EFFECT OF FLUID DISPERSION COEFFICIENTS ON PARTICLE-TO-FLUID MASS TRANSFER COEFFICIENTS IN PACKED BEDS

CORRELATION OF SHERWOOD NUMBERS

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Abstract—Gas-phase and liquid-phase mass transfer data published in the literature are corrected for the axial fluid dispersion coefficient values proposed by Wakao et al. [1]

The corrected Sherwood numbers in the range of Reynolds number from about 3 to 10,000 are correlated by

\[ Sh = 2 + \frac{1}{13} Sc^{1/3} Re^{0.3} \]

A large number of mass transfer studies in packed beds have been reported for the past three and a half decades, since Gamson et al. [2] determined in 1943, the particle-to-gas mass transfer coefficients from the measurements of the rates of evaporation of water from wet porous particles.

The question we like to raise in this paper is about the effect of axial fluid dispersion on the mass transfer coefficients. Most investigators ignored the dispersion effect in the analysis of the experimental results. As shown later, however, this may cause considerable errors in the evaluation of transfer coefficients at low flow rates, particularly when fluid is gas.

In fact, Petrovic and Thodos [3] corrected various earlier gas-phase mass transfer data of Thodos et al. [4] for axial fluid mixing. In the recalculation they assumed the axial Peclet number of two in the range of Reynolds number from 3 to 900.

In a recent article, however, Wakao et al. [1] proposed the following equation for the axial effective dispersion of fluid species reacting with first order kinetics:

\[ D_{ax} = \frac{E^0}{\varepsilon_b} + 0.5 D_p U \]  

and have shown in chart the stagnant contribution \( E^0 \) as a function of intraparticle diffusivity \( D_p \), molecular diffusivity in fluid phase \( D_e \), and Jüttner modulus \( \phi = R\sqrt{(k_c/D_e)} \), where \( R \) is a particle radius and \( k_c \) is a reaction rate constant.

The chart indicates \( E^0/D_e = 20 \) at \( k_c = \infty \) or \( \phi = \infty \). When \( k_c \) is sufficiently large, the reaction proceeds only at the particle surface and consequently the mass transfer between the particle surface and a fluid surrounding the particle becomes the rate-controlling step. In fact, Satterfield and Resnuck [4] obtained the mass transfer coefficients from the experiments of the fast chemical reaction, catalytic decomposition of hydrogen peroxide on metal spheres.

Therefore, in packed beds where mass transfer, such as evaporation, sublimation, dissolution, etc., or a diffusion-controlled chemical reaction is taking place at the particle surface under the condition that the inside of the particle is not involved in the mass transfer or chemical reaction process, the axial fluid effective dispersion coefficient is

\[ \frac{\varepsilon_b D_{ax}}{D_e} = 20 + 0.5 Sc Re \]  

As far as the authors know, the dispersion coefficient for a mass transferring species has not been distinguished from that under inert condition. The stagnant term of the dispersion coefficient has been considered as \( E^0/(\varepsilon_b D_e) = 0.6 - 0.8 \). According to Wakao et al., however, this is the value when no mass transfer takes place between the particles and fluid in packed beds. The axial fluid dispersion coefficient \( D_{ax}^* \) under inert condition is

\[ \frac{\varepsilon_b D_{ax}^*}{D_e} = (0.6 - 0.8)\varepsilon_b + 0.5 Sc Re \]  

In this paper, the published mass transfer data are corrected for the axial fluid effective dispersion coefficient \( D_{ax} \) under mass transfer condition.

RANGE OF THE DATA COLLECTION

Out of the numerous measurements reported in the literature, we limit the data collection to the work with

1. Particles in bed being all active Distended and diluted bed data are not included.
2. Number of particle layers in mass transferring bed being greater than two.

The experimental methods and operating conditions of the collected work are listed in Table 1 (for gas-phase) and Table 2 (for liquid-phase).

