APPLICATION OF A COMPREHENSIVE DYNAMIC CUTTING FORCE MODEL TO ORTHOGONAL WAVE-GENERATING PROCESSES

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Abstract—The dynamics of a cutting process are very complex in nature. They involve not only the changes of plastic state in the intensive shear zone of the chip formation process but also the elastic behaviour of work material surrounding the plastic deformation zone, especially in the vicinity of the tool nose region. As an extension to the previous developments in formulating the shear angle oscillation in dynamic cutting (D. W. Wu, Development of dynamic shear angle model for wave-generating processes based on work-hardening slip-line field theory. Int. J. Mech. Sci. 29, 407-424, 1987; D. W. Wu, Governing equations of the shear angle oscillation in dynamic orthogonal cutting. Trans. ASME J. of Engrg for Indust. 108, 280, 1986), a comprehensive dynamic cutting force model has been developed from the mechanics of the cutting process by taking into account the equilibrium of forces in the primary and secondary plastic deformation zones and the redistribution of the contact stress inside the workpiece in the vicinity of the tool nose region.

The model has been tested through a computer simulation for orthogonal wave-generating processes. By reference to existing experimental evidence, the theoretical predictions show generally good agreement with the test results.

NOTATION

- $E$: Young's modulus of elasticity (p.s.i.)
- $F_s$: cutting force component normal to the cutting direction (lb)
- $F_r$: cutting force component along the cutting direction (lb)
- $f_{sp}$: specific contact force (lb/in$^2$)
- $f_x$: contact force component normal to the cutting direction (lb)
- $f_y$: contact force component along the cutting direction (lb)
- $k$: shear flow stress (p.s.i.)
- $s_o$: mean uncut chip thickness (in.)
- $V$: volume of work material displaced by the tool penetration (in$^3$)
- $v_o$: present cutting speed (ft/min.)
- $w$: width of cut (in.)
- $X$: amplitude of the tool vibration (in.)
- $x$: displacement of the tool tip (in.)
- $x$: oscillation velocity of the tool tip (in/sec)
- $\alpha_o$: tool rake angle (deg.)
- $\gamma_o$: tool clearance angle (deg.)
- $\eta$: depth of tool penetration (in.)
- $\mu$: mean frictional coefficient at the tool-chip interface
- $\mu_e$: mean frictional coefficient around the tool nose region
- $M(\ )$: functional relationship for the mean frictional coefficient
- $\nu$: Poisson's ratio
- $\rho$: size of the elastic-plastic zone beneath the tool nose region (in.)
- $\phi$: dynamic shear angle (deg.)
- $\Phi(\ )$: functional relationship for the shear angle (deg.)
- $\phi_0$: mean shear angle (deg.)
- $\omega$: angular velocity of the tool vibration (rad sec$^{-1}$)

INTRODUCTION

Metal cutting is the complex process of deforming work material in a localized asymmetric area as a result of the relative motion between the workpiece and the tool. The elastic-plastic state of the work material within the deformation area is dependent upon its boundary
kinematic conditions, such as the cutting speed, uncut chip thickness etc., which are commonly known as the cutting conditions of the process.

In dynamic cutting, the oscillation of the relative motion between the workpiece and the tool changes the configuration of the process, and causes instantaneous changes in the boundary kinematic conditions. These changes affect the elastic-plastic state in the deformation area, and result in a variation of the cutting force. Figure 1 shows such relationships between the causes and effects, which can be described as a reaction chain. It is clear that to establish analytically the governing law for the dynamic cutting force in response to the tool vibration, each of the relationships in the reaction chain must be precisely defined.

Based on this modelling concept, a new comprehensive dynamic cutting force model is developed and presented in this paper. Unlike the existing theories [1–5], the model is derived from the mechanics of the cutting process instead of from purely geometric descriptions. This allows more insight into the physics of the cutting dynamics and provides a better understanding of the causes of machining instability.

