Magnetic Design Aspects of the Trans-Rotary Magnetic Gear

Siavash Pakdelian, Student Member, IEEE, Nicolas W. Frank, Member, IEEE, and Hamid A. Toliyat, Fellow, IEEE

Abstract—This paper studies several magnetic design aspects of the trans-rotary magnetic gear (TROMAG). The TROMAG is a magnetic device that converts linear motion to rotation, and vice versa, through magnetic field while doing a gearing action. This means that a high-force, low-speed translation can be converted to a high-speed low-torque rotation. Once coupled with a conventional rotary machine, the resultant system may be used as a compact electromechanical system for high-force linear-motion applications, such as wave energy conversion. The paper investigates the magnetic design of the TROMAG by means of either two-dimensional (2-D) or three-dimensional (3-D) finite element analysis (FEA), or an accurate analytical model. The impact of nonideal discretized helix and dimensions of magnets and air gap are investigated, as well as scaling rules of the TROMAG and the possibility of demagnetization. Experimental results obtained from a lab prototype are also presented to verify the concept and analysis.

Index Terms—Finite element analysis (FEA), lead screw, linear permanent-magnet (PM) machines, magnetic gear, trans-rotary magnetic gear (TROMAG).

I. INTRODUCTION

THE IDEA of employing a helical structure in an electromechanical energy conversion system may be traced back to more than a century ago. For example, Keller and Sibley [1] reveal a “reciprocating motor” for drilling and pumping applications. In the proposed system, the ferromagnetic moving object, on which a thread is grooved, exhibits rotational and translational motions simultaneously as it interacts with magnetic field of a winding. In [2], a “magnetic screw” is driven by a rotary machine through a worm gear. Two sets of windings are employed along with two sets of cores with opposite thread directions to convert unidirectional rotation of the motor to reciprocating motion.

The “magnetic transmission” device presented in [3] seems to be the first to use permanent-magnet (PM) material for converting translation to rotation. The apparatus, whose inventor had already patented a magnetic spur gear, was proposed as a gauge for measuring the liquid level in a tank. The device consists of a short outer translator, moving up and down with the liquid level, and a long inner rotor, attached to a pointer rotating against a graduated dial. The term “magnetic gearing” was used in the patent to show that a small displacement of the translator can result in a large rotation of the rotor.

Several similar attempts were also made during the 70s, the purpose of most of which was to create linear motion from a rotating magnetic field [4]–[6]. In the absence of commercially viable high-energy-density PMs, they all chose to take advantage of the reluctance variation originating from a helical structure.

In [7], a “mag-screw machine” was proposed to drive an artificial heart. The system comprises a brushless dc (BLDC) machine and a magnetic screw, which is integrated into the machine. The magnetic screw consists of a rotating component and a translating component, both of which are made of ferromagnetic iron cores furnished by helically disposed alternating PMs. The BLDC motor drives the rotor, and rotation of rotor, in turn, makes the translator move back and forth to drive a pump.

The idea of magnetically converting linear motion to rotation, and vice versa, by using helical magnets has been revived recently. In [8], a “magnetic helical screw drive” is proposed for wave energy application. In [9], the device is named “magnetic screw” and an analytical model has been developed for calculating the magnetic field and force. The model is similar to that of a PM linear tubular machine [10]. In [11], the same device is called “magnetic lead screw” (MLS), and design and manufacture of a 17 kN prototype has been reported, as well as a discussion comparing the force density of the MLS with a rotary magnetic coupling. Moreover, the authors have performed a structural analysis to determine the amount of rotor deflection caused by the attraction force arising from eccentricity. The efficiency measurements are carried out in [12].