The collected data are briefly reviewed in the following sections.
<table>
<thead>
<tr>
<th>Year</th>
<th>Reference No</th>
<th>Investigator</th>
<th>Experimental method</th>
<th>Steady or nonsteady</th>
<th>Particle</th>
<th>Fluid</th>
<th>Sc</th>
<th>Re</th>
<th>Fluid dispersion considered</th>
<th>Remarks</th>
</tr>
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<td>[2]</td>
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<td>Vanadium</td>
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<td>H$_2$</td>
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<td>Misauchi et al</td>
<td>Pulse chromatography, adsorption</td>
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Diluted beds, distended beds, and data with single particle layer are not included.
Table 2: Liquid-phase mass transfer work

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<th>Particle Material</th>
<th>Shape</th>
<th>Size (mm)</th>
<th>Fluid</th>
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<th>$Re$</th>
<th>$D_G/\mu$</th>
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<td>Succinic acid</td>
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<td>Cylinder</td>
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<td>Granule</td>
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<td>Lead</td>
<td>Sphere</td>
<td>2, 0, 2, 1, 22, 24</td>
<td>Mercury</td>
<td>120-140</td>
<td>$D_G/\mu = 32-1,500$</td>
<td>No</td>
<td></td>
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<tr>
<td>1956</td>
<td>[30]</td>
<td>Selke et al</td>
<td>Ion exchange</td>
<td>Nonsteady</td>
<td>Amberlite IR-120</td>
<td>Sphere</td>
<td>0, 4, 0, 5, 0, 6, 0, 9</td>
<td>Copper sulfate solution</td>
<td>520 &amp; 1,130</td>
<td>$D_G/\mu = 27-120$</td>
<td>No</td>
<td></td>
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<tr>
<td>1958</td>
<td>[31]</td>
<td>Wakao et al</td>
<td>Dissolution of 2-naphthol</td>
<td>Steady</td>
<td>2-naphthol</td>
<td>Cylinder</td>
<td>8, 0, 8, 1, 8, 5</td>
<td>Water</td>
<td>1,460-1,760</td>
<td>$D_G/\mu = 0.4-3.000$</td>
<td>No</td>
<td></td>
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<tr>
<td>Year</td>
<td>Authors</td>
<td>Experiment Type</td>
<td>Conditions</td>
<td>Data</td>
<td>Notes</td>
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<tr>
<td>1963</td>
<td>Williamson et al</td>
<td>Dissolution of benzoic acid</td>
<td>Steady Benzoic acid Sphere 61, 63 Water 940–1,140 $\frac{D_a G}{\mu} = 0.0463$ No</td>
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<td>1966</td>
<td>Wilson and Geankophis</td>
<td>Dissolution of benzoic acid</td>
<td>Steady Benzoic acid Sphere 64 Water 860–1,100 $\frac{D_a G}{\mu} = 0.0016–11$ No</td>
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<tr>
<td>1969</td>
<td>Kasaoka and Nitta</td>
<td>Dissolution of benzoic acid coated on particles</td>
<td>Steady Steel Sphere 28, 41, 64 Water, benzoic acid aqueous solution 350–2,850 $\frac{D_a G}{\mu} = 1–100$ No</td>
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<tr>
<td>1975</td>
<td>Upadhyay and Tripathi</td>
<td>Dissolution of benzoic acid</td>
<td>Steady Benzoic acid Cylinder 60, 77, 81, 86, 90, 112 Water 720–1,350 $\frac{D_a G}{\mu(1 - \epsilon_0)} = 2–2.410$ No</td>
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<tr>
<td>1975</td>
<td>Miyauchi et al</td>
<td>Pulse chromatography, chemical reaction</td>
<td>Nonsteady Sulfonic acid ion exchange resin Sphere 09, 15 Water 510–640 $\frac{D_a G}{\mu} = 0.01–5$ Yes</td>
<td></td>
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<tr>
<td>1976</td>
<td>Appel and Newman</td>
<td>Cathodic reduction of ferricyanide to ferrocyanide</td>
<td>Steady Stainless steel Sphere 40 Mixture of aqueous solution of ferrocyanide, ferricyanide and potassium nitrate $1.390–1,450 \frac{D_a G}{\mu} = 0.008–0.17$ No</td>
<td></td>
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<td>1977</td>
<td>Kumar et al</td>
<td>Dissolution of benzoic acid</td>
<td>Steady Benzoic acid Cylinder 5.5 × 2.5, 88 × 3.4, 88 × 4.5, 96 × 2.8, 128 × 3.1, 128 × 3.8, 128 × 4.9 Water, 60% propylene glycol 770–42,400 $\frac{D_a G}{\mu} = 0.01–600$ No</td>
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</table>

Diluted bed data are not included.
Evaporation of water into air stream—steady state measurements

Since Hougen et al [2] and Hurl[5], the water evaporation has been studied by many investigators. The mass transfer coefficients have been determined from the measurements during the constant rate period of drying.