Owing to the complexity of the cutting dynamics, only a single degree-of-freedom wave-generating process is discussed in this paper. Figure 2 represents such a system, in which a flexible tool, vibrating normal to the cutting direction, is set to remove a layer of workpiece with an originally flat top surface. A coordinate frame fixed at the equilibrium position of the tool tip is selected to describe the dynamic behaviour of the system. As shown in Figs 2 and 3, the tool tip has displacement $x$ with respect to the origin and oscillation velocity $\dot{x}$ along the vibration axis. The cutting speed, the mean uncut chip thickness, the width of cut, the tool rake angle and the tool clearance angle of the process are pre-set to be $v_0$, $s_0$, $w$, $\alpha_0$ and $\gamma_0$, respectively.

![Fig. 1. Relationships between causes and effects in dynamic cutting.](image1)

![Fig. 2. Vibration model of a single degree-of-freedom machining system.](image2)
The cutting process is said to be orthogonal when the cutting edge of a wedge-shaped tool is perpendicular to the direction of relative motion between the workpiece and the tool. In the case of a continuous chip formation process, this geometry represents a two-dimensional plane strain process, as shown in Fig. 4. Three principal deformation zones can be identified in the process:

1. the primary deformation zone where the work material is continuously sheared to form the chip;
2. the secondary deformation zone where large chip–tool frictional forces cause further deformation of the chip material;
3. the contact deformation zone where the work material is deformed as a result of the contact process between the tool nose and the workpiece.

The activities occurring in these deformation zones generate a resultant force exerted on the cutting tool. To simplify the analysis, this resultant force is divided into four independent components: the cutting forces $F_x$ and $F_y$ acting on the tool–chip interface, and the contact forces $f_c$ and $f_f$ on the tool nose and its adjacent flank face, as shown in Fig. 3. These force components fluctuate during a dynamic cutting process. The behaviour of these force components in response to the tool vibration is formulated as follows.

**Cutting forces $F_x$ and $F_y$ on the tool–chip interface**

As mentioned earlier, the chip is formed in the primary and secondary deformation zones. If the cutting conditions are favourable, the primary deformation zone can be regarded as a
plane called the shear plane, and the secondary deformation zone can be treated as a contact surface called the tool-chip interface [6]. The forces exerted on these two deformation fronts are always balanced to maintain the equilibrium of the chip even though the boundary kinematic conditions continuously change during a dynamic cutting process. This equilibrium condition results in a force diagram [6], shown in Fig. 5. The magnitudes of the cutting force components $F_x$ and $F_y$ can therefore be determined by

$$F_x = \frac{(s_0 - x)wk \sin (\tan^{-1} \mu - \alpha_0)}{\sin \phi \cos (\phi + \tan^{-1} \mu - \alpha_0)},$$

$$F_y = \frac{(s_0 - x)wk \cos (\tan^{-1} \mu - \alpha_0)}{\sin \phi \cos (\phi + \tan^{-1} \mu - \alpha_0)}.$$

In orthogonal wave generation, $\mu$ and $\phi$ continuously vary according to the tool vibration. Their behaviour can be defined by the following simultaneous governing equations of the shear angle oscillation [7, 8]:

$$\tan (\phi + \tan^{-1} \mu - \alpha_0) - \tan (\phi' + \tan^{-1} \mu' - \alpha_0 + \delta) = \frac{3}{2} (\phi' - \phi) + \frac{3}{2} \delta,$$

$$\mu = M(v_1, s_1, \alpha_1),$$

$$\mu' = M(v_2, s_2, \alpha_2),$$

$$\phi = \Phi(v_2, s_2, \alpha_2),$$

$$v_1 = \frac{v_0}{\cos \delta} - \dot{x} \cos \alpha_0 \cos (\phi - \alpha_0)/\sin (\phi - \delta),$$

