Sensorless Torque/Speed Control of Induction Motor Drives at Zero and Low Frequencies With Stator and Rotor Resistance Estimations

Mohamed S. Zaky, Member, IEEE, and Mohamed K. Metwaly

Abstract—Stability, robustness, and estimation accuracy of the adaptive flux observer (AFO) for sensorless induction motor (IM) drives are the most critical issues at zero and very low frequencies. In this paper, the design of speed, stator resistance, and rotor resistance estimators, to improve the robustness of AFO to parameters variation, is proposed. These estimators are arranged to have a cascade multi-input multi-output structure, and simplified to a single-input single-output structure for stability analysis and gain selections. To design both the observer feedback gains and adaptive proportional-integral gains, the stability conditions of the estimators are derived to guarantee a stable AFO in all the four quadrants of operation. The sensitivity analysis against stator and rotor resistance variations is also provided. The detailed analytical, simulation, and experimental results are presented to validate the proposed AFO of sensorless IM drives in torque- and speed-controlled modes of operation, particularly at zero and very low frequencies.

Index Terms—Adaptive flux observer (AFO), robustness, sensitivity analysis, sensorless control, stability analysis, torque/speed control.

I. INTRODUCTION

VARIOUS model-based speed estimation methods are available in the literature, since they are simple and neither additional losses nor torque ripples are caused by signal injections. In addition, they present a sufficient performance at medium and high speeds. However, the behavior of the drive at low speed is still inadequate. This is due to the fact that some of the motor states become unobservable from the stator side at low speed. Also, parameter uncertainty due to stator resistance mismatch and effect of inverter nonlinearity are other issues. Therefore, the main problems of sensorless drives are the observer’s performance and stability at zero and low stator frequencies [1]–[7]. However, this problem is maximized at zero and low stator frequencies during the regenerating mode of operation.

Adaptive flux observer (AFO) is considered one of the machine model-based methods, and it is characterized by robustness, accuracy, reduced complexity, and computational cost. Extensive research efforts have been presented to improve the stability of AFO under low-speed regenerating mode. However, an instability problem under low-speed regenerating mode has been considered one of the key issues for AFO-based sensorless induction motor (IM) drives in addition to parameters variation. Different algorithms to design the feedback gains as well as proportional-integral (PI) adaptive gains have been presented either for the full-order and reduced-order observers [8]–[20]. Traditionally, the feedback gains of AFO have been designed by pole placement technique to place the observer’s poles proportionally to the IM’s poles [8], or left to the IM’s poles [9]. However, the speed estimation using the above feedback gains suffers from a well-known instability problem in the low-speed regenerating mode. Numerous research works have been presented to clarify the unstable phenomenon of AFO [10], [11]. In [12] and [13], the necessary and sufficient stability conditions of AFO have been derived. The conditions of feedback gains and adaptive PI gains for a stable AFO have been presented using Lyapunov theory [14], positive-real property [15], or a unified state-space representation [16]. The feedback gains and adaptive PI gains of AFO have been derived in [17]–[20] to overcome the unstable region in the low-speed regenerating mode.

The main problems of AFO are the design of its feedback gains, adaptive PI gains, and parameters variation, especially in the operation at low-speed regenerating mode. Since the speed estimation of AFO is based on the machine model, it is highly sensitive to machine parameters variation. Stator resistance variation with machine temperature is the most serious problem at low speed. Therefore, continuous adaptation of the stator resistance is required to maintain a stable operation at low speeds. On the other hand, the rotor circuit model of IM has been used to estimate the rotor flux, and the mismatch in the stator resistance does not affect the rotor flux. However, the mismatch of the rotor resistance will deviate the estimated rotor flux and will cause an error in the estimated slip frequency [21]–[26].

As a result of its simplicity and fast dynamic response, indirect field-oriented control (IFOC) of the IM has been commonly used in high-performance applications. However, it is dependent on the rotor resistance, for calculation of the slip frequency, which undergoes considerable variation with the operation of the drive. This causes that the rotor flux orientation is lost, and consequently, the complete decoupling between the $d$ and $q$ axis variables of IFOC is not
achieved completely. Therefore, great attention has been paid to compensate for the rotor resistance change to enforce field orientation through online estimation of the machine parameters [27].

