Bidding Strategies in Sealed-Bid Reverse Multi-attribute Auctions

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Abstract—This study considers the reverse multi-attribute, or procurement auction in which the item for sale is defined by several quality attributes in addition to the price, the buyer is the auctioneer, and the sellers are the bidders. We focus on a variation of the first-price sealed-bid protocol termed first-score sealed-bid auction. We analyze a specific model for the protocol, and we provide optimal strategies for both the auctioneer and the bidders participating in the auctions. In addition, we conclude that the auctioneer can manipulate the auction to yield better outcome, however, as the number of the bidders participating in the auction increases, the buyer is motivated to announce a real utility function.

Key words: Multi-attribute Auctions, Electronic Procurement, Reverse Auctions

I. INTRODUCTION

Many firms within the B2B sector have recognized the opportunities that exist for lowering their costs by participating in online reverse auctions. As implied by their name, reverse auctions are simply traditional auctions in reverse [1]. In traditional auctions, a bidder (auctioneer) offers an item for sale to the highest bidder (bidder). By contrast, in a reverse auction, a buyer (auctioneer) offers a tender or a contract for the supply of specific goods or services to the bidders (bidders).

Reverse auctions, or procurement usually require the bid to specify several characteristics of the contract to be fulfilled, often allow the contracts to be determined on the basis of a variety of attributes, involving not only the price, but also quality, lead time, contract terms, supplier reputation, and incumbent switching costs [2]. A multi-attribute item is defined as an item characterized by several negotiable dimensions. Multi-attribute reverse auctions allow negotiation over price and qualitative attributes such as color, weight, or delivery time. They promise higher market efficiency through a more effective information exchange of the buyer’s preferences and supplier’s offerings [3]. In contrast to the single-attribute auction, where each side of the auction knows the preferences of the other side regarding the price, in multi-attribute reverse auctions, the bidders (bidders) do not necessarily have any information about the auctioneer’s (buyer’s) preferences regarding these additional attributes. To overcome this problem, we analyze a scoring function or explicitly guide the auction by revealing if a given bid is better than the best bid yet offered. The scoring function enables the auctioneer to articulate its preferences regarding the various attributes which are made public to all bidders at the beginning of the auction. Bidders use this scoring function to value specific configurations and thus can understand how changes to the various attributes will affect the overall desirability of the bid.[4]

Given the formulation of the utility function of the buyer and the cost function of the bidders, two questions arise when attempting to analyze the concept of sealed-bid multi-attribute reverse auctions, in which the bidders have no visibility of the value regarding the attributes, The first one is, should the buyer (auctioneer) reveal all her preferences, or part of them at the beginning of the auction? The second one is, how should a bidder formulate her bid considering the various attributes? Addressing these issues, we analyze an auction protocol for the case of multi-attribute items, a variation of first-price sealed-bid protocol termed first-score sealed-bid protocol in this paper.

The outline of the paper is as follows. In Section II we will first review the related work of multi-attribute reverse auctions. And in Section III we will describe in detail the model we assume for the auctions. Then, in Section IV we present the optimal strategies for the bidders, and we discuss the buyer’s behavior in Section V. In Section VI, we will describe and discuss some results from the specific model in the setting. Finally, We will present our conclusions in Section VII.