$$v_2 = \frac{v_0}{\cos \delta},$$

$$s_1 = s_2 = (s_0 - x) \sin (\phi - \delta)/\sin \phi,$$

$$\alpha_1 = \alpha_2 = \alpha_0 - \delta,$$

and

$$\delta = \tan^{-1} \left( \frac{\dot{x}}{v_0} \right).$$

**FIG. 5. Cutting force diagram.**
where $v_0$, $s_0$, and $\alpha_0$ are the pre-set cutting speed, uncut chip thickness and rake angle respectively; $x$ and $\dot{x}$ are the displacement and velocity of the tool tip, $M(...)$ and $\Phi(...)$ are the functional relationships for the mean frictional coefficient and the shear angle respectively in terms of the cutting speed, the uncut chip thickness and rake angle; $v_1, v_2, s_1, s_2, \alpha_1, \alpha_2$, and $\delta$ are intermediate variables for calculating $\mu, \mu'$ and $\phi'$, and $\phi$ is the instantaneous shear angle. $M(...)$ and $\Phi(...)$ are two commonly used machining databases which can be established from a limited number of machining tests under steady-state conditions [9]. Hence, by solving equations (1)-(11) simultaneously the transfer relationships between the cutting force components $F_x$ and $F_y$ and the vibration variables $x$ and $\dot{x}$ can be determined.

**Contact forces $f_x$ and $f_y$ on the tool nose region**

Experimental evidence has shown that the clearance angle of the cutting tool has a strong damping effect on the cutting process. This effect is attributed to the actual contact between the workpiece and the tool nose region which includes the tool cutting edge and its adjacent flank face [10]. In order to determine the dynamic force induced by such a contact, it is necessary to understand what happens to the small portion of work material that approaches the contact area.

It is known that the cutting edge of the tool is never perfectly sharp [11], and that even a very carefully treated tool edge cannot retain its original sharpness after engaging a cut. It is also known that in metal cutting the material moving in front of the tool faces is severely retarded [12] owing to the friction between material layers. As a result, the separation of material around the tool nose region may effectively occur along a segment of a circular arc with a finite radius [13, 14], as shown in Fig. 6. The size of this effective radius depends on the tool geometry and the tool and work materials, as well as the cutting conditions: for example, cutting at either a low cutting speed or a small uncut chip thickness usually creates a large effective radius.

Because of the existence of this finite effective radius, the material approaching the tool cutting edge may be deformed in one of two ways. In the upper portion, the material is deformed and removed as a part of the chip [15]. In the lower portion, due to an effective large negative rake angle, the material cannot move upward to become part of the chip but instead is extruded and pressed under the tool [14, 16]. This extruded material flows under the tool nose region and eventually departs from under the tool flank face.

During this extrusion process, the material is displaced downward different distances according to the shape of the contact surface. The maximum surface displacement here is called the depth of tool penetration, which refers to the penetration of the tool into the workpiece. The total volume of material displaced at any given instant is dependent not only on the sharpness of the tool tip but also on the instantaneous cutting conditions. Since this volume of displaced material must be restrained by its surrounding material [17], a

![Fig. 6. Separation of material around the tool nose region.](image)
very complex elastic–plastic stress field is established under the tool nose region, as shown in Fig. 7.

This stress field is then in equilibrium with the normal and the tangential contact forces exerted on the contact surface where the separation of material around the tool nose region occurs. The directions of these contact forces alter continuously in accordance with the tool vibration. However, under a small vibration, the tangential load would act approximately in the mean cutting direction, i.e. along the y axis, and the normal load along the x axis. To simplify the analysis, the normal and tangential contact forces are treated in this paper as equal to the x and y components of the resultant mean contact force.

These two force components are interrelated in such a way that the ratio of the tangential load to the normal load is assumed to be a constant. The constant, which is denoted by $\mu_c$, is often regarded as the mean frictional coefficient around the tool nose region. The value of $\mu_c$ can be determined by experimental means. The relationship between $f_x$ and $f_y$ is defined therefore as

$$f_y = \mu_c f_x.$$  \hspace{1cm} (12)

Since the tangential load on the contact surface produces very little volumetric strain inside the workpiece, the majority of the material displaced by the tool penetration would be accommodated by the strain system induced by the normal load. It is assumed therefore that the normal contact force $f_x$ is proportional to the total volume of displaced material $V$, i.e.

$$f_x = f_{sp} V.$$  \hspace{1cm} (13)

where $f_{sp}$, called the specific contact force, is dependent on the mechanical and thermal properties of the work material. In order to predict the behaviour of the contact forces $f_x$ and $f_y$ in relation to the tool vibration, both the specific contact force $f_{sp}$ and the total volume of displaced material $V$ must be determined.