In the work of Gamson et al [2] (Re = 100-4000), and Wilke and Hougen[6] (Re = 45-250), the particle surfaces were assumed at wet-bulb temperatures. Even in the two runs at Re = 150 and 370, Hurl[5] was the first to measure the particle surface temperatures. Galloway et al [7] (Re = 150-1200) found that the differences between the measured surface temperatures and the saturation adiabatic (or wet-bulb) temperatures were less than 5°F.

De Acetis and Thodos[8] (Re = 60-2100), however, pointed out that considerable temperature differences existed between the measured surface temperatures and the wet-bulb values when flow rates were low.

Bradshaw and Myers[9] (Re = 200-4000) also measured the surface temperatures in some runs to find the particle surface at wet-bulb.

Since De Acetis and Thodos, Thodos et al [3,10-14] all measured the surface temperatures and used them in the determination of transfer coefficients.

In the work of Hougen et al [2,6] who assumed wet-bulb surface temperatures question may remain for the data at lower Reynolds numbers, probably in the range Re = 45-150. However, their data are not substantially different from those of Thodos et al who measured surface temperatures.

We, therefore, conclude that all of the data may be used for our data correlation, if information on bed height and void fraction which are required for the correction for axial fluid dispersion is given.

Evaporation of organic solvent—steady state measurements

Thodos et al have studied the evaporation of organic solvents from particle surfaces into gases. The surface temperatures were also measured. The systems employed by Hobson and Thodos[10] are n-butanol, toluene, n-octane and dodecane in air, nitrogen, carbon dioxide and hydrogen, those of Petrovic and Thodos[3] are n-octane, n-decane, n-dodecane and n-tetradecane in air, and that of Wilkins and Thodos[15] is n-decane in air.

Sublimation of naphthalene—steady state measurements

The rates of sublimation of naphthalene were measured by Hurl[5], Resnick and White[16], Chu et al[17], and Bradshaw and Bennett[18]. Compared to liquid–gas system, the naphthalene–gas system has the advantage that adiabatic temperature drop is small. But the disadvantage of the system is that the vapor pressure has not been thoroughly investigated.

As compared in Table 3, different vapor pressures have been measured or assumed in different papers. The disagreement in the measured vapor pressures may be considered as an indication of the difficulty in naphthalene sublimation study. In fact, small errors in the vapor pressure values as well as in the measured pressures are seriously magnified in the calculation of mass transfer coefficients, particularly when the difference between the two pressure values is small. This is very serious in the experimental determination of mass transfer coefficients at low flow rates.

Table 3 indicates that considerably high vapor pressures were assumed by Hurl. Probably this is the reason he has obtained the low transfer coefficients. Resnick et al have determined anomalously low coefficients. The reason for the anomaly is not very clear, but the data at low flow rates are, as mentioned already and also as pointed out by Bar-Ilan and Resnick[42], not enough reliable.

The data of Hurl and Resnick et al are rejected, but those of Chu et al (Re = 20-2000) and Bradshaw et al (Re = 440-9900) for spheres and cylinders are included in our data correlation.

Diffusion-controlled catalytic reaction at particle surface—steady state measurements

Satterfield and Resnick[4] conducted the experiments of catalytic decomposition of hydrogen peroxide at metal surface. The reaction was so fast that the mass transfer between the particle surface and bulk fluid was the rate-determining step. The mass transfer coefficients were easily evaluated from the measurements of the overall rates.

Pulse gas chromatography—nonsteady state measurements

Wakao and Tanisho[19] (Re = 0.06-18) and Wakao et al[21] (Re = 0.1-1) determined the mass transfer coefficients from the pulse chromatographic measurements, respectively, in non-adsorptive and adsorptive systems. They obtained anomalously low transfer coefficients, but Wakao[43] has found that the assumption of concentric intraparticle concentration which is widely made in heat and mass transfer analysis is responsible for the low coefficient values. Their original data are, therefore, not included in our data correlation.