This paper focuses on the torque/speed control of sensorless IM drives using AFO to assure the stability, robustness, and good estimation accuracy at zero and very low stator frequencies. Thus, the main objectives are as follows:

1) design the speed, stator resistance, and rotor resistance estimators for improving the robustness of the AFO-based sensorless drive to parameters variation;
2) simplify a complicated multi-input multi-output (MIMO) structure of the three estimators into a single-input single-output (SISO) structure for a simple stability analysis procedure to design both the observer feedback gains and adaptive PI gains of the proposed estimators;
3) investigate the problem of instability of the sensorless IM drive with zero feedback gains during the low-speed regenerating mode;
4) propose new observer feedback gains to improve the observer’s performance and stability in the operation at low-speed regenerating mode;
5) examine the robustness and sensitivity of the proposed estimators under the IM parameters change using the stability analysis of pole-zero map;
6) prove the effectiveness of the proposed analytical results using simulations and experiments in all the four quadrants of operation, particularly at zero and very low frequencies.

II. MATHEMATICAL MODELS
A. Model of the IM
The IM model in the stationary reference frame is described as given in [22]

\[
\begin{align*}
\frac{d\vec{i}_s}{dt} &= \frac{1}{\sigma L_s} \left( \vec{v}_s - R_i \vec{i}_s - \frac{L_m}{L_r} \frac{d\vec{\lambda}_r}{dt} \right) \quad (1-a) \\
\frac{d\vec{\lambda}_r}{dt} &= \frac{L_m}{L_r} \vec{i}_s - \left( \frac{1}{T_r} - J\omega_r \right) \vec{\lambda}_r. \quad (1-b)
\end{align*}
\]

The dynamic model can be described by the following state equation:

\[
\frac{d}{dt} \begin{bmatrix} \vec{i}_s \\ \vec{\lambda}_r \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \vec{i}_s \\ \vec{\lambda}_r \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \end{bmatrix} \vec{v}_s \quad (1-c)
\]

where

\[
\begin{align*}
A_{11} &= aI, & A_{12} &= cI + dJ, & A_{21} &= eI, & A_{22} &= -\varepsilon A_{12} \\
b_1 &= bI, & I &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & J &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
a &= -\left( \frac{R_i}{\sigma L_s} + \frac{L_m^2}{\sigma L_s T_r L_r} \right), & c &= \frac{1}{\varepsilon T_r}, & d &= -\omega_r \\
e &= \frac{L_m}{T_r}, & \varepsilon &= \frac{\sigma L_s L_r}{L_m}, & b &= \frac{1}{\sigma L_s}, & \sigma &= 1 - \frac{L_m^2}{L_s L_r} \\
T_r &= \frac{L_s}{R_r}.
\end{align*}
\]

B. Adaptive Flux Observer
The equations of AFO are derived from (1-c) as follows [8]:

\[
\begin{align*}
\frac{d\hat{\vec{i}}_s}{dt} &= \hat{A}_{11} \hat{\vec{i}}_s + \hat{A}_{12} \hat{\vec{\lambda}}_r + b_1 \hat{\vec{v}}_s - K (\hat{\vec{i}}_s - \vec{i}_s) \quad (2) \\
\frac{d\hat{\vec{\lambda}}_r}{dt} &= \hat{A}_{21} \hat{\vec{i}}_s + \hat{A}_{22} \hat{\vec{\lambda}}_r \quad (3)
\end{align*}
\]

where \( \hat{\vec{i}}_s = [\hat{i}_{ds} \hat{i}_{qs}]^T \) is the actual stator current vector, \( \vec{i}_s = [i_{ds} i_{qs}]^T \) is the stator voltage vector, \( \vec{\vec{v}}_s = [v_{ds} v_{qs}]^T \), \( \hat{\vec{\lambda}}_r = [\hat{\lambda}_{dr} \hat{\lambda}_{dq}]^T \) is the estimated stator current vector, \( \hat{\vec{\lambda}}_r = [\hat{\lambda}_{dr} \hat{\lambda}_{dq}]^T \) is the actual rotor flux vector, \( \vec{\vec{v}}_s = [v_{ds} v_{qs}]^T \) is the stator voltage vector, \( \vec{\vec{v}}_s = [\hat{\vec{\lambda}}_r + \hat{\vec{\lambda}}_s]^T \) is the error between the estimated and actual stator current vectors, \( L_m \) is the magnetizing inductance, \( L_s \) and \( L_r \) are the stator and rotor self-inductances, \( R_s \) is the stator resistance, \( T_r \) is the rotor time constant, \( \omega_r \) and \( \hat{\omega}_r \) are the actual and estimated rotor speeds, and \( \sigma \) is the leakage inductance coefficient.