Calculating the total volume of material displaced

In dynamic cutting, the volume of material displaced by the tool through penetration varies as the tool vibrates and changes the instantaneous cutting direction. The relationship between the volume and the tool position is developed below.

Figure 8 shows the nose region of a cutting tool with a finite effective edge radius. In front of the rounded portion of the tool nose there is a singular point A which separates the
material approaching it into two regions, one flowing up with the chip over the tool rake face and the other flowing under the tool nose. The latter causes an elastic-plastic deformation of the sublayer of the work material and forms a localized stress system beneath the tool nose. Any material element moving across this deformation zone will experience a loading-unloading cycle. It is assumed that the side spread of work material near the contact area is negligible. For a continuous flow of work material under the tool nose region, the elastic strain developed in a material element during the loading cycle will be fully recovered during the unloading cycle. Let \( L \) be the locus of the tool motion traced out by point \( A \) while the vibrating tool progresses in cutting. Then the segment of \( L \) to the right of the tool flank face also represents the workpiece surface after machining, as indicated in Fig. 8.

Point \( A \) here is the beginning of the contact around the tool nose region. Point \( C \), which is the intersection of \( L \) and the tool flank face, then represents the end of the contact. Let \( B \) be the lowest point on the contact surface, as shown in Fig. 8. Then the area enclosed by the reference \( L \) and the profile ABC should be proportional to the total volume of material displaced by the tool penetration. In orthogonal wave-generation, as referred to in Figs 2 and 8, the volume of displaced work material can thus be calculated by multiplying the cross-sectional area \( ABCD \) by the width of cut \( w \) normal to the area.

Further, the area \( ABCD \) can be divided by the vertical line \( BD \) into two sub-areas \( ABD \) and \( BCD \), as shown in Fig. 8. The length of the segment \( BD \) is known as the depth of tool penetration. During dynamic cutting, sub-area \( BCD \) varies significantly according to the tool position or the phase of the tool vibration, as illustrated in Fig. 9. Sub-area \( ABD \), on the other hand, remains relatively constant when a small tool vibration takes place. This constant volume of displaced material would produce a constant contact force exerted on the tool, and thus have little influence on the dynamics of the cutting process. Therefore \( V \), the total effective volume of material displaced by the tool penetration, can be reduced to:

\[
V = w \text{ (sub-area } \ BCD), \tag{14}
\]

where \( w \) is the width of cut.

As illustrated in Fig. 9, the size of sub-area \( BCD \) is dependent on the tool vibration. For convenience in the analysis, it is assumed that the tool vibrates sinusoidally with an amplitude...
X and an angular velocity $\omega$. The displacement of the tool tip at time $t$ can thus be described as

$$x = X \sin \omega t.$$  \hspace{1cm} (15)

Since point D is very close to the tool tip, the displacement of D at time $t$ is approximately equal to $x$. Let E be an arbitrary point on the locus L which has a horizontal distance $\zeta$ from the vertical line BD, as shown in Fig. 8. The displacement $u$ of point E at time $t$ can be described by a wave function

$$u(t, \zeta) = X \sin \omega t \left( t + \frac{\zeta}{v_0} \right),$$  \hspace{1cm} (16)

where $X$ and $\omega$ are the amplitude and the angular velocity of the tool vibration respectively and $v_0$ is the pre-set cutting speed. For example, at time $t$ the displacement at point D equals $X \sin \omega t$ for $\zeta = 0$, and the displacement at point C equals $X \sin \omega (t + \zeta_c/v_0)$ for a horizontal distance $\zeta_c$, as shown in Fig. 8.