The present authors have already explained how this device can be developed from elementary magnet arrays by studying the characteristics of force transmission in orthogonal directions between two arrays of magnets [13]. The authors chose the name trans-rotary magnetic gear (TROMAG) instead of magnetic screw (or the like) to further emphasize the gearing capability of the device and to categorize it as a type of magnetic gear. With contact-free force transmission capability, magnetic gears, in general, exhibit inherent overload protection, minimal friction, low wear and tear, low maintenance, and high reliability. The same features are expected from the TROMAG as well, making it a strong contender for high-force, low-speed linear motion applications. This paper is mainly devoted to magnetic design of the TROMAG. First, in Section II, an overview on the device structure and characteristics is given. Then, several
aspects of magnetic design are presented in Section III. The impact of a nonideal helix on the force and torque characteristics, variation of shear stress with dimensions of magnets and air gap, scaling rules of the TROMAG, and possibility of demagnetization are discussed. Finally, the experimental results from a prototype are presented.

II. TROMAG STRUCTURE

Fig. 1 shows different views of a two-pole TROMAG, specifications of which are given in Table I. Red and blue colors indicate alternating magnet polarities. As viewed from the Z-axis, the device resembles two concentric rotors each of which having two PM poles. The TROMAG consists of two main parts: the translator and the rotor. While the translator moves back and forth along the Z-axis, the rotor rotates about the same axis. In this picture, the outer part is identified as the translator and the inner part as the rotor; whereas in general either of inner part or outer part can play the role of rotor or translator, depending on the mechanical assembly. Both rotor and translator are made of radially magnetized, alternating helical PMs mounted on cores. Ferromagnetic iron cores are employed to improve the force density.

In general, one of the two parts needs to be longer than the other one, so the magnets of the rotor and translator remain engaged as the translator moves. The length of the shorter part is called the active length, and the extra length added to provide the engagement is referred to as the stroke.

Similar to mechanical screws, the thread can be left-handed or right-handed; and that will determine the direction of rotor rotation as the translator moves. Moreover, the number of poles is equivalent to the number of starts in a threaded rod, that is, for example, the arrangement of magnets on a two-pole TROMAG looks like the thread of a double-start screw.

Mechanical rotation of the PM helix about an axis results in corresponding translation of the field along the same axis. In a two-pole TROMAG, analogous to a double-start threaded rod, as the rotor rotates one complete revolution its field translates by one “lead.” As the number of poles increases, the number of leads traversed due to one revolution of rotor increases proportionally. Therefore, by using a small lead and low number of poles it becomes possible to convert low speed translation to high speed rotation. A gear ratio $G$ can then be defined as the ratio of the rotor angular speed $\omega$ (rad/s) to the translator linear speed $v$ (m/s)

$$G = \frac{\omega}{v} = \frac{2\pi}{P\tau_p} \quad (1)$$

in which $\tau_p$ is the thread width (pole pitch) as shown in Fig. 1(a). In this figure, the thread width is equal to the magnet width, $w$; whereas in general magnet does not need to cover the whole pitch. $P$ denotes the number of poles as viewed from the XY plane, and the product $P \times \tau_p$ signifies the lead. For example, in a two-pole TROMAG with 10 mm wide threads, as the translator moves at 1 m/s linear speed, the rotor will rotate at 3000 r/min. Considering the power balance between rotor and translator, the ratio of translator (axial) force $F_t$ to rotor torque $T_r$ would equal the gear ratio. It is worth noting that the gear ratio would better be defined as $G = F_t/T_r$ instead of $\omega/v$ because the former definition holds in all operating conditions as long as the helices are ideal, whereas in a TROMAG with nonunity efficiency or during transient states, the relation between $\omega$ and $v$ may not simply stated by (1). However, the former definition provides an intuitive way of understanding the device operation, as explained in [13].