II. RELATED WORK

There are two primary goals in the auctions theory literature, efficiency and utility maximization. An efficient auction mechanism captures the case where there is no other solution that is better for both the auctioneer and the bidders [5], whereas utility maximization, on the part of auctioneer, is strived for private sector auction, and is the appropriate objective in many industrial procurement settings. In complex auctions, these two objectives are often in conflict [6], because the auctioneer can usually increase his utility by either withholding items for sale or by allocating items to those who do not value it the most.
so far, a number of theoretic work and experimental work have been done on multi-attribute auctions. Among of them, Che [7] provided a thorough analysis of design of multi-attribute auctions in which a two-attribute model in a sealed-bid setting is developed and analyzed, by using a utility function to score bids. He defined three auction protocols: the first-score, second-score, and the second-preferred-offer. The first-score is a variation of the first-price sealed-bid auction, where each bidder submits a sealed bid, and the winner is the bidder who achieves the highest score. The winner is then obliged to produce the goods with the preferred quality at the offered price. The second score is a generalization of the second-price sealed-bid auction where the winner is the bidder who achieves the highest score but is only required to provide goods with a combination of attributes that yields the score of the second highest bid. The second preferred-offer is another variation of the second-price auction in which the winner is required to exactly match the attributes combination of the second highest bid. Given these auction protocols, Che proposed an optimal design for the auction protocols, based on an announced scoring rule.

Branco[8]'s analysis, extending Che’s work, is based on Che’s independent cost model and derives an optimal auction mechanism for the case when the bidding firms’ costs are correlated. Bichler [9] discussed MAUT as an algorithm for bid evaluation in single-sourcing, multi-attribute reverse auctions. Teich et al. [10] developed a Proto-type online trading system designated as Negoti-Auction to enable the execution of multiple-sourcing reverse auctions, and to provide pricing guidelines for the bidders. Bichler and Kalagnanam[3] shows that multi-attribute auctions can produce higher gains1 for participants because of the bidding flexibility it offers. Similar to Che’s[7] analysis, E.David[11] analyzed a multi-attribute auction in which an arbitrary number of attributes (m) are negotiated.

Recently, C-B Cheng[12] formulated the bid construction process as a fuzzy multi-objective programming problem, and adopted an exhausted enumeration algorithm to solve a sealed-bid reverse auction. A more detailed review of multiple-issue reverse auctions and their design features can be found in the work of Teich et al. [13]. Most related work to this paper is E.David’s [14] study, in which the author used an English auction protocol for a three-dimensional procurement multi-attribute auction.

III. THE MODEL

This section describes a model of three-dimensional bidding on two quality attributes and price. The auction model consists of one buyer (auctioneer) and a fixed number of n sellers (bidders). We assume that the auctioneer is buying a single item and the item is sold to a single bidder. At the beginning of the auction, the buyer announces the bidding attributes (consisting of price \( p \), and two quality factors, \( q_1 \) and \( q_2 \)), the auction protocol, and the scoring function \( S(p,q_1,q_2) \) that describes her preferences concerning the item’s properties. Bidders that decided to send a sealed bid has to specify the full configuration he offers. The bidder that has the highest score wins the auction and should provide the bid he offered. Such an auction is called the first-score auction in Che[7].

In addition, we assume both the buyer and the bidders to be rational in a sense that they are trying to maximize their utility and that they will not do an action that yields them a negative utility, and assume the multi-attribute auction is utility independent, namely, the utility of one attribute does not depend on the value of any other attributes[15].

A. Buyer’s utility function and scoring rule

Given the background, the buyer and bidders are characterized as follows: The buyer derives utility from a bid \((p,q_1,q_2) \in \mathbb{R}^3\),

\[
U_{buyer}(p,q_1,q_2) = -p + V(q_1,q_2)
\]  

(1)

where \( V(*) \) is the function of announced bid’s value for the buyer. In the past research it is assumed that \( V'(*) > 0 \), \( V''(*) < 0 \), and \( \lim_{q \to q'} V(q) = 0 \) to ensure an interior solution. We proposed to use the additive weighting function to combine the different attributes into a decision rule. For simplicity, we assume that as \( q_i \) increases, the quality of the item increases, for both \( q_1 \) and \( q_2 \).

By contrast with E.David et al’s work [14], which has the same expression of the function of bid’s value for all different attributes, we examine a more complex and practical formula of utility function

\[
U_{buyer}(p,q_1,q_2) = -p + (W_1 \cdot \sqrt{q_1} + W_2 \cdot \ln q_2)
\]  

(2)

The increase in the bidder’s attributes and the decrease in the price increase the buyer’s utility. In addition, the influence of \( q_1 \) and \( q_2 \) is assumed to be independent and heterogeneous: the influence of \( q_1 \) is square rooting, whereas the influence of \( q_2 \) is logarithmic. This assumption is valid in many domains, and our interests are focused on the different influence of each attribute.