By using this wave function, the surface displacement of point E at time $t$, i.e. EF in Fig. 8, can be defined as a function $h(\zeta)$ such that

$$h(\zeta) = u(t, \zeta) - u(t, 0) + \eta - \zeta \tan \gamma_0,$$

or

$$h(\zeta) = X \sin \omega t \left( t + \frac{\zeta}{v_0} \right) - X \sin \omega t - \eta - \zeta \tan \gamma_0,$$  \hspace{1cm} (17)

for $0 \leq \zeta \leq \zeta_c$, where $\eta$ is the depth of tool penetration equal to BD, $\gamma_0$ is the pre-set clearance angle of the tool and $\zeta_c$ defines the location of point C. It is noted that in dynamic cutting $\zeta_c$ is not constant but rather a time variable. To satisfy the boundary condition of $h(\zeta)$ at C, i.e. $h(\zeta_c) = 0$, the following equation is established from equation (17) to solve for $\zeta_c$:

$$X \sin \omega t \left( t + \frac{\zeta_c}{v_0} \right) - X \sin \omega t + \eta = \zeta_c \tan \gamma_0.$$  \hspace{1cm} (18)

Once $\zeta_c$ is determined, the sub-area BCD can be calculated by integrating $h(\zeta)$ with respect to $\zeta$ from 0 to $\zeta_c$. By equations (14) and (17), the effective volume of displaced material can therefore be defined as

$$V = w \int_0^{\zeta_c} n(\zeta) d\zeta$$

$$= w \left\{ \left( \frac{\eta_0}{\omega_0} \right) \left[ X \cos \omega \left( t + \frac{\zeta_c}{v_0} \right) + X \cos \omega t \right] - \frac{\zeta_c^2}{2} \tan \gamma_0 + (\eta - X \sin \omega t) \zeta_c \right\}.$$  \hspace{1cm} (19)

**Estimating the specific contact force**

In equation (13), the specific contact force is defined as the normal contact force per unit volume of material displaced by the tool penetration. This contact characteristic is assumed to be independent of the tangential load, since the volumetric strain produced by the tangential load is negligibly small compared with that induced by the normal load. Therefore, in calculating the specific contact force, the penetration process is considered equivalent to an indentation process of elastic work material by a frictionless tool-shaped indenter, as shown in Fig. 10. Comparison of Figs 7 and 10 shows that this is a reasonable assumption even though the contact stress fields produced by the two processes are not identical.

Furthermore, since there exists an effective radius of the tool nose, the shape of the contact surface in the indentation model can be approximated by a cylindrical surface. This approximation can be justified if the depth of the indentation remains relatively small. This means that the replacement of the tool-shaped indenter by frictionless cylinder will not significantly change the stress field inside the work material, as shown in Figs 10 and 11; thus the specific contact force calculated from either indentation model is practically the same.
A comprehensive dynamic cutting force model

Fig. 10. Maximum shear stress distribution induced by the normal contact force alone, 
\[ \zeta = 2f_x (n^2 aw) \] for the width of cut = w.

Fig. 11. Maximum shear stress distribution beneath a frictionless cylindrical indenter, 
\[ \zeta = 2P/(n^2 aw) \] for width of the indenter = w.

From all these considerations, the specific contact force for the cutting process can be calculated from the equivalent system described in Fig. 11, i.e. a frictionless cylinder in contact with a semi-infinite solid. The lines of maximum shear stress in the figure indicate the non-uniform distribution of the stress field. The theoretical solution for such a system has been well-established from the Hertz theory. Having examined a two-dimensional punch problem in 1970, Shaw and DeSalvo [17] proposed an equation to describe the relationship between the normal load and the total volume change for a similar type of indentation. The equation was given as

\[ V_i = 1.29 P \left( \frac{1 - 2v}{E} \right) \rho, \]  

where \( V_i \) is the volume change due to the indentation, \( P \) the normal load, \( E \) the Young's modulus of elasticity, \( v \) the Poisson's ratio and \( \rho \) the extent of the deformation zone as
indicated in Fig. 11. When applying the above equation to the equivalent cutting situation, 
P is interpreted as the normal contact force \( f_c \) and \( V' \) is regarded as the total volume of 
material displaced by the tool penetration. The ratio of \( P \) to \( V' \) then represents the specific 
contact force \( f_{sp} \) which depends on the values of \( E, v \) and \( \rho \). For machining of steel, \( E \) and \( v \) are 
estimated to be \( 30 \times 10^6 \) p.s.i and 0.3 respectively. The value of \( \rho \), on the other hand, needs to 
be determined from actual cutting tests.