It is shown in [14] that if the TROMAG of Fig. 1(a) is viewed from the Z-axis, as shown in Fig. 1(b), then it may be treated as a traditional rotary machine in terms of number of poles and the rotor length. In such analogy the rotor torque is linearly proportional to the aforementioned parameters. However, it is also possible to look at the TROMAG as a tubular PM linear machine instead of a rotary PM machine, as done in [9] and shown in Fig. 1(c). In this case, the thread width would serve as the pole pitch and the number of alternating magnets on the rotor, as viewed from the YZ plane, would resemble the number of poles of a linear machine. In this case, both rotor torque and
translator force would be proportional to the number of poles. Such an analogy is equally valid and useful for analysis and design purposes. However, it cannot explain multistart threads because the cutaway views of TROMAGs with different number of helix starts would all look alike. Moreover, such an approach makes it difficult to explain rotor lengths that are not an integer multiple of the thread width, because traditional understanding of electric machines would not allow noninteger number of poles.

If the translator is kept stationary and the rotor is rotated 360 electrical degrees, which in the case of a two-pole TROMAG is equivalent to one complete mechanical revolution, the torque and force characteristics of the device can be obtained. The results of 3-D finite element analysis (FEA), for the TROMAG of Table I, are shown in Fig. 2. Similar results would be obtained if the rotor was kept stationary and the translator was moved by two pole pitches. The results resemble the torque (force) characteristic of a rotary (linear) synchronous machine. The peak values of the torque and force profiles can then be analogously called pull-out torque and pull-out force.

III. DESIGN ASPECTS

A. Effect of Nonideal Helix

Although an ideal TROMAG would employ ideal helical magnets, in practice the TROMAG may need to be manufactured using blocks of rare earth material magnets that are commercially available. A turn of an ideal helix can then be replaced by a discretized helix, which consists of two half-helices, each spanning 180 electrical degrees. The two half-helices are placed such that the second one begins from the end of the first one, without being shifted longitudinally. However, each half helix is formed by a number of segments, each of which displaced longitudinally with respect to its neighbors. The amount of displacement equals the helix width divided by the number of segments minus one. For example, if 20 PM blocks with 9° arc and 10 mm length in Z-direction are used to form one turn of the helix, then each segment of a half helix is displaced 10/19 mm with respect to its neighbor.

The more segments which are used, the closer the approximation of an ideal helix will be attained. Nonetheless, employing a lower number of segments will ease the fabrication process. Therefore, it is essential to investigate the effect of helix non-ideality on the device characteristic to enable making a wise decision on the number of segments. To study the effect of non-ideal helices, several segment arcs in the range of 9°–60° are studied. Fig. 3 shows the resultant arrangement of magnets in four cases. In all cases, the segments arc for both the rotor and translator is assumed to be the same.
By using 3-D FEA, it has been shown that, as pointed out in [7], the pull-out force of the device is proportional to its active air gap area, i.e., the area enclosed between the rotor and translator. The pull-out force per active air gap area, the so called shear stress $\sigma$, however, is a function of magnet thickness $h_M$, pole pitch $\tau_p$, magnet width $w$, magnet material (remanent flux density $B_r$ and coercivity $H_C$), and core material (relative permeability) as well as the air gap length $g$. In this section, it is assumed that the helix is ideal.

Using an analytical model instead of 3-D FEA would expedite thorough investigation of the relationship between shear stress and dimensions of magnet and air gap. Moreover, such a model can help automate the design process. The analytical model developed in [9] has been adopted here for this purpose and is presented in the Appendix. The model assumes an ideal helix and starts with 2-D approximation of the TROMAG. In addition to infinite permeability for iron core, it is also assumed that the device length in Z-direction is infinite. The latter assumption results in a symmetric field distribution in Z-direction. The first step would be to calculate the field due to rotor magnets in the absence of translator magnets. To that end, Laplace and Poisson equations have to be solved in the air gap (the physical air gap and the region appearing after removing rotor magnets) and magnet regions, respectively. Then the translator magnets are replaced with equivalent current-carrying sheets. Finally, the axial component of force exerted on the current carrying sheets can be obtained by applying Lorentz’ force law and integrating the force over the surface of the current sheet. The model has proved to be accurate; error as compared to 2-D FEA, not exceeding a few percent [9], [15].