Gangwal et al[22] also performed chromatographic adsorption measurements and found the limiting Sherwood number not less than unity.

imposed as a pulse on the carrier gas flowing in the packed bed. They measured the overall mass transfer resistance, comprising gas-phase and solid-phase diffusion resistances, and have mentioned that the limiting Sherwood number was considered to be 12.5 at bed void fraction 0.5

**REVIEW OF THE PUBLISHED LIQUID-PHASE DATA**

In some papers measurements were made at very low Reynolds numbers. However, the problem at low flow rates is a natural convection. The convection effect becomes increasingly important as Reynolds number decreases. The critical Reynolds number above which the natural convection effect may be ignored is not clear. According to Dryden et al. [28], the critical Reynolds number for 6.3 mm particles is at \( Re/e_b = 5 \).

In order to avoid the possible natural convection effect, the liquid-phase data for \( Re < 3 \) are not included in our data correlation.

**Dissolution of solid into liquid stream—steady state measurements**

The rates of dissolution of benzoic acid particles into water flowing through the beds of spherical or cylindrical particles were measured by Ishno et al. [26], Dryden et al. [28], Williamson et al. [32], Wilson and Geankoplis [33], Kasaoka and Nitta [34], Upadhyay and Tripathi [35], and Kumar et al. [38]. Evans and Gerald [27] used granules ground in mill.

- Dissolution of benzoic acid into propylene glycol was studied by Wilson and Geankoplis [33], and Kumar et al. [38].
- Dissolution of 2-naphthol into water was investigated by McCune and Wilhelm [24], Dryden et al. [28], and Wakao et al. [31].
- Dissolution of succinic acid and salicylic acid into acetone, \( n \)-butyl alcohol and benzene was studied by Gaffney and Drew [25].
- Dissolution of lead into mercury was investigated by Dunn et al. [29].

**Electrochemical reaction—steady state measurements**

Appel and Newman [37] applied a limiting current method to obtain the mass transfer coefficients at very low flow rates, \( Re < 0.17 \).

**Evaporation of water**

Evaporation of organic solvent [2, 3, 5–14].

**Sublimation of naphthalene**

Sublimation of naphthalene [17, 18].

**Diffusion-controlled reaction on particle surface**

Diffusion-controlled reaction on particle surface [4].

**Dissolution of solid** [24–29, 31–35, 38].

The dissolution data are available in the very wide range of Reynolds number 0.0016–3,000, but to avoid possible natural convection effect, as mentioned already, the data [28] at Reynolds number less than about three are not used for our data correlation.

Incidentally, the above measurements are those made under steady state condition with solid particles having constant concentration of mass transferring species at the surface.

To the contrary, the data obtained from nonsteady state measurements have not passed the criteria. In general, the two rate-parameters are involved in the analysis of steady state measurements: particle-to-fluid mass transfer coefficient and fluid dispersion coefficient. In the analysis of nonsteady state measurements more parameters are involved. It is conceivable that this makes the determination of transfer coefficient far complicated and even difficult in some nonsteady cases.

**Ion exchange—nonsteady state measurements**

Ion exchange in Amberlite particles was studied by Selke et al. [30]. Their mass transfer coefficients in the range of Reynolds number 1–40 are considerably larger than those of many other investigators. The graphical technique they applied for the determination of transfer coefficients does not seem to have given accurate coefficient values. The data are not included in our correlation.

**Pulse liquid chromatography—nonsteady state measurements**

Miyauchi et al. [36] performed chromatographic measurements of the reaction between sodium hydroxide imposed as a pulse on water stream and sulfonic acid in ion-exchange resin particles. They measured the overall mass transfer resistance, similarly to the gas chromatographic measurements [20], and have mentioned that the limiting liquid-phase Sherwood number was considered to be 16.7 at bed void 0.4.

**Selected data**

As a result, the data which have passed our criteria are as follows:

- Evaporation of water [2, 3, 5–14]
- Sublimation of naphthalene [17, 18]
- Diffusion-controlled reaction on particle surface [4]
- Dissolution of solid [24–29, 31–35, 38]

**Reevaluation of mass transfer data obtained from evaporation, sublimation, dissolution and diffusion-controlled chemical reaction**

Let us consider a mass transfer between a flowing fluid and particle surface on which the concentration of the species under consideration is constant. The resistance to mass transfer is considered to reside only in the fluid phase, so the steady state mass balance gives:

\[
U \frac{dC}{dx} - D_{xx} \frac{d^2C}{dx^2} + \frac{a}{e_b} k_\phi (C - C_\phi) = 0
\]  

(4)
Two types of packed beds, (a) and (b), have been used in the published experimental works:

