Thermal radiation role in conjugate heat transfer across a multiple-cavity building block

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Abstract

Accurate calculation of the heat transfer rate across building blocks may lead to significant energy savings. Conjugate heat transfer analysis is carried out numerically to compute the heat transfer rate/R-value as the number/layout of air-filled cavities is changed. Conduction heat transfer in the block material and both natural convection and radiation in the cavity were considered. It is found that increasing the number of cavities keeping the block width unchanged decreases the heat flux significantly. Five cavities can fit the building block under investigation without compromising the strength. Furthermore, changing the surface emissivity can increase the R-value substantially so that no insulation would be needed to fill the spaces. Thermal radiation plays a considerable role in the heat transfer process of this application. Through this study, the heat transfer characteristics and the gains in the R-value were calculated for different number of cavities, for different cavities layouts for the conjugate contribution of conduction, convection and/or radiation across the building block. Results are useful for designers and manufacturers of building blocks for better energy savings of end users.

1. Introduction

Significant attention was focused on the problem of heat transfer in enclosures during the last few decades due to its significant role in energy savings especially in residential and industrial buildings. The literature reveals several numerical [1–27] and experimental [7,11,12,19,20] studies of heat transfer through the rectangular enclosures/cavities. These investigations in general considered natural convection heat transfer in an air layer enclosed between two isothermal vertical plates at different temperatures while the horizontal surfaces of the enclosures are adiabatic. Limited studies focused on the conjugate problem where more than one heat transfer mode; or different boundary conditions were involved.

Partitioning the cavities is an effective way to reduce the heat transfer rate [1–5]. On the other hand, this effect becomes less significant at high values of Rayleigh number [6]. To reduce energy losses, Bilgen [1] recommended to place the partition away from the hot wall whereas Tong and Gerner [4] recommended to place it midway between vertical walls. In a numerical study, Turkoglu and Yucel [5] indicated that the mean Nusselt number is inversely proportional to \((1 + N)\) for less than five partitions.

Decreasing the cavity aspect ratio also contributes to further heat transfer rate reduction [1]. On the other hand, in another study [5], it was reported to have an insignificant effect on the heat transfer rate. Al-Hazmy [2] reported that the insertion of polystyrene bars in the air filled cavities reduces the heat rate by a maximum of 36%. Moreover, using hollow polystyrene bars reduces the heat rate by 25% only due to the air motion inside cells. Furthermore, it was reported by Tong and Gerner [4], Lacarriere [7] and Baig and Antar [8] that partitioning the cavities eliminates the need to fill them with insulation in order to inhibit convection. The introduction of air gaps to the walls were also studied by Mahlia and Iqbal [9] who investigated the optimum insulation thickness in building walls on cost benefit. They reported that up to 77% saving can be achieved using air gaps of different sizes.

A one-dimensional analytical model of coupled heat transfer (conduction, convection, radiation) in enclosures was developed by Sambou et al. [10]. However, Gao et al. [14] indicated that one-dimensional approach is impractical and that it requires cautious numerical implementation. They obtained accurate results with less numerical computation by introducing low order models based on Moore’s balanced method.

Kangni et al. [6] investigated laminar natural convection and conduction in enclosures having multiple (1–5) partitions with...
finite thickness. For a range of the thermal conductivity ratio of the partition to the fluid of $1 \leq k \leq 10^2$, heat transfer decreases with increasing partitions at high Rayleigh number. Furthermore, they reported that heat transfer dominates by conduction up to a threshold Ra above which convection effect becomes significant.

Baig and Antar [8] investigated numerically the conjugate conduction-natural convection heat transfer to compute the heat leak R-Value for different numbers of air-filled cavities and for different cavity layouts. They reported that not only the increased number of Cavities in the layout can significantly reduce heat transfer rate, but also changing the cavities layout can be effective in reducing the heat flux without compromising the building blocks strength.

Diaz et al. [11,12] carried out an experimental and numerical study to investigate the thermal transmittance coefficient, $U$, of a wall made of air-block bricks. They reported that wall insulation decreases with the increase in the mortar and material conductivities. They also reported that changing the profiles of the holes alters the rate of the heat transfer through the hollow blocks. Yang et al. [13] emphasized the importance of the material properties of the hollow blocks used.

Antar and Thomas [25] showed the two dimensional analysis may be considered a good approximation using a lower-order analysis. Antar and Thomas [25] also investigated the two dimensional approximation is sufficient for aspect ratios (depth to width of cavity) that exceed 1.8.