The symbol “\( \Delta \)” signifies the estimated variables and values. \( K \) is the observer feedback gain matrix defined as \( K = K_1 I + K_2 J \). \( \hat{A}_{11}, \hat{A}_{12}, \hat{A}_{21}, \) and \( \hat{A}_{22} \) are the coefficient matrices, where the estimated variables \( \hat{\vec{\omega}}_r, \hat{\vec{\lambda}}_r, \) and \( \hat{\vec{\lambda}}_r \) are used instead of the actual variables \( \vec{\omega}_r, \vec{\lambda}_s, \) and \( \vec{\lambda}_r \). The block diagram for the proposed sensorless torque/speed control of IFOC for the IM drive with parameters estimation is shown in Fig. 1. In this scheme, the unit vector \( \theta_e \), required for axes transformation, can be calculated from the estimated speed \( \hat{\vec{\omega}}_r \) and slip speed \( \vec{\omega}_s \) as, \( \theta_e = \int (\vec{\omega}_r + \vec{\omega}_s)dt \).

III. DESIGN OF THE ESTIMATORS
The error matrix \( \Delta \) is derived as described in (4) by considering the speed, stator, and rotor resistances as variable parameters

\[
\begin{align*}
\Delta A_{11} &= \Delta a_{11r} I + \Delta a_{11s}, & \Delta A_{12} &= \Delta a_{12r} I + \Delta a_{12s} \quad (4) \\
\Delta A_{21} &= \Delta a_{21r} I, & \Delta A_{22} &= \Delta a_{22r} I + \Delta a_{22s}
\end{align*}
\]

where

\[
\begin{align*}
\Delta a_{11r} &= \Delta R_s L_m^2 \frac{\sigma L_s L_r}{\sigma L_s L_r}, & \Delta a_{11s} &= \Delta \frac{R_s}{\sigma L_s}, & \Delta a_{12r} &= \frac{\Delta R_r}{\varepsilon L_r} \\
\Delta a_{12s} &= -\Delta \omega_r, & \Delta a_{21r} &= \Delta R_r L_m, & \Delta a_{22s} &= -\Delta R_r \frac{L_m}{L_r} \\
\Delta a_{22s} &= -\Delta \omega_r, & \Delta a_{21s} &= \omega_r - \hat{\omega}_r, & \Delta R_s &= R_s - \hat{R}_s \\
\Delta R_r &= R_r - \hat{R}_r.
\end{align*}
\]

A. Stator Current and Rotor Flux Error Equations
The error of rotor flux and stator current is derived by subtracting (1-c) from (2) and (3) as

\[
\begin{align*}
\frac{d\hat{\vec{\epsilon}}_s}{dt} &= (A_{11} - K) \hat{\vec{\epsilon}}_s + A_{12} \hat{\vec{\epsilon}}_r + \Delta A_{12} \hat{\vec{\lambda}}_r + \Delta A_{11} \hat{\vec{i}}_s \quad (5) \\
\frac{d\hat{\vec{\epsilon}}_r}{dt} &= A_{21} \hat{\vec{\epsilon}}_s + A_{22} \hat{\vec{\epsilon}}_r + \Delta A_{21} \hat{\vec{i}}_s + \Delta A_{22} \hat{\vec{\lambda}}_r \quad (6)
\end{align*}
\]

where \( \hat{\vec{\epsilon}}_s = \hat{\vec{\lambda}}_s - \hat{\vec{\lambda}}_s, \) and \( \Delta A = \hat{A} - A = \begin{bmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{bmatrix} \).
By taking the Laplace transform of (5) and (6) and substituting with the coefficients of the matrix \( \Delta A \), the correlation between the errors in the speed, the stator resistance, and the rotor resistance is described in the equation of stator current in terms of the errors in the speed, stator resistance, and rotor resistance is derived. Hence

\[
[sI - (A_{11} - K)]\ddot{e}_i = A_{12}\dot{e}_s + (\Delta a_{12}I + \Delta a_{12o}I)\dot{\lambda}_r^s
+ (\Delta a_{11}I + \Delta a_{11o}I)\dot{\lambda}_s^s
\]