2) Shear Stress Versus Magnet Dimensions: The analytical model mentioned in section B1 is adopted to study the influence of magnet thickness and width, pole pitch, and air gap length on the shear stress. Fig. 5(a) shows variation of maximum shear stress with pole pitch for several values of magnet thickness. In this section, magnets are assumed to be full pitch, i.e., magnet width $w$ is equal to pole pitch $\tau_p$ (half of the “lead” for a two-pole TROMAG). The magnet material is given in Table I. The air gap length is fixed at 1 mm while the magnet thickness is increased from 1 to 10 mm with steps of 1 mm. Two main conclusions can be drawn from this diagram. First, for any given air gap length and magnet thickness there would be an optimum pole pitch for which the shear stress is maximized. For example, for the case of 1 mm air gap and 5-mm-thick magnets the optimum pitch is 10 mm; and the resultant shear stress is about 230 kN/m$^2$. As a second conclusion, for a given air gap length and pole pitch, increasing the magnet thickness will increase the shear stress; however, the effect will saturate beyond some point. As seen, by moving up on the figure the distance between consecutive curves decreases although the increments of magnet thickness are constant. This result was not unexpected, as increasing the magnet thickness increases the reluctance of the flux path as well.

Due to manufacturing tolerances, the air gap length has to be increased as the TROMAG is designed for larger sizes. The effect of increasing the air gap length on the shear stress is shown in Fig 5(b). To obtain this diagram, the pole pitch is kept constant at 10 mm, the air gap length is increased from 1 to 5 mm, and the shear stress is calculated for values of magnet thickness in...
the range of 1–10 mm. It is observed that increasing the air gap length at a constant magnet thickness reduces the shear stress. The curves presented in Fig. 5(b) may also be looked upon as another form of demonstrating the second conclusion made from Fig. 5(a): for a given air gap length, increasing the magnet thickness increases the shear stress up to some point and then the effect becomes saturated. Therefore, increasing the magnet thickness may not be an effective solution to overcome the reduction of shear stress due to increased air gap length.

To maintain the shear stress constant as the air gap length increases, both magnet thickness and pole pitch have to be increased proportionally. However, increasing the magnet thickness adversely affects the cost. Moreover, increasing the pole pitch may have two potentially adverse consequences. A larger pole pitch means, according to (1), lower gear ratio. If dynamic effects associated with the load are neglected, it can be stated that a lower gear ratio results in higher rotor torque and therefore higher demand on the machine side when the TROMAG is used in combination with a rotary machine. In addition, an increased magnet width requires thicker core to carry the resultant flux.

3) Effect of Magnet Coverage: The effect of magnet width to pole pitch ratio \(w/\tau_p\), the so-called magnet coverage, on the shear stress is examined by using the analytical model and is illustrated in Fig. 6. The air gap length is fixed at 1 mm, and the magnet coverage has increased from 40% to 100% of the pole pitch, set at 10 mm, for different values of magnet thickness from 1 to 10 mm. As seen, for each magnet thickness there is a range of magnet coverage over which the shear stress increases almost in linear proportion with magnet coverage. The extent of that range depends on the magnet thickness. Moreover, the rate of rise of shear stress due to increase of magnet coverage tends to saturate as the coverage approaches unity because the leakage flux between adjacent magnets increases. As an example, when the magnet thickness is 5 mm reducing the coverage from 1 to 0.75, and consequently saving 25% on magnet material results in only 11% reduction of shear stress, from 230 to 205 kN/m\(^2\). However, if the coverage is further reduced to 50%, the drop of shear stress compared to the case of unity coverage will be almost 50%. It is therefore concluded that for any given pole pitch (corresponding to a given gear ratio) and air gap length a certain combination of magnet thickness and magnet coverage will maximize the resultant shear stress with respect to the amount of PM material consumed. Fig. 7 shows variations of the force per active magnet volume versus magnet thickness and magnet coverage for the case of 10 mm pole pitch and 1 mm air gap length. The highest force per active magnet volume achieved in this case is about 40 MN/m\(^3\) and occurs when magnet thickness and coverage are almost 1.6 mm and 66%, respectively. In this case, the shear stress works out to 85.4 kN/m\(^2\). As an example, under the circumstances mentioned earlier, to make a TROMAG with pull-out force of 100 kN, the required active air gap area would be 1.17 m\(^2\). The product of air gap radius and the active length then needs to be 0.1864 m\(^2\).
4) Scaling: According to the definition of shear stress, as long as the shear stress is kept constant the pull-out force can be elevated by increasing the active air gap area $A$ in linear proportion with the force. If $R_m$ denotes the outer radius of the inner part, including magnets, and $L_{ac}$ denotes the active length, then the pull-out force $F$ equals