It is understood that radiation heat transfer effects become significant when the temperature difference between the surfaces is high, or when compared with natural convection heat transfer case where the heat transfer coefficient is small. Rabhi et al. [21] reported that the heat transfer rate increases as we include thermal radiation effects whereas it decreases if the number of partitions increases. Li et al. [27] addressed the significance of thermal radiation on the total heat transfer rate across clay bricks and reported that it ranges from 4.6 to 25.8% for different cavities layouts. Hence, it is important to mention that neglecting radiation heat transfer amongst cavity walls is definitely not acceptable for hollow blocks used for building applications.

The effects of thermal radiation and natural convection in a two dimensional, air-filled square enclosure were also considered by Mezrhab and Bchir [22], Mahapatra et al. [23] who discussed the conditions under which the radiation heat transfer may be neglected. Antar [24] investigated the significance of multi-dimensional effects in estimating the rate of heat loss and identified the cases where simple one-dimensional convection/radiation analysis may be considered a good approximation using a lower-order analysis. Antar and Thomas [25] showed the two-dimensional effects of the heat transfer estimation across the block. They indicated significance of convection and radiation resistances through a simplified model. Then, they [26] developed a numerical finite-difference analysis for steady-state heat transfer in a composite wall with a two-dimensional rectangular gray body radiating cavity with and without natural convection circulation of air. They evaluated the accuracy of the first-order two-dimensional method for modeling the effects of radiation in practical applications of this type.

It is important to mention that Kumar [28] specified that the thickness of any shell shall not be less than 11 mm and that of any web not be less than 8 mm to maintain strength of the structure. This criterion is used in the current investigation as a constraint to satisfy the minimum requirements to sustain the entire structure.

It is noticed that many of the previous studies include neither the effects of radiation in addition to both “detailed” convection and conduction modes in multiple cavities applications, nor the realistic boundary conditions that considers the interaction between the three heat transfer modes at the inner cavities’ surfaces. This is considered in the present study where the values of the temperature at the cavities inner surfaces are unknown and are determined from the solution.

This work is aimed at investigating the effect of thermal radiation on the heat transfer (or R-value) across hollow building blocks of different internal layouts. Including thermal radiation effects is expected to result in an increase in the heat transfer rate. The effect of the surface emissivity on the heat transfer rate (and R-value) is also investigated. The accuracy of estimating the heat transfer rate is of great importance since it leads to significant energy savings in buildings.

2. Problem formulation

2.1. Conduction heat transfer

Steady two dimensional conduction heat transfer in the block solid material is governed by the following equation: (note that symmetry is used as indicated in Fig. 1)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$  \hspace{1cm} (1)

$$T = T_o \quad \text{at } x = 0 \quad T = T_i \quad \text{at } x = L$$ \hspace{1cm} (2)

$$\frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0 \quad \frac{\partial T}{\partial y} = 0 \quad \text{at } y = w$$ \hspace{1cm} (3)

$$-k \frac{\partial T}{\partial x} = q_{c,x} + q_{r,x} \quad \text{at } x = L_1 \quad \text{and } \quad x = L_1 + L_2$$
for $w_1 \leq y \leq w_1 + w_2$ \hspace{1cm} (4)

$$-k \frac{\partial T}{\partial y} = q_{c,y} + q_{r,y} \quad \text{at } y = w_1 \quad \text{and } \quad y = w_1 + w_2$$
for $L_1 \leq x \leq L_1 + L_2$ \hspace{1cm} (5)

where $L = (L_1 + L_2 + L_1), \quad w = (w_1 + w_2 + w_1)$ (Refer to Fig. 1 for details).

2.2. Convection heat transfer

The governing differential equations for the free convection within each of the enclosures are given for constant properties, no heat dissipation, applying Boussinesq approximation as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (6)
\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (7) \]

\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \rho g \beta (T - T_0) \quad (8) \]

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (9) \]

At the inner walls of the gaps, the no slip condition applies

\[ (U = V = 0) \quad (10) \]

The boundary conditions at the interface of the air gap include continuity of temperature and heat flux. However, temperatures are unknown and hence an iterative solution will be adopted to guess the temperature at the interface and apply the heat balance equation (refer to equations (4) and (5))

\[ q''_{\text{conduction}} = q''_{\text{convection}} + q''_{\text{radiation}} \quad (11) \]

Note that the boundary conditions will be applied for all the air gaps in the material, i.e. once for the arrangement given in Fig. 1, and as many as the number of gaps. The proposed layouts are numbered as follows: case 1, one cavity, case 2 two cavities and so on until case 6 that has six cavities. Then cases 7 to 10 are given as shown in Fig. 2.