(7)

\[
[sI - A_{22}]\ddot{e}_s = A_{21}\dot{e}_i + \Delta a_{21}\dot{\lambda}_s^s + (\Delta a_{22r} + \Delta a_{22o})\dot{\lambda}_r^s
\]

(8-a)

\[
\ddot{e}_r = [sI - A_{22}]^{-1}[A_{21}\dot{e}_i + \Delta a_{21}\dot{\lambda}_s^s + (\Delta a_{22r} + \Delta a_{22o})\dot{\lambda}_r^s]
\]

(8-b)

Substituting the value of \( \ddot{e}_s \) from (8-b) into (7), the error equation of stator current in terms of the errors in the speed, the stator resistance, and the rotor resistance is described in

\[
\ddot{e}_i = \ddot{e}_1 + \ddot{e}_2 + \ddot{e}_3
\]

(9)

where

\[
\ddot{e}_1 = G_{o}(s) \cdot J\ddot{\lambda}_r^s \Delta \omega_r
\]

(10)

\[
\ddot{e}_2 = G_{Rs}(s) \cdot \ddot{\lambda}_s^s \Delta R_s
\]

(11)

\[
\ddot{e}_3 = (G_{Rr1}(s) \cdot \ddot{\lambda}_s^s + G_{Rr2}(s) \cdot J\ddot{\lambda}_r^s) \Delta R_r
\]

(12)

and

\[
G_{o}(s) = \frac{s}{\epsilon} \left[ s^2 I + s(K - A_{11} - A_{22}) - A_{22} \left( K - A_{11} - \frac{A_{21}}{\epsilon} \right) \right]^{-1}
\]

(13)

\[
G_{Rs}(s) = \frac{[sI - A_{22}]}{\sigma L_s} \left[ s^2 I + s(K - A_{11} - A_{22}) - A_{22} \left( K - A_{11} - \frac{A_{21}}{\epsilon} \right) \right]^{-1}
\]

(14)

\[
G_{Rr1}(s) = \frac{s}{\epsilon L_r} \left[ s^2 I + s(K - A_{11} - A_{22}) - A_{22} \left( K - A_{11} - \frac{A_{21}}{\epsilon} \right) \right]^{-1}
\]

(15)

\[
G_{Rr2}(s) = L_m G_{Rr1}(s)
\]

(16)

The speed, stator resistance, and rotor resistance estimators, given by (13)–(16), are structured to have a cascade MIMO scheme. In Fig. 2(a), the block diagram of a closed-loop MIMO is constructed to illustrate the three estimators and the coupling between them as expressed in (10)–(12). As shown, the three estimators constitute a cascade-decoupled MIMO structure. This decoupled structure can be simplified into three separate SISO diagrams to design the adaptive PI gains for speed, stator, and rotor resistance estimators to guarantee the stability and good tracking performance of the drive system. This can be achieved by considering that the error of each estimator is a bounded external disturbance, which is neglecting in the stability analysis. Fig. 2(b) shows the block diagram of the speed estimator by neglecting \( \ddot{e}_2 \) and \( \ddot{e}_3 \). In Fig. 2(c), the block diagram of the stator resistance estimator is constructed by ignoring \( \ddot{e}_1 \) and \( \ddot{e}_3 \). Finally, the block diagram of the rotor resistance estimator is constructed by ignoring \( \ddot{e}_1 \) and \( \ddot{e}_2 \), as shown in Fig. 2(d).

Remark 1: In a steady state, the following assumptions are made:

1) the motor inductances \( L_m, L_r, \) and \( L_s \) during the IM operation are constant and measured offline except stator and rotor resistances that are estimated online;
2) the stator currents and voltages, rotor speed, and load torque are constant.