The scoring function \( S(p,q_1,q_2) \) may be different from the real utility function in the sense that the announced weights \( w_i \) may be different than the actual weights \( W_i \). In particular, the scoring function is of the form

\[
S_{bidder}(p,q_1,q_2) = -p + (w_1 \cdot \sqrt{q_1} + w_2 \cdot \ln q_2)
\]  

(3)

where \( w_i \) are the weights that the buyer assigns to attribute \( q_i \). Let \( s(p,q_1,q_2) \) denotes the value that the announced bid for the buyer,

\[
s(p,q_1,q_2) = w_1 \cdot \sqrt{q_1} + w_2 \cdot \ln q_2
\]  

(4)
The weights $W_i$, can be different from the real weights $W_i$, therefore, the buyer can manipulate the auction through adjust the value.

B. Bidder’s utility function

Each bidder $i$ has private information about the cost parameters $\theta_i$ (type). Similar to the model in Che[7], we assume that the cost parameter $\theta_i$ is independently and identically distributed over $[\theta, \bar{\theta}]$, according to a distribution function $F$ (We used the uniform distribution) for which there exists a positive, continuously differentiable density function $f$. Because of complete symmetry among the bidders, the subscript $i$ is dropped in the reminder of this paper.

Consider the cost function of the bidders. Bidder $i$, upon winning, earns from a bid $(p, q_1, q_2)$ profits,

$$\pi(p, q_1, q_2) = p - c(q_1, q_2, \theta)$$

(5)

We assume that the bidder’s cost function is additive across attributes, increasing, convex, and twice continuously differentiable in $q_i$. Also, the influence of $q_1$ is linear and that of $q_2$ are non-linear.

$$c(q_1, q_2, \theta) = \theta (\alpha_1 \cdot q_1 + \alpha_2 \cdot q_2^2)$$

(6)

where $\alpha_i$ are fixed coefficients for each of the quality attributes.

In the following, we will analyze the above model. We first present the optimal bids to be suggested by each bidder, and then we will discuss what the auctioneer adjusts the announced scoring rule to gain higher outcome.

IV. BIDDERS’ STRATEGIES

Bidders should compete only over the price in single attribute auction. However, in multi-attribute the bidder has to decide all the quality attributes in addition to the price. The equilibrium under the auction scheme is characterized for a scoring rule that can be viewed as inducing a Bayesian game, where each bidder picks a bid as a function of his cost parameter. Given the bidders’ type and a particular scoring rule, the following lemma proved by Che[7] specify how each bidder will choose the values of $q$ for the case of one quality dimension.

Lemma[7]. With first-score auctions, quality is chosen at $q^*(\theta)$ for all $\theta \in [\theta, \bar{\theta}]$, where

$$q^*(\theta) = \arg \max_c s(q) - c(q, \theta)$$

(7)

Easy to prove, the results of lemma 1 hold also for the case of multiple dimensions [11]. The lemma 1 describes how the bidder will decide the quality dimension of a bid.

Given the announced scoring rule and the model described in Section 3, we developed our equations for optimal non-price attributes and optimal price to be offered by a bidder, which was adjusted to fit the multiple quality dimensions from Che’s result.

Theorem 1. Given the scoring rule and the bidder’s profit, and assuming the model described in Section 3, the non-price attributes of the bidder $i$ can be written as

$$\hat{q}_i^*(\theta) = \left(\frac{W_i}{2 \cdot \alpha_2 \cdot \theta}\right)^2$$

(8)

Proof. Using the above condition and the result of lemma 1, we can derive the optimal attribute when

$$\frac{\partial}{\partial \theta} s(q, q_2, \theta) - c(q, q_2, \theta) = 0$$

(9)

Therefore, the bidders can maximize their utility by bidding with the optimal attributes. Notice that the result is different from that of E.David et al. Moreover, the optimal attributes depend only on the private cost parameter of the bidder, the coefficients for each of the quality attributes, and the weights of the scoring rule announced by the buyer, whereas are independent from the price the bidder will submit. In the following theorem, we will derive the optimal value of the price that the bidder bid.