2.3. Radiation heat transfer

For an opaque surface with graybody conditions, radiation heat transfer \( dq_s \) from a differential surface area \( dA_s \) is expressed in terms of the local radiosity \( J_s \) and the local irradiation \( G_s \) by

\[ dq_s = dA_s (J_s - G_s) \quad (12) \]

where

\[ J_s = \varepsilon_s F_{bs} + \rho_s G_s \quad (13) \]

Note that

\[ \rho_s = 1 - \varepsilon_s \quad (14) \]

the energy balance requirement is,
\[dA_s G_s = \sum_{j=1}^{4} \int_{A_i} J_j dA_j \ dF_{dj} \ ds\]  
(15)

where \(dA_s G_s\) represents the total rate of energy incident upon surface \(dA_s\).

The radiation shape factor is calculated using the relation

\[F_{i,j} = \frac{1}{A_i A_j} \int \frac{\cos \theta_i \cos \theta_j}{\pi R^2} L_{ij} dA_i dA_j\]  
(16)

It is important to note that the differential shape factors \(dF_{ds,j}\) for this two-dimensional arrangement are specified using the adaptive method used by Fluent for non-complex geometries. Hottel’s crossed-string method is one of the analytic methods that are used for accurate estimation of the radiation shape factors for this application [26,30].

3. Grid independence and numerical solution

Numerical methods must be employed to solve this problem due to the nonlinear nature of the governing equations. In order to numerically solve the governing partial differential equations (PDEs), approximations are made to convert partial derivatives to algebraic expressions using finite volume method. The set of these equations are subsequently solved in the entire domain to obtain the solution. The hollow brick system under study is discretized into small cells. These equations are solved using SIMPLE as a pressure-velocity solution algorithm [29].

To minimize the computation time, the grid was refined at the corners so that a dense grid is used where higher gradients are expected whereas the grid is kept coarser away from the walls for all the studied cases. Fig. 1c shows a sample of the numerical grid. The Control volume approach is employed in the numerical scheme. Variables are computed at ordinary nodal points, except the velocities, which are determined at staggered grid centered on the faces of the cells.

Grid independent tests are conducted as shown in Fig. 4 for a selected layout (case 5). The number of nodes was increased and the heat transfer rate was calculated for each case. Grid selection was made when the percent change in the calculated heat transfer rate does not exceed 0.5% between the selected grid and the finer one. Based on this criteria, the total number of nodes considered for this layout is 12720 where the minimum and maximum cell volumes are 6.88 \(\times\) 10^{-7} and 7.99 \(\times\) 10^{-6} m^3, respectively. The grid independence study was repeated for all other configurations and the same criteria were applied. The number of nodes increases as we use more cavities within the block to take care of the boundary layer growth within each cavity.

4. Validation of the numerical results

Since the current layouts are not reported in the open literature, the numerical results are validated by calculating the Nusselt number using the layouts employed in other investigations. Note that in these analyses, both upper and lower surfaces were considered adiabatic and the vertical surfaces were assumed isothermal [3,6], or an average temperatures was used as representative values [25,26]. The current model was adjusted to match these boundary conditions and the value of Nusselt number or \(R\)-values were compared to the reported values.

The percentage difference in the \(R\)-value computed for the case of a single and double cavity block compared with the conjugate heat transfer simplified low-order analysis of Antar and Thomas [26,27] are 5.9 and 6.8% respectively. However, due to the coarse
grid of this analysis and geometrical limitation, it is believed that
the current investigation provides a more accurate estimation of
the R-values considering their work as a limiting case for coarse
mesh. In the absence of thermal radiation, the percentage differ-
ence in calculating the Nusselt number (at \( Ra = 10^5 \) and \( H/W = 1 \))
compared with Ho and Yih [3] was 2% whereas the Nusselt number
calculations compared with Kangni et al. [6] for a different case
(\( Ra = 10^6 \) and \( H/W = 5 \)) showed a deviation of 0.2%.

5. Results and discussions

In this study, the effect of thermal radiation in calculating the
heat transfer rate (or the R-value) across a building block is
considered. It has been shown in a previous study [8] that
increasing the number of cavities within the block increases the R-
value significantly. These results were based only on conduction
and convection calculations and it was reported that R-value
increase reached 43.16% by increasing the number of cavities from 1
to 5 (Fig. 2a–d). A further increase of 17.65% was reported when
case 7 (Fig. 3a) was considered.

In order to account for the effects of radiation heat transfer, the
radiation boundary condition was included assuming the inner
surfaces of the cavities to be diffuse. Surface-to-surface radiation
model is considered with the air within the cavities as a non-
participating medium.