So, using these assumptions, there are generally two possible attraction points of \( (\ddot{R}_s, \ddot{R}_r) \) that match the circuit equations. Therefore, the estimated \( \ddot{R}_s, \ddot{R}_r \) may converge to the false values, or the persistency condition may not be satisfied in some cases. This conclusion is applicable for any simultaneous estimation of the stator and rotor resistance schemes. Thus, to overcome this limitation, the injection of excitation voltage with different frequencies is sufficient (not necessary) to make the convergence of the estimated resistances to their true values. However, the estimation of the stator and rotor resistances of the IM under test converges to the real values without any signal injection using these assumptions [28]–[30].

Remark 2: The relationship between the estimated variables \( \dot{\omega}_r, \dot{R}_s, \) and \( \dot{R}_r \) and the errors \( \Delta \omega_r, \Delta R_s, \) and \( \Delta R_r \) represents...
a transfer function of the $3 \times 3$ MIMO system, which is difficult to be used in the stability analysis. Then, the simplification proposed in this paper to investigate the stability analysis under the justification that the estimated variables match their actual values in the steady state. The design of each estimator is performed independently, with each estimator being treated as an SISO closed-loop control system. As an example, the rotor speed estimator is designed by assuming that the effect of the errors $\hat{\omega}_r$ and $\hat{\omega}_s$ is a bounded external disturbance, which is ignored for the purpose of the stability analysis. This can be justified as a result that the poles of the speed estimator using the proposed feedback gains are moved to the stable region of the $s$-plane. This ensures the convergence of $\Delta \omega_r$ to zero, whatever the value of the disturbance signal. This agrees with the physical interpretation that, at steady state, the speed error is exactly equal to zero, since $\hat{\omega}_r$ is set equal to $\omega_r$. Similar to the speed estimator, the same is done for the stator and rotor resistance estimators.

**Remark 3:** It is assumed that during a short transient period, if slow motor resistances are considered to track only the thermal variation of stator and rotor resistances. Then, the cross coupling effect from the stator and rotor resistance estimations can be neglected, and the sensorless drive behaves as if the stator and rotor resistances are fixed. However, there is only a speed estimation error between the actual and estimated speeds. The stability property during this short transient period can be neglected in practice. It is found that the observed instability is due to the zero feedback gains, and then, the speed, stator, and rotor resistance estimations can be stabilized by the designed feedback gains and the error feedback control system. So, the proposed design finds a specific feedback gain, which stabilizes each the speed, stator, and rotor resistance estimations independently. This specific gain stabilizes the whole system according to the analysis results of this paper. To confirm this justification, stability analysis under parameters change is presented in Section V.

### B. Design of Speed Estimator

To study the stability analysis of the speed estimator, the transfer function $G_{\omega 23}(s)$ given in (17), as shown at the bottom of this page is used for this purpose to determine the locations of the poles and zeros of the speed estimator (see [22] for more detailed derivation).

In (17), $\omega_o$ is the frequency of rotor flux and $a_1, a_2, a_3,$ and $a_4$ are defined as follows:

\[
a_1 = \left( \frac{R_s}{\sigma L_s} + \frac{L_m^2}{\sigma L_s \epsilon T_r L_r} + \frac{1}{T_r} + K_1 \right),
\]

\[
a_2 = (K_2 - \omega_o),
\]

\[
a_3 = \left[ \frac{1}{T_r} \left( \frac{R_s}{\sigma L_s} + \frac{L_m^2}{\sigma L_s \epsilon T_r L_r} - \frac{L_m}{\epsilon T_r} + K_1 \right) + K_2 \omega_o \right],
\]

\[
a_4 = \left[ \frac{K_2}{T_r} - \omega_o \left( \frac{R_s}{\sigma L_s} + \frac{L_m^2}{\sigma L_s \epsilon T_r L_r} - \frac{L_m}{\epsilon T_r} + K_1 \right) \right].
\]

### C. Design of Stator Resistance Estimator

To investigate the stability analysis of the stator resistance estimator, the transfer function $G'_{R_s 11}(s)$ shown in (18), as shown at the bottom of this page is derived in the rotor flux reference frame (see [22] for more detailed derivation).