$$F = \sigma A = \sigma 2\pi R_m L_{ac}. \quad (2)$$

Let $d_c$ denote the thickness of iron core for both rotor and translator. The total volume of the active iron works out to

$$V_{Fe,ac} = 2\pi (2R_m + g)L_{ac}d_c \cong 4\pi R_m L_{ac}d_c \quad (3)$$

and the total volume of the active magnets is going to be

$$V_{PM,ac} = 2\pi (2R_m + g)L_{ac}h_M \cong 4\pi R_m L_{ac}h_M. \quad (4)$$

Iron and magnet volumes are indications of TROMAG active material consumption. The term $R_m L_{ac}$ appears in expressions for force, iron volume, and magnet volume. Therefore, as long as the shear stress is constant what affects the active material consumption is the product $R_m L_{ac}$ not the individual values of $R_m$ and $L_{ac}$. However, the individual values of $R_m$ and $L_{ac}$ would affect the overall force per active volume. The device volume is proportional to second power of radius and first power of length. Hence, the force per active volume remains constant with variation of active length but it linearly drops as the radius increases.

On the other hand, if the stroke of translator $L_S$ is accounted for, the total volume of iron and magnet material yields to

$$V_{Fe,tot} \cong 2\pi R_m (2L_{ac} + L_S)d_c \quad (5)$$

$$V_{PM,tot} \cong 2\pi R_m (2L_{ac} + L_S)h_M. \quad (6)$$

At constant shear stress, if the rotor radius is increased both the pull-out force and total iron and magnet volume increase in linear proportion with the radius. If the active length is increased, although the force increases in linear proportion with the length, the material consumption increases at a lower rate whose value depends on the stroke.

C. Demagnetization

Due to their high coercivity, high-energy-density PM materials are known to withstand large demagnetizing fields. However, confrontation of two sets of rare-earth PM poles in a TROMAG could result in demagnetization if the issue is not properly addressed in the design.

Some amount of irreversible demagnetization occurs when the operating point of the PM drops below the knee point of its $B-H$ characteristic. At the room temperature, the flux density corresponding to the knee point, $B_D$, may fall in the third quadrant, i.e., it is negative. $B_D$, however, climbs up toward the second quadrant as the temperature increases. Other than temperature, the value of $B_D$ depends on the type of magnets (Nd-Fe-B or Sm-Co) and the materials grade; however, 0–0.1 T could be an approximate range of $B_D$ at 40 °C for the conventional Nd-Fe-B magnets.

Intuitively speaking, confrontation of two sets of magnets with unequal thickness could result in demagnetization of the thinner magnets. Consider a TROMAG with 20 mm pole pitch and unity magnet coverage, with the thicknesses of rotor PMs $h_r$ and translator PMs $h_t$ being set at 5 and 4 mm, respectively. Fig. 8 shows the 2-D model of this TROMAG and the corresponding flux lines at three different relative positions of rotor and translator. Boundary conditions are imposed to exclude the end effects originating from the open-ended structure of the device. At both aligned and unaligned positions, the force and torque are zero while at the half-way position the TROMAG operates at its pull-out values of force and torque.

