces that are monotone: $\varphi \leq \eta \implies I(\varphi) \leq I(\eta)$ and vertically invariant: $I(\varphi + r) = I(\varphi) + r$ for all $\varphi$ and $r \in R$—see Dolecki and Greco (1995) for additional discussion. $\Phi$ is a convex subset of $B(M)$ and by Schmeidler’s (1989) Axiom 5, $\Phi$ is quasi-concave on $\Phi$. We also assume Axioms 1–4, so Lemma 28 in MMR holds for the niveloid $J$ in the ADM representation theorem. By Axiom 5, $J$ is quasi-convex on $\Phi$. MMR show in Lemma 25 that $J$ is concave if and only if $J$ is quasi-concave. Hence $J$ is convex if and only if $J$ is quasi-concave, since $J$ is convex (quasi-convex) if and only if $-J$ is concave (quasi-concave). MMR extend $I$ to a concave niveloid $I$ on all of $B(\Sigma) \subseteq \Phi$ in Lemma 25 and 22 in MMR. Epstein et al. (2007) show in Axiom 5.7 that niveloids are Lipschitz continuous on any convex cone of an AM-space with unit and concave (convex) if and only if they are quasi-concave (convex). Hence, since $B(\Sigma)$ is a convex cone in an AM-space with unit, $I$ is Lipschitz continuous. It follows from the theorem of the biconjugate for continuous, concave functionals that $I(\varphi) = \inf_{p \in \text{bar}(\Sigma)} \{\int \varphi dp - \hat{J}(p)\}$. where $\hat{J}(p) = \inf_{\varphi \in B_k(\Sigma)} \{\int \varphi dp - \hat{I}(\varphi)\}$ is the concave, conjugate of $\hat{I}(\varphi)$—see Rockafellar (1970), p. 308 for finite state spaces. MMR show on p. 1476 that we can restrict attention to $\Delta$, the family of positive, finitely additive measures of bounded variation in $\text{bar}(\Sigma)$. Hence $I(\varphi) = \min_{p \in \Delta} \{\int \varphi dp - \hat{J}(p)\} = \min_{p \in \Delta} \{\int u(f) dp - J^*(p)\}$, where $\varphi = u(f)$ and $J^*(p) = -\hat{I}^*(p)$. $J^*(p)$ is convex since $\hat{I}^*(p)$ is concave.

Extending $-J$ to $\hat{J}$ on $B(\Sigma)$, using Lemma 25 in MMR, it follows from the theorem of the biconjugate for continuous, convex functionals that

$$J(\varphi) = \max_{p \in \Delta} \{\int \varphi dp - \hat{J}^*(p)\},$$

where

$$\hat{J}^*(p) = \max_{\varphi \in B_k(\Sigma)} \{\int \varphi dp - \check{J}(\varphi)\}$$

is the convex, conjugate of $\hat{J}(\varphi)$—see Rockafellar (1970), p. 104 for finite state spaces and Zălinescu (2002), p. 77 for infinite state spaces. Again it follows from MMR that

$$J(\varphi) = \max_{p \in \Delta} \{\int \varphi dp - \hat{J}^*(p)\} = \max_{p \in \Delta} \{\int u(f) dp - J^*(p)\} = W(f),$$

where $\varphi = u(f)$ and $J^*(p) = \hat{J}^*(p)$. $J^*(p)$ is convex since $\hat{J}^*(p)$ is convex.

$$f \succeq g \iff J(u(f)) \geq J(u(g)) \iff W(f) \geq W(g).$$

Hence $\arg \max_{f \in F, p \in \Delta} \{\int u(f) dp - J^*(p)\}$ is set of pure strategy Nash equilibria of the ADM intrapersonal game, where $u(\cdot)$ is the Bernoulli utility function of the rational process and $J^*(\cdot)$ is the cost function of the emotional process. It follows that the axioms for ambiguity-seeking preferences also characterize the ADM intrapersonal games with a unique pure strategy Nash equilibrium: $\arg \max_{f \in F, p \in \Delta} \{\int u(f) dp - J^*(p)\}$. □