(a) No inert packed bed before the mass transferring packed bed \((0 < x < L)\)

The Danckwerts boundary conditions are

\[
\text{at } x = 0, \quad U(C - C_{\text{in}}) = D_{\text{ax}} \frac{dC}{dx}
\]

\[
\text{at } x = L, \quad \frac{dC}{dx} = 0
\]

(b) Inert packed bed \((-L < x < 0)\) before the mass transferring packed bed \((0 < x < L)\)

Same-sized particles are usually packed in both beds, but the fluid dispersion coefficient \(D_{\text{ax}}^*\) in the inert bed is different from \(D_{\text{ax}}\) in the mass transferring bed. The system is described as

\[
\text{at } x = -L, \quad U(C^* - C_{\text{in}}) = D_{\text{ax}}^* \frac{dC^*}{dx}
\]

\[
\text{at } -L < x < 0, \quad U \frac{dC^*}{dx} - D_{\text{ax}}^* \frac{d^2C^*}{dx^2} = 0
\]

\[
\text{at } x = 0, \quad C^* = C
\]

\[
D_{\text{ax}}^* \frac{dC^*}{dx} = D_{\text{ax}} \frac{dC}{dx}
\]

\[
\text{at } x = L, \quad \frac{dC}{dx} = 0
\]

In either case the effluent concentration is given by

\[
\frac{C_s - C_{\text{exit}}}{C_t - C_{\text{in}}} = \frac{4A \exp \left[ \frac{UL}{2D_{\text{ax}}} \right]}{(1 + A)^2 \exp \left[ \frac{AUL}{2D_{\text{ax}}} \right] - (1 - A)^2 \exp \left[ -AUL \right] \frac{UL}{2D_{\text{ax}}}}
\]

where

\[
A = \sqrt{1 + \frac{4\Delta_k D_{\text{ax}}}{\epsilon b U^2}}
\]

If we assume \(D_{\text{ax}} = 0\), eqn (7) reduces to

\[
\frac{C_s - C_{\text{exit}}}{C_t - C_{\text{in}}} = \exp \left[ -\frac{Sh'}{ScRe} aL \right]
\]

where \(Sh'\) means a Sherwood number supposed to be evaluated under the assumption of \(D_{\text{ax}} = 0\).

We are interested in the mass transfer coefficients reevaluated with \(D_{\text{ax}}\) given by eqn (1) in general form, or eqn (2) for the mass transfer systems under consideration. The Sherwood number based on the reevaluated mass transfer coefficient is denoted by \(Sh\).

From the \(Sh'\) data reported in the literature, we are able to calculate \(Sh\) values by equating eqns (7) and (8), if information on bed height (or number of particle layers) and bed void fraction is given.

In some papers \([3, 11, 15, 18]\), mass transfer coefficients have been obtained under the assumption of \(Pe = 2\). These data are also easily converted into \(Sh\) values.

The \(Sh\) values are calculated from all of the steady state measurements listed in the preceding section except the papers \([2, 5, 9]\) in which no detailed data on \(L\) and/or \(\epsilon_b\) are given.

**CORRELATION OF SHERWOOD NUMBERS**

The \(Sh\) values calculated from the data of Petrovic and Thodos \([3]\) and Satterfield and Resnick \([4]\) are shown together with their \(Sh'\) data in Fig 1. Considerable difference is seen between the two Sherwood values. The difference increases with the decrease of Reynolds number.

Figure 2 illustrates the \(Sh\) values reevaluated from the evaporation data with Schmidt number 0.6.

For mass (and heat) transfer for single spheres the following Ranz and Marshall \([44]\) equation is popular

\[
Sh = 2 + 0.6 Sc^{1/3} Re^{1/2}
\]

Figure 2 shows that the \(Sh\) values for packed beds are higher than the dotted line for single spheres, but the difference decreases as Reynolds number decreases. It seems that the \(Sh\) data for packed beds also approach the same limiting value for single spheres.