In (18), $\omega_o$ is the slip speed and $a_5 = \omega_o^2 + \omega_o a_2 - a_3$.

\[
G'_{\omega 22}(s) = \frac{s^3 + a_1 s^2 + (\omega_o^2 + a_3) s + \omega_o^2 a_1 + \omega_o a_4}{\epsilon \left[ \left( s^2 + a_1 s - \omega_o^2 - \omega_o a_2 + a_3 \right)^2 + \left( 2 \omega_o + a_2 \right) s + \omega_o a_1 + a_4 \right]^2}
\]

\[
G'_{R_s 11}(s) = \frac{s^3 + (a_1 + 1/T_r) s^2 + (a_1 + a_1 s + a_3) s + \omega_o \left( 2 \omega_o + a_2 \right) s + \omega_o a_1 + a_4 - a_5/T_r}{\sigma L_s \left[ \left( s^2 + a_1 s - \omega_o^2 - \omega_o a_2 + a_3 \right)^2 + \left( 2 \omega_o + a_2 \right) s + \omega_o a_1 + a_4 \right]^2}
\]
D. Design of Rotor Resistance Estimator

Simplifying (15) and (16), one obtains

\[
G_{Rr_1}(s) = \frac{s}{\varepsilon L_r} \left[ s^2 I + s(a_1 I + a_2 J) + (a_3 I + a_4 J) \right]^{-1}
\]  

(19)

\[
G_{Rr_2}(s) = \frac{L_m s}{\varepsilon L_r} \left[ s^2 I + s(a_1 I + a_2 J) + (a_3 I + a_4 J) \right]^{-1}.
\]  

(20)

Therefore, the error \( \tilde{e}_3 \) is derived in (21)–(24), as shown at the bottom of the next page, where

\[
G_{c\omega} = K_{P\omega} + \frac{K_{I\omega}}{s}, \quad G_{cRs} = K_{PRS} + \frac{K_{IRe}}{s}
\]

are the transfer functions of adaptive PI gains for the speed estimator, stator resistance estimator, and rotor resistance estimator, respectively.

It should be noted that transfer functions \( G'_{Rr_{11}}(s) \) and \( G'_{Rr_{12}}(s) \) are derived in the rotor flux reference frame. Thus, the block diagram of an SISO rotor resistance estimator is shown in Fig. 2(d). The block diagram to show how the estimated stator and rotor resistances are used in speed estimation and flux observation is shown in Fig. 3.

IV. DESIGN OF THE OBSERVER FEEDBACK GAINS

A. Stability Conditions of Speed Estimation

The stability conditions of a speed estimation are governed by the locations of poles and zeros of the transfer function \( G'_{\omega_{22}}(s) \). The estimator is considered a minimum phase system when the poles and zeros are located in the stable region of the \( s \)-plane. Alternatively, the estimator is considered a nonminimum phase if there are one or more zeros in the unstable region of the \( s \)-plane. Therefore, the observer feedback gains and adaptive PI gains must be carefully designed to guarantee a stable estimator in all the four quadrants of operation.

The IM parameters, which are given in Table I in the Appendix, were used in Figs. 4–7. The pole–zero locations correspond to the regenerating-mode operation, where the sign of the slip angular frequency was changed in this operating mode. The rated slip angular frequency is 16.75 rad/s, the stator angular frequency varies from 0 to 5 rad/s, and then, the rotor speed (mechanical) varies from 16.75 to 21.75 rad/s.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>RATED VALUES AND PARAMETERS OF TYPE 1 IM</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rated output power</th>
<th>1.5 HP</th>
<th>Stator resistance</th>
<th>7.4826 Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated voltage</td>
<td>380 V</td>
<td>Rotor resistance</td>
<td>3.684 Ω</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>50 Hz</td>
<td>Stator self-inductance</td>
<td>0.4335 H</td>
</tr>
<tr>
<td>No. of pole pairs</td>
<td>2</td>
<td>Rotor self-inductance</td>
<td>0.4335 H</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mutual inductance</td>
<td>0.4114 H</td>
</tr>
</tbody>
</table>
K with erating mode. (a) Using K frequency of 16.75 rad/s.