It is known that the work material in the vicinity of the contact surface is plastically 
deformed during the cutting process. A considerable amount of residual stress caused by 
such plastic deformation would remain in the sublayer of the machined workpiece. Through 
the measurement of the residual stress, the size of the plastic deformation zone can be 
determined. Several experimental investigations [18-20] into the residual stress in machining 
have been reported. The depth of the plastic zone was found to range from 0.01 to 0.1 in., 
depending on not only the mechanical load but also the temperature rise in the region [19]. 
Since equation (20) is developed from an indentation process involving little heat generation, 
it may not be suitable for describing a situation at elevated temperature. For this reason, a 
particular set of test data in [20], conducted at low cutting speed, was chosen for the analysis. 
In this experiment, specimens of annealed mild steel (1018 grade) were machined by HSS 
tools without lubrication. Residual stress on the machined surface was measured by using a 
layer removal method. The cutting conditions were carefully selected to minimize the effect of 
the heat generated in the cutting process. For the given test conditions listed in Table 1, the 
depth of the plastic zone under the tool nose region was found to be approximately 0.02 in. 
Further, it is assumed that the depth of the surrounding elastic zone is roughly equal to the 
size of the plastic zone. Then, the total distance across the elastic–plastic deformation zone, 
 i.e. \( \rho \), should be approximately equal to 0.04 in. With this estimation of \( \rho \), the specific contact 
force \( f_{sp} \) for machining of steel can be calculated from equation (20) as 
\[
{f_{sp}} = 1.5 \times 10^9 \text{ (lb/in}^3)\). \hspace{1cm} (21)
\]

**SUMMARY**

The comprehensive dynamic cutting force model has been developed from the mechanics 
of the cutting process. It consists of two sets of equations. The first set includes equations 
(1)–(11), which describe the dynamic behaviour of the cutting force components \( F_x \) and \( F_y \) in 
response to the vibration variables \( x \) and \( \dot{x} \). The second set, composed of equations (12), (13), 
(15), (18) and (19), determines the relationships between the contact force components \( f_c \) and 
\( f_r \) and the vibration variables \( x \) and \( \dot{x} \). Both sets of equations are nonlinear in nature, and must 
be solved by iterative methods.

**VERIFICATION**

The comprehensive dynamic cutting force model was tested through computer simulations 
for the wave-generating process by reference to existing experimental evidence. First, the 
machining data-base published in [21] was chosen to formulate the functional relationships 
\( M(\ldots) \) and \( \Phi(\ldots) \) described in equations (4)–(6). In this particular data-base, tests were 
carried out for orthogonal dry turning of hot finished mild steel with standard ISO P30 
carbide inserts. Using dimensional analysis, the chip thickness ratio \( p \) and the cutting force 
ratio \( C \) were obtained as two explicit functions in terms of the cutting conditions:

\[
p = 0.56 \sin \alpha + (0.074 - 0.06 \sin \alpha) \left[ \frac{\ln (\sigma v^{0.9}) + 2.3}{5^{0.1}} \right] \hspace{1cm} (22)
\]

**TABLE 1. CUTTING CONDITIONS FOR RESIDUAL STRESS ANALYSIS**

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Cutting speed (ft/min)</th>
<th>Uncut chip thickness (in.)</th>
<th>Rake angle (deg.)</th>
<th>Depth of plastic zone (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>0.011</td>
<td>15</td>
<td>0.020</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.009</td>
<td>30</td>
<td>0.020</td>
</tr>
</tbody>
</table>
A comprehensive dynamic cutting force model

\[ C = 0.075 - 0.87 \sin \alpha - (0.07 - 0.05 \sin \alpha) \left( \ln \left( \frac{\sin \theta}{0.5} \right) + 4.6 \right), \]  
(23)

where \( v \) is the cutting speed in ft/min, \( s \) the uncut chip thickness in in./rev and \( \alpha \) the rake angle in degrees. These machining data-bases then determine the functional relationships \( M(...), \Phi(...) \) according to

\[ M(v, s, \alpha) = \frac{C + \tan \alpha}{1 - C \tan \alpha}, \]  
(24)

\[ \Phi(v, s, \alpha) = \tan^{-1} \left( \frac{P \cos \alpha}{1 - P \sin \alpha} \right). \]  
(25)