Strictly speaking, radiation heat transfer may have a signifi-
cant effect when the convection heat transfer coefficient is relatively
small, or when the temperature difference between the surfaces
increases significantly. Since the main idea in this work is to
to decrease the convection heat transfer coefficient, it is expected that
radiation heat exchange would play a significant role in the total
heat transfer calculations. When conduction and convection are
combined with radiation heat transfer for an opaque surface,
radiation interaction is usually considered to occur only at the
surface. Heat balance of conduction with radiation and convection
form a boundary condition at the cavities surfaces whereas heat
conduction will be the main heat transfer mode within the solid.

For a single cavity block (case 1, or the base case in Fig. 1),
calculations indicate that the total heat transfer rate is increased by
30% if radiation is considered. This shows the significance of radia-
tion heat transfer in this application. It is worth mentioning that the
increase in the heat transfer rate due to radiation was also reported
in the first order simplified analysis of Antar and Thomas [26]. The
percentage increase reported in their work was ranging from 23% to
31% which supports the results obtained in the current investigation.

Increasing the number of cavities plays a significant role in
increasing the R-value of the building block as shown in Fig. 5. Fig. 5a
depicts the percentage change in the R-value as the number of
cavities is increased from 1 to 6 whether radiation heat transfer
mode is considered or neglected. The increase in R-value with the
number of cavities is more significant when radiation heat transfer
is considered due to the additional resistance for radiation heat
transfer through each of the cavities. On the other hand, Fig. 5b
shows that the heat transfer rate is increased due to radiation heat
transfer (less R-value). The figure shows that the difference between
the heat transfer rate calculations with and without radiation
reduces as the number of cavities is increased due to the increase in
the radiation resistance as the number of cavities increase. That is,
more cavities lead to more radiation resistances. To provide guide-
lines for designers, the R-value was fitted as a function of the
number of cavities resulting in the following equations:

\[
R\text{-value} = 18.02 - 7.426N + 2.27N^2 - 0.335N^3 + 0.019N^4 \text{Km}^2/\text{W} \tag{17}
\]

if radiation is considered, where \( N \) is the number of cavities.
However, if the effect of thermal radiation is neglected, the
following equation results

\[
R\text{-value} = 13.32 - 6.55N + 2.29N^2 - 0.36N^3 + 0.02N^4 \text{Km}^2/\text{W} \tag{18}
\]
Including radiation heat transfer calculations has resulted in qualitatively the same trend as we investigate the different cases considered in this study. The highest heat transfer rate (lowest $R$-value) is found to be in case 1 (single, wide cavity) and the lowest heat transfer rate (highest $R$-value) in case 7. The effect of increasing the number of partitions is not only found to reduce the convective heat transfer but also radiation heat transfer by acting as a radiation barrier. This is evident by comparing the rate of heat transfer between different cases with and without radiation to the base case (case 1) as shown in Fig. 5b. For example, for case 2, heat transfer rate is reduced by 1.19 W if radiation is neglected whereas the reduction becomes 2.7 W if radiation is considered.

Consider the same comparison for case 9, which has the highest number of cavities. The reduction in heat transfer by radiation can be found to be 1.92 W. It is important to note that this layout is not the best one since it has more thermal bridges to transfer heat through conduction heat transfer mode in addition to the “close to unity” aspect ratio of each cavity that promotes convection heat transfer. Therefore, net $R$-value is less than case 7.

Fig. 6a and b shows the percentage change in $R$-value and the heat transfer rate respectively for the proposed designs that represent cases 7 to 10. The heat transfer rate in these configurations is found to be lowest in case 7 (highest change in the $R$-value). It is interesting to note that the percentage reduction of the $R$-value due to the inclusion of thermal radiation calculations is found to be small in configurations 7 and 8, Whereas it increases (around 5%) in cases 9 and 10. This could be attributed to presence of more radiation barriers in the form of several cavities in case 9 and due to low aspect ratio of the cavities in case 10 that shows the dominant effect of heat convection compared to radiation as a result of the increase in the heat transfer coefficient.

The effect of each of the convection and radiation heat transfer modes was separated and presented in Fig. 7 to indicate the relative significance of each mode for the cases investigated in this study. It is interesting to note the significance of radiation heat transfer mode that cannot be neglected in this application. Moreover, the trend shown in cases 1–6 is qualitatively the same. However, for case 7, convection effects are more dominant compared to radiation effects. This means that although convective/conduction heat transfer decrease significantly as a result of introducing more conduction barriers, the changes in radiation heat transfer are less significant due to limited number of cavities compared to later cases (8–10).