Fig. 5. Dominant pole/zero allocations of \( G_{Rs11} \) at a low-speed regenerating mode. (a) Using \( K = 0 \). (b) Using the proposed feedback gains (26) with \( K_1 = 1 \) and \( K_2 = -180 \). The direction of arrows indicates the change of the stator angular frequency from 5 to 0 rad/s at negative-rated slip angular frequency of 16.75 rad/s.

the s-plane with reducing the stator angular frequency from 5 to 0 rad/s. Hence, the speed estimation process will be unstable regardless of the adaptation PI gains, since one of the dominant poles will move toward the unstable zero. These analytical results of Fig. 4(a) show that the speed estimator has an unstable region in the low-speed regenerating mode using \( K = 0 \). Accordingly, the observer feedback gain \( K \) should be properly designed to ensure the stability of the speed estimator in various operating modes. This can be achieved by applying the Routh–Hurwitz criterion on \( G_{Rs22}(s) \).

The stable zero conditions of the speed estimator are derived as follows:

\[
\begin{align*}
   &a_1 > 0 \\
   &a_1 a_3 - \omega_o a_4 > 0 \\
   &\omega_o a_1 + a_4 > 0
\end{align*}
\]

(25)

Using (25), the new observer feedback gains \( K \) are designed as given in

\[
\begin{align*}
   &K_1 > 0 \\
   &K_2 > k_o r
\end{align*}
\]

(26)

where \( k \) is a negative constant gain.

The transfer function \( G_{Rs22}(s) \) with the pole/zero allocations in the very low stator frequency regenerating mode with the proposed new observer feedback gains (26) is shown in Fig. 4(b) during the stator angular frequency range from 5 to 0 rad/s. It is found that the best value of \( k \) is ranging from −20 to −50. This range is validated for different operating conditions of the sensorless drive by investigating the locations of the poles and zeros. As shown, the speed estimator becomes stable using the proposed new feedback gains (26), and the unstable zero is moved into the stable region of the s-plane.

\[
\begin{align*}
   e_{3d} &= \begin{bmatrix} G'_{Rs11}(s) & G'_{Rs12}(s) \\ G'_{Rs21}(s) & G'_{Rs22}(s) \end{bmatrix} \begin{bmatrix} \hat{2}_a \\ \hat{2}_s \end{bmatrix} \Delta R_r \\
   e_{3d} &= \left[ \begin{bmatrix} \hat{2}_a \\ \hat{2}_s \end{bmatrix} G'_{Rs11}(s) + L_m \begin{bmatrix} \hat{2}_a \\ \hat{2}_s \end{bmatrix} G'_{Rs12}(s) \right] \Delta R_r \\
   G'_{Rs11}(s) &= G'_{Rs22}(s) = \frac{1}{\varepsilon L_r} \left[ \frac{1}{s^3 + a_1 s^2 + (\omega_o^2 + a_3) s + \omega_o a_1 + \omega_o a_4} \right] \\
   G'_{Rs12}(s) &= \frac{L_m}{\varepsilon L_r} \left[ \frac{(\omega_o + a_2) s^2 + a_4 s + \omega_o^2 a_2 - \omega_o a_3 + \omega_o^3}{(s^2 + a_1 s - \omega_o^2 - \omega_o a_2 + a_3)^2 + ((2\omega_o + a_2) s + \omega_o a_1 + a_4)^2} \right]
\end{align*}
\]
C. Stability Conditions of Rotor Resistance Estimation

To provide a fast convergence and accurate estimation of the rotor resistance, adaptive PI gains of the rotor resistance estimator were selected as $K_{PRr} = 250$ and $K_{IRr} = 2800$.

V. Sensitivity Analysis to Parameters Variation

A. Analytical Sensitivity Conditions

Stability analysis using pole–zero map to test the robustness and sensitivity of the estimators to the IM parameters variation is also provided. A good advantage of the estimators using the proposed new feedback gain design is their small sensitivity to the IM parameters variation. The root-locus results for the sensitivity tests of the estimators to incorrect values of stator and rotor resistances than their nominal values are shown in Figs. 6 and 7 (points corresponding to nominal values of parameters are marked with cross).

Pole–zero map of $G_{R_{111}}(s)$ depending on rotor resistance variations from $-60\%$ to $+60\%$ of its nominal value during the regenerating mode of operation with stator angular frequency of 1 rad/s and the rated slip angular frequency of 16.75 rad/s with negative sign is shown in Fig. 6. Although rotor resistance variations influence the pole and zero locations of $G_{R_{111}}(s)$, these poles and zeros are located in the stable region of the $s$-plane, even for an extreme variations of the rotor resistance as observed in Fig. 6.