Theorem 2. In the above auction setting where, the optimal price of bidder $i$ can be denoted as

$$p^*(\theta) = \frac{w_i^2}{4 \alpha_1 \theta} \left[\frac{1}{\theta} + \frac{1}{(\theta - \bar{\theta})^{-1}} \int_{\bar{\theta}}^{\theta} \frac{(\theta - t)^{-1}}{t^2} dt \right]$$

(9)

Proof. The price $p^*(\theta)$ can be calculated by following Che’s method. Firstly, let $F(x) = \frac{\theta}{\bar{\theta}}$, then according Che

$$p^*(\theta) = \frac{w_i^2}{4 \alpha_1 \theta} \left[\frac{1}{\theta} + \frac{1}{(\theta - \bar{\theta})^{-1}} \int_{\bar{\theta}}^{\theta} \frac{(\theta - t)^{-1}}{t^2} dt \right]$$

The optimal values of $q_i$ are assigned, that is
Resulting with
\[ p^*(\theta) = \frac{w_2}{4\alpha_1} \left[ 1 + \frac{1}{(\theta-\theta)^{\gamma}} \cdot \frac{\hat{e}^\gamma (\theta-t)^{\gamma-1}}{t} dt \right] + \frac{w_2}{2} \left[ 1 + \frac{1}{(\theta-\theta)^{\gamma}} \cdot \frac{\hat{e}^\gamma (\theta-t)^{\gamma-1}}{t} dt \right] \]

From Theorem 1, we can see that the bidder will decide about his bid according to the scoring rule, his private cost parameter and his belief about other competitors. Moreover, from Theorem 2, the bidder’s belief about other competitors only influences the price he will suggest. As the announced weights increase, the quality of the proposed item increases, and the price of the bid will increase too. As the private cost parameter increases, the bidder’s efficiency decreases, it will suggest lower quality items.

V. BUYER’S BEHAVIOR

The buyer can increase her gain by manipulating the scoring rule. To maximize her utility, the buyer will decide which bidder will be awarded and if all her preferences, or part of them will be revealed at the beginning of the auction. Given the scoring rule, the utility functions of the buyer and the bidders, the number of bidders, and the distribution and the range of the bidders’ types, the buyer’s expected utility can be written following E. David[11] as
\[ E(U_{buyer}) = \int U_{buyer}(p^*, q_i^*, q_j^*) (1-F(t))^{\gamma-1} \cdot n \cdot f(t) dt \]
where \( F(t) \) and \( f(t) \) are the distribution and the density function of the cost parameter, respectively. The winning bid is \((p^*, q_i^*, q_j^*)\) whose probability of winning against other bidders is given by \((1-F(t))^{\gamma-1} \cdot n \cdot f(t) \), and the expected utility of the buyer from this bid is \( U_{buyer}(p^*, q_i^*, q_j^*) \). Using equation (2) and (8), we can get the utility of the buyer due to the winning bidder with the highest score. After the assignment of the full function, the buyer’s expected utility results in
\[ E(U_{buyer}) = -\frac{n}{(\theta-\theta)^\gamma} \left[ \frac{w_1}{4\alpha_1} \int (\theta-t)^{\gamma-1} \frac{\hat{e}^\gamma}{t} dt + \int (\theta-z)^{\gamma-1} \frac{\hat{e}^\gamma}{z^2} dz dt \right] + \frac{w_2}{2} \int (\theta-t)^{\gamma-1} \frac{\hat{e}^\gamma}{t} dt + \frac{w_2}{2} \ln \frac{w_2}{2\alpha_2} \int (\theta-t)^{\gamma-1} \frac{\hat{e}^\gamma}{t} dt + \frac{w_2}{2} \ln \frac{w_2}{2\alpha_2} \int (\theta-z)^{\gamma-1} \frac{\hat{e}^\gamma}{z^2} dzdt \]