It is important to note that the layout of case 7 is better than that of case 8 since it has high aspect ratio cavities and therefore has less convection effects. On the other hand, no improvement in the $R$-value ($R$-0.507 K m$^2$/W) is noticed for case 10. This is due to pronounced role of conduction heat transfer in the solid surrounding the cavities thus providing unnecessary thermal bridges. In addition, more cavities in this case with lower aspect ratio results in an enhanced air circulation that promotes convection heat transfer rate.
Based on this analysis, it is clear that the reduction in heat transfer may be attained by partitioning the building block as well as changing its internal layout such that thermal bridges are minimized. The most promising configuration that could provide effective insulation is case 7, in which we have the advantages of both effects. To conclude, one may say that changing the layout of the brick can play a substantial role in increasing its thermal resistance.

The heat transfer by radiation depends also on the emissivity of the surfaces. It is known that the thermal resistances can be increased by simply decreasing the surface emissivity values. To quantify this information, emissivity values are varied from 0.1 to 1 and results are presented for the first five cases in addition to the seventh one. The $R$-values obtained are presented in Fig. 8 that shows that the $R$-value increases significantly by changing the emissivity; an increase of almost 38% is observed in the case of the hollow block with no partitions (case 1) only by changing the value of emissivity from 1 to 0.1. In case of one partition (Case 2), the $R$-value, which was already reduced to 27.5% due to the introduction of a partition, is further reduced by nearly 28% due to reducing the emissivity from 1 to 0.1. The maximum percentage increase in $R$-value occurs in case 7, is found to be 30% compared with the case of $\varepsilon = 1$ and 60.6% compared to case 1.

It is recommended that building block designers should avoid wide cavities, and replace them by multiple narrow cavities and reduce surface emissivity, $\varepsilon$, in order to increase the $R$-value and improve the thermal characteristics of their products toward better insulation and effective energy saving.
6. Conclusion

A detailed two-dimensional numerical analysis is carried out to explore the effect of changing the building block layout in reducing the heat leak across the block leading to a proposed ceramic brick layout with better heat resistance characteristics. The three modes of heat transfer were considered, conduction through building material and both convection and radiation within the cavities. Increasing the number of air-filled cavities keeping the total width unchanged leads to significant reduction in the heat transfer rate (increase in R value) of about 43.16% if five cavities are considered due to the reduction in the effect of natural convection (reduced Ra). Furthermore, changing the layout of the cavities can play a substantial role in decreasing the heat leak (increasing the R-value) and hence provides better thermal insulation without affecting the structural characteristics of the blocks. Changing the layout (to case 7) further increases the R-value by 13.5%.

Considering radiation heat transfer results in a significant increase in the heat transfer rate that reflects significant radiation effects that contribute to the higher heat transfer rate (less thermal resistance). For example, 30% increase in the heat transfer rate in the case of non-partitioned block was computed due to thermal radiation effects. In addition, The R-value increases significantly by changing the emissivity; an increase of almost 38% is observed in the case of the hollow block with no partitions by changing the value of emissivity from 1 to 0.1. The R-value changes by 41% in case 7 by reducing the emissivity from 1 to 0.1.

Decreasing the thickness of the solid material used (within the safe limits) decreases the thermal bridges and thus avoids unnecessary conduction heat leak. In addition, using cavities of high aspect ratio decreases the convection heat transfer effects. This provides an economic suggestion for engineers in the selection of the building block patterns for effective energy conservation.

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References


Nomenclature

A: aspect ratio of the cavity (–/H/L)
α: specific heat capacity at constant pressure, kJ/(kg·K)
E_b: blackbody radiation surface heat flux, W/m²
F: radiation shape factor
G: irradiation, W/m²
g: acceleration due to gravity, m/s²
J: radiosity, W/m²
k: thermal conductivity, W/mK
L: dimension of the cavity, m
Nu: Nusselt number
p: pressure, kPa
Pr: Prandtl number
φ: heat transfer density. Energy 2006;31:620
q_e: and q_e*: convection heat transfer flux at the interface in both x and y directions
q_r: and q_r*: radiation heat transfer flux at the interface in both x and y directions
Ra: Rayleigh number
T: temperature, C
u: velocity component in the x-direction, m/s
v: velocity component at the y-direction, m/s
W: height of the block under investigation, m
Greek symbols

$\beta$: coefficient of thermal expansion, K$^{-1}$
$
\varepsilon$: surface emissivity

$\mu$: dynamic viscosity, kg/ms

$\rho$: density, kg/m$^3$

$\rho_s$: surface reflectivity

$\sigma$: Stefan-Boltzmann constant, W/m$^2$K$^4$

Subscript

$c$: cold

$h$: hot

$i$: inner surface

$o$: outer surface

$\infty$: ambient