Normal (radial) component of the flux density $B_r$ on the lines spanning two pole pitches over the surface and middle of the thinner magnets are obtained from 2-D FEA and are shown in Fig. 9 for the three positions mentioned earlier. As expected, the unaligned position is the worst case for demagnetization because in that position the opposite poles of the rotor and translator fully confront. Therefore, only the unaligned position will be considered in the rest of this study. It is observed in Fig. 9 that when the TROMAG is in the unaligned position $B_r$ clearly becomes negative at the surface of the thinner magnets, and close to zero (but still negative) at their middle, thereby demagnetization could occur. If the thicknesses of the magnets of both arrays are set at 5 mm, $B_r$ will be about 0.02 T at the surface. This means that even with equally thick magnets demagnetization may be possible, depending on the magnet grade and the temperature.
To investigate the effect of magnet coverage on the extent of demagnetization, the system of Fig. 8(c) is simulated for different values of magnet coverage. The maximum demagnetizing flux density (radial) at the surface of the rotor magnets $B_r$, shown in Fig. 10, implies that lower magnet coverage results in smaller demagnetizing field. Unity magnet coverage, thus, is the worst design regarding demagnetization.

To study the influence of pole pitch and magnet thickness on demagnetization, the air gap length $g$ is kept constant at 1 mm and the pole pitch is varied over the range of 5–30 mm assuming unity magnet coverage. In one case, whose corresponding results are presented in Fig. 11(a), the magnets of rotor and translator have the same thickness ($h_r = h_t = h$). In another case, whose corresponding results are depicted in Fig. 11(b), the thickness of translator magnets is fixed at 5 mm while the thickness of rotor magnets holds several smaller values ($h_r \neq h_t$). In both cases, $B_r$ is obtained versus the pole pitch.

Scrutiny of the results reveals that demagnetization is a function of magnet dimensions ratio, as well as the air gap length. It is observed that thinner and wider magnets are more prone to demagnetization. In other words, at a given magnet thickness to air gap length ratio $h/g$, increasing the ratio of pole pitch to magnet thickness $\tau_P/h$ results in increase of the demagnetizing field.

In order to prevent demagnetization of PMs, therefore, it is advisable to employ magnets of similar material grade and thickness for both the rotor and translator to ensure that all over the magnets the flux density remains in the second quadrant at any relative position of rotor and translator. Moreover, the knee point of the PM $B–H$ characteristic at practical operating temperatures must be checked against the value of demagnetizing flux density.

**IV. EXPERIMENTAL SETUP**

A TROMAG, with the inner part being the long rotor and the outer part being the short translator, is prototyped and its force characteristic is measured. Both the rotor and translator cores are made of 1020 steel material. To ease the fabrication, flexible
strips of magnets, made of a mixture of Nd-Fe-B and some elastic material(s) are employed. The remanent flux density of the employed magnets is roughly 0.45 T. Shallow double start threads are grooved on both the rotor and translator cores to facilitate placing and pasting the magnet strips. The translator is press fit into a flange that is mounted on linear ball bearings sliding on rails. Fig. 12 shows a picture of the setup assembly in which the rotor is partially covered by magnet strips. Principal dimensions of the fabricated TROMAG are given in Table II.

To obtain the force characteristic, the rotor is rotated manually by 90° while the translator is prevented from motion through an obstacle (not shown in the figure) against which the force transducer is pressed. The measured force, whose peak value is 670 N, is presented in Fig. 13 along with the results of 3-D FEA. With 0.45 T as the remanent flux density, the pull-out force of 675 N is obtained from a 3-D FEA model with discretized helix, 700 N from 2-D FEA, and 717 N from the 2-D analytical model. A close agreement is observed between the experiment and analysis.