Thus, the expected utility of the buyer depends on the range and the distribution of the bidder’s cost parameters, the scoring rule, and the number of bidders participating in the auction. Moreover, the increase of the coefficients of the attributes will decrease the buyer’s expected utility. In the following theorem, we offer a method for calculating the scoring rule by the buyer using the foregone conditions.

Theorem 3. Given the model described in section 3, the distribution and the range of the bidder’s cost function, and the number of the bidders participating in the auction, the optimal values of the announced weights by the buyer, \( w_i \) \((i = 1, 2)\) satisfy
\[ w_i^* = W_i^* \left[ 1 + \frac{\int (\theta-z)^{\gamma-1} \frac{\hat{e}^\gamma}{z^2} dz dt}{\int (\theta-t)^{\gamma-1} \frac{\hat{e}^\gamma}{t} dt} \right] \]
and
\[ w_2^* = W_2^* \left[ 1 + \frac{\int (\theta-z)^{\gamma-1} \frac{\hat{e}^\gamma}{z^2} dz dt}{\int (\theta-t)^{\gamma-1} \frac{\hat{e}^\gamma}{t} dt} \right] \]

Proof. We find the differential of \( E(U_{buyer}) \) with respect to the weights, and compare the results to zero.
\[ \frac{\partial E(U)}{\partial w_i} = 0 \]
\[ \frac{\partial E(U)}{\partial w_2} = 0 \]
So the optimal weights of the scoring rule are identified by (11) and (12).

The buyer can optimize the scoring rule, based on the above results, if the number of the bidders, the distribution of the bidder’s cost parameter and the strategies are known to the buyer.
VI. AUCTION RESULT

Notice that the format varies in the expression of \( w_1 \) and \( w_2 \), that is caused by the difference of the value function of different attributes for the buyer. Moreover, the value of the weights \( w_i \) announced by the buyer is different from the real value of the weights \( W_i \). Easy to prove, the value of the announced weight \( w_1 \) is lower than the value of weight \( W_1 \), and the value of weight \( w_2 \) is much lower than the value of weight \( W_2 \) when the lower bound of the cost parameter, \( \theta \), satisfy the condition \( \theta > 1 \). So we can get the following corollary.

**Corollary 1.** Based on the results from theorem 3, and in the above setting, the optimal weights of the scoring rule announced by the buyer for the different attributes, comparing to the real weights of the buyer’s utility function, satisfy the following conditions:

\[
\begin{align*}
  w_1 \leq W_1, & \quad \text{for } \forall \theta \in [\theta, \Theta] \\
  w_2 \leq W_2, & \quad \text{if } \theta > 1
\end{align*}
\]

met only if the buyer has the highest estimation of the bidder’s cost parameter.

**Corollary 2.** By the theorem 3, the value of the optimal weights of the scoring rule for the different attributes, \( w_i \), is very near to the value of the real weights of the buyer’s utility function \( W_i \) when the number of the bidders participating in the auction \( n \) tends to infinity, namely

\[
w_i \to W_i (n \to \infty) \quad \text{for} \quad i = 1, 2.
\]

**Proof.** See the Appendix.

From these results we can realize, that the buyer can manipulate the auction by not telling the truth about the preferences to yield better outcomes. However, as the number of bidders increases, the buyer is more motivated to announce a scoring function closer to its real utility function.