In the same publication [21], the shear flow stress \( k \) in p.s.i. was found to vary slightly with the cutting conditions, and can be determined according to the following regression equation:

\[ k = C_0 + C_1 \left( \frac{\sin \phi_0}{s_0} \right), \]  
(26)

with

\[ \phi_0 = \Phi(v_0, s_0, \alpha_0), \]

where \( s_0 \) is the pre-set uncut chip thickness in inches, \( \phi_0 \) the mean shear angle in degrees and \( \Phi(...) \) is as defined in equation (24). \( C_0 \) and \( C_1 \) are the two speed-dependent coefficients which can best be determined from the graphs shown in Fig. 12.

Further, the computer simulations, marked as Series A–D, were carried out to resemble the wave-generating tests described in a previous experimental investigation [22]. These tests were designed to study the behaviour of the dynamic force components \( (F_x + f_x) \) and \( (F_y + f_y) \) in relation to the pre-set cutting speed, mean uncut chip thickness and the tool rake angle, and the frequency of the tool vibration. The conditions for the simulation series are summarized in Table 2.

In the simulation study, a sinusoidal time series with an amplitude of 0.002 in. was generated in the computer to resemble the harmonic motion of the tool. The mean frictional

![Fig. 12. Coefficients in equation (15) for mean shear flow stress (after Nigm [21]).](image-url)
coefficient around the tool nose region $\mu_c$ was chosen to be 0.3 [23]. The depth of penetration (i.e. BD in Fig. 8) is estimated to be 0.00025 in., as suggested by [16]. Using a numerical method, the computer program calculated $F_z, F_y, f_x$ and $f_y$ by solving equations (1)–(13), (15), (18) and (19) simultaneously. Some examples of the simulation results are shown in Fig. 13.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Series A</th>
<th>Series B</th>
<th>Series C</th>
<th>Series D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Feed (in./rev)</td>
<td>0.0037–0.0096</td>
<td>0.0075</td>
<td>0.0075</td>
<td>0.0075</td>
</tr>
<tr>
<td>2. Speed (ft/min)</td>
<td>460</td>
<td>200–740</td>
<td>460</td>
<td>460</td>
</tr>
<tr>
<td>3. Rake angle (deg.)</td>
<td>5</td>
<td>5</td>
<td>0–10</td>
<td>5</td>
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<tr>
<td>4. Frequency (Hz)</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>60–300</td>
</tr>
<tr>
<td>5. Clearance (deg.)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>6. Objective of test</td>
<td>Effect of feed</td>
<td>Effect of speed</td>
<td>Effect of rake angle</td>
<td>Effect of frequency</td>
</tr>
</tbody>
</table>

FIG. 13(A)
The solid lines in the figure represent the variations of the uncut chip thickness, i.e. \( x \), while the dashed lines indicate the force variations \( \Delta (F_x + f_x) \) and \( \Delta (F_y + f_y) \) with respect to the mean values of \( (F_x + f_x) \) and \( (F_y + f_y) \) respectively. These force variation signals follow an approximately sinusoidal pattern, and clearly show some phase lag behind the displacement \( x \). Using Fourier analysis, the amplitude ratios of the force variation to the uncut chip thickness variation and the phase differences between these varying signals were calculated and compared with the experimental results in [22].

Figures 14–21 show the outcome of the analysis. In the bottom figures, the ordinate axis represents the amplitude ratio per unit width of cut, while the ordinate axis in the top figures represents the phase angle. The abscissa for both sets of figures is the relevant parameter for each of the four series of simulations. The solid lines in the figures represent the predicted results for the dynamic force components \( (F_x + f_x) \) or \( (F_y + f_y) \), while the dashed lines represent the special cases in which only the cutting force components \( F_x \) and \( F_y \) are considered. The differences between these two types of line indicate the effect of the contact force around the tool nose region. Comparing these predicted results with the experimental results from [22], it is found that there is generally a good agreement between the experimental data and the theoretical predictions except that the amplitude ratios for \( (F_x + f_x) \) in all four simulation series are lower than the experimental results.
FIG. 14. Effect of feed on dynamic forces in $x$-direction.