![Figure 1](image1)

**Fig 1** \(Sh\) and \(Sh'\) vs \(Re\), for Petrovic and Thodos \([3]\), and Satterfield and Resnick \([4]\).  

![Figure 2](image2)

**Fig 2** \(Sh\) vs \(Re\), for water evaporation, solid line shows eqn (12).
We, therefore, assume the following form of empirical equation

\[ Sh = 2 + \alpha Sc^{1/3} Re^0.6 \]  

(10)

It should be noted that Petrovic and Thodos[3] have obtained the correlation for the gas-phase data corrected for the axial Peclet number of two

\[ \epsilon Pe = 0.357 \, Re^{-0.359} \text{ for } 3 < Re < 900 \]  

(11)

As shown in Fig 2, at higher Reynolds numbers the data are satisfactorily correlated by the Petrovic-\-Thodos equation. However, at lower Reynolds numbers the data are obviously higher than their correlation. This is quite natural from the fact that the data shown in Fig 2 are those corrected for the axial effective dispersion coefficients of eqn (2). As mentioned already, the correction for larger axial effective dispersion coefficients gives higher mass transfer coefficients particularly at lower flow rates.

In liquid-phase system Sh values are large and good to be used for the determination of \( \alpha \) and \( \beta \) values. Figure 3 is a plot of the liquid-phase Sh data as \((Sh - 2)/Sc^{1/3}\) against Re. In the wide range of Reynolds number from about three the Sherwood data are well correlated by a single line. The correlation is

\[ Sh = 2 + 1.1 \, Sc^{1/3} Re^0.6 \]  

(12)

As far as the liquid-phase data for Re > 3 are concerned, the Sh values are inappreciably different from the Sh’ data. This is obvious from the fact that eqn (7) reduces to eqn (8), when not only \( D_{ax} = 0 \) but also \( ScRe \) is sufficiently large. This means that the liquid-phase data (high Sc) are little affected by the axial fluid dispersion unless Re is very low, and the gas-phase data (low Sc) are considerably affected by the dispersion as long as Re is not high.

In Fig 4 both liquid-phase and gas-phase Sh data are plotted against \((Sc^{1/2} Re^0.6)^2\). The reason the square of \( Sc^{1/2} Re^0.6 \) is taken is to enlarge the plot in the x-axis direction. It is seen that the data are well correlated by eqn (12).

**CONCLUSIONS**

1. The published mass transfer data in the range of Reynolds number greater than about three are correctly for the axial fluid effective dispersion coefficient given by eqn (2). The reevaluated gas-phase data are considerably higher than those obtained under the assumption of \( D_{ax} = 0 \). The reevaluated liquid-phase data are not appreciably different from those with \( D_{ax} = 0 \).

2. Except for the data obtained in the range of Reynolds number less than about three which may have been affected by natural convection, the reevaluated gas- and

**Fig 4** Relation between Sh and \( Sc^{1/2} Re^0.6 \), for gas- and liquid-phase data.

**Fig 5** Comparison of Sh and Sh’, for gas- and liquid-phase data.
liquid-phase data are correlated by eqn (12) over the wide range of Reynolds number up to 10,000

3 Together with eqn (12) for mass transfer coefficients the axial fluid effective dispersion coefficients given by eqn (1) should be used in the design and analysis of packed bed reactors

**NOTATION**

- $a$: particle surface area per unit volume of packed bed
- $C$: concentration
- $C_{ex}$: exit concentration
- $C_{in}$: inlet concentration
- $C_{p}$: concentration on particle surface
- $C_{w}$: concentration in inert packed bed
- $D_{ax}$: axial fluid effective dispersion coefficient
- $D_{ax}^*$: axial fluid effective dispersion coefficient in inert bed
- $D_{in}$: intraparticle diffusivity
- $D_{p}$: particle diameter
- $D_{m}$: molecular diffusivity
- $E_{0}$: stagnant contribution of fluid effective dispersion coefficient
- $k_{r}$: chemical reaction rate constant
- $k_{f}$: mass transfer coefficient
- $L$: height of mass transferring packed bed
- $I$: height of inert packed bed
- $Pe$: $D_{in}U/D_{ax}$, Peclet number for axial fluid dispersion
- $Re$: $D_{in}Ue_{p}/v$, Reynolds number
- $Sc$: $\nu D_{m}$, Schmidt number
- $Sh$: $k_{r}D_{in}/D_{m}$, Sherwood number based on $k_{f}$ determined with $D_{ax}$ of eqn (2)
- $Sh'$: Sherwood number determined with $D_{ax} = 0$
- $U$: interstitial fluid velocity
- $x$: axial distance variable

**Greek symbols**

- $\varepsilon_{b}$: bed void fraction
- $\nu$: fluid kinematic viscosity
- $\phi$: Jittner modulus

**REFERENCES**

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