VII. CONCLUSION

This paper has studied a specific model in sealed-bid reverse multi-attribute auction. The auction model consists of one buyer, who is auctioneer, and a fixed number of \( n \) bidders. We assume that the auctioneer is buying a single item, consisting of two quality attributes, from the bidders. Comparing to the past research, in which the buyer’s utility function has the same format for different attributes, we focus on the buyer’s utility function in which the influences of the quality attributes are different. We present the optimal strategies for both the buyer and the bidders when the utility function of the buyer and the cost function of the bidders are given. Moreover, we conclude that the buyer can manipulate the auction by announcing a lower value of the weights for different quality attributes than the value of the real weights of his utility function, and he has the motivation to reveal his true preferences when the number of the bidders participating in the auction is increasing.

**APPENDIX**

Proofs of **Corollary 1** and **Corollary 2** follow.

**Proof of Corollary 1.** Since the integrals of (11) have positive values and the integrals of (12) have positive values when \( \theta > 1 \), it is easy to prove the validity of corollary 1.

**Proof of Corollary 2.** To prove \( w_i \to W_i (n \to \infty) \), we can achieve it by showing that the quotient of two integrals of (11) tends to zero. The rest of the proof follows, since

\[
\frac{1}{\theta} \int_0^\theta (\theta-t)^{n-1} dt \leq \frac{1}{\theta} \int_0^\theta (\theta-t)^{n-1} dt \leq \frac{1}{\theta} \int_0^\theta (\theta-t)^{n-1} dt
\]

Moreover,

\[
\int_0^\theta (\theta-t)^{n-1} dt = \frac{1}{n} (\theta-\theta)^n
\]

Then, using the mean value theorem,

\[
\int_0^\theta (\theta-t)^{n-1} dt = \frac{(\theta-\eta)^{n-1}}{\eta} (\theta-\xi)(\theta-\theta)
\]

where \( \theta < \eta < \xi \) and \( \theta < \xi < \theta \).

Hence,

\[
\int_\theta (\theta-t)^{n-1} dt = \frac{(\theta-\eta)^{n-1}}{\eta} (\theta-\xi)(\theta-\theta) - \frac{1}{\theta} \int_0^\theta (\theta-t)^{n-1} dt
\]

Using squeeze rule, and when \( n \to \infty \),

\[
\frac{(\theta-\eta)^{n-1}}{\eta} (\theta-\xi)(\theta-\theta) \to 0
\]

and

\[
\frac{1}{\theta} \int_0^\theta (\theta-t)^{n-1} dt \to 0
\]

Thus,

\[
\frac{(\theta-\eta)^{n-1}}{\eta} (\theta-\xi)(\theta-\theta) \to 0
\]

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\[
\int_0^\infty \frac{(\theta - z)^{n-1}}{z^2} dzdt \rightarrow 0 \quad (n \rightarrow \infty)
\]

\[
\int_0^\infty \frac{(\theta - t)^{n-1}}{t} dt
\]

From this it follows that

\[w_1 \rightarrow W_1(n \rightarrow \infty)\]

Similar to the above method, we show that \(w_2 \rightarrow W_2\) when \(n \rightarrow \infty\).

Since

\[
\int_0^\infty (\theta - t)^{n-1} \ln t dt = \frac{1}{n} (\theta - \theta)^n
\]

\[
\int_0^\infty (\theta - t)^{n-1} dt = n \ln \left( \frac{\theta - \xi}{\theta - \theta} \right) \rightarrow 0 \quad (n \rightarrow \infty)
\]

And

\[
\int_0^\infty \frac{(\theta - z)^{n-1}}{z} dzdt \in \left( \theta - \theta, \frac{1}{n+1}, \frac{1}{n+1} \right)
\]

It is easy to see that

\[
\lim_{n \rightarrow \infty} \frac{1}{n+1} = \lim_{n \rightarrow \infty} \frac{1}{n+1} = 0
\]

Thus

\[
\int_0^\infty \frac{(\theta - z)^{n-1}}{z} dzdt \rightarrow 0 \quad (n \rightarrow \infty)
\]

\[
\int_0^\infty (\theta - t)^{n-1} dt
\]

It follows that

\[w_2 \rightarrow W_2(n \rightarrow \infty) \quad \Box\]

REFERENCES