V. CONCLUSION

A number of key observations were made through studying the TROMAG design. It was shown how deviation from an ideal helix can change both the capacity and behavior of the device. Employing a lower number of segments reduces the pull-out force and results in a load-dependent gear ratio. The impact of magnet dimensions and air gap length on the force was studied as well; concluding that an optimum pitch exists for any given gap length and magnet thickness. In addition, it was observed that a certain combination of magnet thickness and magnet coverage maximizes the force per volume of magnet material. Moreover, the variation of force per weight of material with overall TROMAG dimensions was discussed. It was shown that the individual choices of air gap radius and active length to achieve a given air gap area does not affect cost and weight of the active material; but nonetheless, by small radius and long length a more compact design can be reached. If the stroke of the translator is considered, then increasing the length instead of radius would lead to more cost and weight effective designs. In addition to application-specific space restrictions, maximum length to radius ratio is mechanically restricted however. Also revealed was that to avoid demagnetization, PMs used for rotor and translator should be of the same thickness. Another measure to be taken in this regard would be to employ magnets having a knee point falling within the third quadrant at the operating temperature. Finally, static measurements from a lab prototype were presented verifying the TROMAG concept as well as accuracy of the analysis methods employed in this study.

APPENDIX

The analytical model calculating the axial force of a TROMAG is adopted from [9] and [10] and is summarized later. As a first step, as depicted in Fig. 14(a), the TROMAG is approximated by a 2-D system [the area enclosed within the dashed rectangle of Fig. 14(a) is zoomed on and shown in Fig. 14(b) and (c)]. Then the translator magnets are removed as indicated by dashed lines in Fig. 14(b). In the resultant system, the magnetic field due to rotor magnets is calculated. It would suffice to obtain the radial component of field only because the axial force, whose calculation is of the interest in this paper, arises from that component. To calculate the field of rotor magnets, Laplace [see (A.1)] and Poisson [see (A.2)] equations have to be solved in the air gap region (both the physical air gap and the air region resulted from removing the translator magnets) and

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TABLE II

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet thickness on translator and rotor (mm)</td>
<td>5</td>
</tr>
<tr>
<td>Magnet width (mm)</td>
<td>8</td>
</tr>
<tr>
<td>Pole pitch (in)</td>
<td>0.4</td>
</tr>
<tr>
<td>Outer radius of rotor core at groove peaks (in)</td>
<td>1.439</td>
</tr>
<tr>
<td>Inner radius of translator core at groove peaks (in)</td>
<td>1.813</td>
</tr>
<tr>
<td>Air gap length (mm)</td>
<td>2</td>
</tr>
<tr>
<td>Translator length (in)</td>
<td>4</td>
</tr>
</tbody>
</table>
functions of first and second kind of order one, and \( m_n \) is defined as \((2n - 1)\pi/\tau_p\). The radial and axial components of flux density in the magnet region \( B_{rI} \) and \( B_{zI} \) turn out to be as follows:

\[
B_{rI} = -\sum_{n=1,2,...} \left[ \left( F_{A_n}(m_n r) + a_{II_n}(m_n r) \right) + \left( -F_{B_n}(m_n r) + b_{II_n}(m_n r) \right) K_1(m_n z) \right] \cos(m_n z) 
\]

(A.6)

\[
B_{zI} = \sum_{n=1,2,...} \left[ \left( F_{A_n}(m_n r) + a_{II_n}(m_n r) \right) - \left( F_{B_n}(m_n r) - b_{II_n}(m_n r) \right) K_0(m_n r) \right] \sin(m_n z). 
\]

(A.7)

To express unknown coefficients, the following definitions are made:

\[
P_n = \frac{4B_r}{\tau_p} \sin \left[ \frac{(2n - 1)\pi}{2\tau_p} \right] 
\]

(A.8)

\[
c_{1n} = I_0(m_n R_s) \quad c_{2n} = K_0(m_n R_s) 
\]

(A.9)

\[
c_{3n} = I_0(m_n R_e) \quad c_{4n} = K_0(m_n R_e) 
\]

\[
c_{5n} = I_0(m_n R_m) \quad c_{6n} = K_0(m_n R_m) 
\]

\[
c_{7n} = I_1(m_n R_m) \quad c_{8n} = K_1(m_n R_m) 
\]