FIG. 15. Effect of feed on dynamic forces in $y$-direction.

Arbitrary dynamic cutting force model

Test Results (Nagm and Sadek [22])

- : Theoretical Prediction with Contact Force

- : Theoretical Prediction without Contact Force
Feed = 0.075 in/rev
Rake = 5 deg
Freq = 120 Hz

Test Results (Nag and Sadek [22])
--- Theoretical Prediction with Contact Force
--- Theoretical Prediction without Contact Force

FIG. 17. Effect of speed on dynamic forces in 1-direction.

FIG. 16. Effect of speed on dynamic forces in x-direction.
Test Results (Nigm and Sadek [22]): Theoretical Prediction with Contact Force

- Test Results (Nigm and Sadek [22]): Theoretical Prediction without Contact Force

Fig. 18. Effect of rake angle on dynamic forces in x-direction.

Fig. 19. Effect of rake angle on dynamic forces in y-direction.
Feed = 0.075 in/rev
Speed = 460 ft/min
Rake = 5. deg

**Test Results (Nigm and Sadek [22])**

- Theoretical Prediction with Contact Force
- Theoretical Prediction without Contact Force

**Fig. 20.** Effect of frequency on dynamic forces in x-direction.

**Fig. 21.** Effect of frequency on dynamic forces in y-direction.
One of the possible causes for these departures is the use of the simplification that the depth of tool penetration $\eta$ in the simulation study is assumed to be invariant during the process. However, as a physical reality, the magnitude of this parameter $\eta$ is not constant, but rather is influenced by the instantaneous cutting conditions, such as the cutting speed and uncut chip thickness. These influences are evident from some steady state machining test results [18, 24] which show a strong correlation between the contact force (also known as the ploughing force) around the tool nose region and the uncut chip thickness (also known as feed-rate). If this phenomenon also occurs in wave generation, the continuous alteration of the uncut chip thickness would change the depth of tool penetration simultaneously, and lead to a variation in the total volume of work material displaced by the tool penetration. Consequently the amplitude of the dynamic normal contact force would increase according to equation (13), and so would the amplitude ratio of the force exerted in the $x$ direction. Furthermore, the mean frictional coefficient $\mu_c$ on the tool nose contact surface is estimated to be 0.2–0.3 [23], depending on the cutting conditions and material properties. This low value of the frictional index explains why there are some departures from the prediction of the amplitude ratio in the $x$ direction but relatively smaller deviations in the $y$ direction.

CONCLUSION

A comprehensive dynamic cutting force model has been developed based on the mechanics of the cutting process. In terms of the vibration variables $x$ and $\dot{x}$, the model develops expressions for the cutting force components $F_x$ and $F_y$ exerted on the tool–chip interface and the contact force components $f_x$ and $f_y$ occurring around the tool nose region.

The cutting forces are associated with the chip formation process. Their relationships with the vibration variables are derived from the equilibrium of the resultant forces exerted on the shear plane and the tool–chip interface.

The contact forces, on the other hand, result from the contact between the tool nose region and the work material adjacent to it. The magnitude of the force normal to the contact surface is assumed to be proportional to the volume of material displaced by the tool penetration. A series of analytical equations has been derived to calculate the volume change, while the specific contact force, i.e. the proportionality, has been estimated from the theoretical solutions of the Hertz theory. With the additional information of the mean frictional coefficient on the contact surface, the contact force components in response to the tool vibration are then determined.

The model has been tested by comparing results from computer simulation of wave-generating processes to existing experimental evidence. The results indicate a generally good agreement between the experimental data and the theoretical predictions, except for the amplitude of the dynamic force component normal to the cutting direction, which appears to be under-estimated. This is possibly due to the simplifying assumption of a constant depth of tool penetration used in the simulation model. This assumption is however inconsistent with some steady state cutting test results, which show a strong influence of the uncut chip thickness on the contact forces around the tool nose region. Further research into this uncut chip thickness variation effect is therefore recommended.

REFERENCES