The coefficients \( F_{A_n}(m_n r) \) and \( F_{B_n}(m_n r) \) are as follows:

\[
F_{A_n}(m_n r) = \frac{P_n}{m_n} \int_{m_n R_s}^{m_n R_e} \frac{K_1(x) dx}{I_1(x) K_0(x) + K_1(x) I_0(x)} 
\]

(A.10)

\[
F_{B_n}(m_n r) = \frac{P_n}{m_n} \int_{m_n R_s}^{m_n R_e} \frac{I_1(x) dx}{I_1(x) K_0(x) + K_1(x) I_0(x)} 
\]

(A.11)

The coefficients \( a_{II_n}, a_{II_n}, b_{II_n}, \) and \( b_{II_n} \) are obtained from (A.12) to (A.14):

\[
\left( \mu_r \left( \frac{c_5n}{c_{2n}} - \frac{c_{1n}}{c_{2n}} \right) - \left( \frac{c_5n}{c_{8n}} - \frac{c_{1n}}{c_{8n}} \right) \right) a_{II_n} 
\]

\[
\left( \mu_r \left( \frac{c_5n}{c_{2n}} - \frac{c_{1n}}{c_{2n}} \right) - \left( \frac{c_5n}{c_{8n}} - \frac{c_{1n}}{c_{8n}} \right) \right) b_{II_n} 
\]

\[
\left( F_{A_n}(m_n R_m) c_{6n} - F_{B_n}(m_n R_m) c_{5n} \right) + \left( F_{A_n}(m_n R_m) c_{7n} - F_{B_n}(m_n R_m) c_{6n} \right) \right) a_{II_n} 
\]

(A.12)

(A.13)

(A.14)

The next step is to use the Lorentz’s force law to calculate the force exerted on translator magnets. To that end, the translator magnets are replaced with equivalent current-carrying sheets as depicted in Fig. 14(c) by crosses and dots. Each sheet is an annulus around the Z-axis with inner radius of \( r = R_m + g \) and outer radius of \( r = R_e \) and an infinitesimal width (length in Z-direction) of \( dz \). The current flowing in each sheet equals the magnetomotive force (mmf) \( F_{PM} \) generated by the magnet it
is replacing

\[ F_{PM} = H_c h_M. \]  \hspace{1cm} (A.15)

In (A.15), \( H_c \) is the PM coercivity and \( h_M \) is the PM thickness. If \( B_r, \mu_0, \) and \( \mu_r \) denote the PM residual flux density, permeability of the vacuum, and recoil permeability of the PM material, then \( H_c = B_r / \mu_0 \mu_r. \)

Finally, the force exerted on each current carrying sheet can be obtained by applying Lorentz’s force law and integrating the force over the surface of current sheet \( S^\text{\(r\)}}.

\[ \vec{F} = \int (\vec{H}^r \times \vec{B}^r) dS. \]  \hspace{1cm} (A.16)

In (A.16), \( \vec{H}^r \) is a vector whose magnitude is \( H_c \) and its direction is the same as that of the flow of current in the equivalent current sheet. \( \vec{B}^r \) is the magnetic flux density vector in region \( I \), and \( dS = 2\pi r dr. \)

The radial component of flux density results in an axial force component \( F_z \) on the translator as follows:

\[ F_z = \frac{4\pi B_r}{\mu_0 \mu_r} \sum_{m=1,2,...} K_{r,m} \sin \left( \frac{m \pi w}{2} \right) \sin (m \pi z_d) \]  \hspace{1cm} (A.17)

in which \( w \) is the magnet width, and \( z_d \) is shown in Fig. 14. The coefficient \( K_{r,m} \) is given by

\[ K_{r,m} = \frac{4}{B_r} \int_{R_s+g}^{R_t} \left[ a_{r,m} I_1 (m \pi r) + b_{r,m} K_1 (m \pi r) \right] r dr. \]  \hspace{1cm} (A.18)

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REFERENCES