Fig. 7 shows the pole–zero map of $G'_{o22}(s)$ depending on stator and rotor resistance variations from $-60\%$ to $+60\%$ of their nominal values during the regenerating mode of operation with the stator angular frequency of 1 rad/s and the rated slip angular frequency of 16.75 rad/s with negative sign. It is found that increasing stator resistance value influences the pole and zero locations that move toward the imaginary axis. But, incorrect values of stator resistance do not bring the poles and zeros to the unstable region of the $s$-plane, even for very big changes in this parameter, within the range of $+60\%$ as shown in Fig. 7(a). On the other hand, increasing the rotor resistance value causes that the poles and zeros move deeply in the stable region of the $s$-plane. However, decreasing this parameter moves the poles and zeros toward the imaginary axis without bring them to the unstable region of the $s$-plane, even for very big variations as noted in Fig. 7(b). It is proved that the changes of the stator and rotor resistances within the range of $-60\%$ to $+60\%$ of their nominal values do not cause the instability of the estimators using the proposed new design of the observer feedback gains.

B. Performance Against Stator and Rotor Resistance Mismatches

Practically, parameter variation is unavoidable due to temperature rise. To examine the performance of the sensorless IFOC of the IM drive for parameters variation, simulation results of Fig. 8 are presented. The results were conducted for the drive running at 2.1 rad/s (20 r/min) under a constant full load torque with and without the two parameters estimation. The first subplot illustrates the actual value of rotor resistance (red line) and the value used in the controller (black dashed line), the second subplot shows the torque, the third subplot shows the rotor flux, the fourth subplot
Fig. 8. Simulation results showing the stator and rotor resistance estimations during low speeds at 2.1 rad/s mechanical (20 r/min) and rated load torque. (a) 50% step change of $R_r$ and $R_s$ without $R_r$ and $R_s$ estimators. (b) 50% step change of $R_r$ and $R_s$ with $R_r$ and $R_s$ estimators. (c) 50% ramp change of $R_r$ and $R_s$ with $R_r$ and $R_s$ estimators. (d) 50% incorrect initial conditions of $R_r$ and $R_s$ with the activation of $R_r$ and $R_s$ estimators at $t = 1$ s.

shows the speeds, the fifth subplot shows the actual stator resistance (red line) and the value used in the controller (black dashed line), and the sixth subplot shows the stator current. Fig. 8(a) shows the simulation results under 50% increase in the stator and rotor resistances at $t = 3$ s, and both the stator and rotor resistance estimators are switched OFF. It is observed that the estimated torque is 3.5% lower than the actual motor torque and recovers after 0.4 s to the steady-state value. The rotor flux in the motor increases by 35% compared with its estimated value due to the error in the rotor resistance. The actual and estimated speeds are changed and recover to their steady state after 0.4 s. The stator current decreases by 12% compared with its value before resistance mismatches. Fig. 8(b) shows the simulation results when both the rotor and stator resistance estimators are switched ON. As shown, the estimated rotor and stator resistances have tracked their actual values well, since the errors encountered between the estimated and real torques and rotor fluxes without the two estimators have largely disappeared in this case. In addition, this eliminates the initial speed estimation error
Fig. 9. (a) Simulation and (b) experimental results showing the performance during slow-speed reversals under the rated load torque of $T_L = 7$ Nm using zero feedback gains.

and the stator current error. It should be noted that the practical variation in resistances is very slow. However, Fig. 8(a) and (b) applies a step change in resistances only for the purpose of investigation and verification of the proposed estimators. This indicates that as long as the estimators are capable of tracking the step changes in the resistances. So, it is able to track the slow changes of the actual resistances. To justify this conclusion, the simulation results of Fig. 8(c) are developed during ramp changes of the two resistances. As shown, the performance of the two resistance estimators is better, since the estimated rotor and stator resistances track their actual values smoothly. Moreover, the errors in the torque, rotor flux, and stator current are negligible. In Fig. 8(d), the simulation results, to demonstrate the ability of the proposed rotor and stator resistance estimators to calculate their real values when the initial conditions are not corrected, are also presented. As shown, these tests validate the effectiveness of the estimation of the machine parameters using the proposed algorithms. Fig. 8 is presented in comparison with the online parameter estimation method presented in [31] and [32] to prove the effectiveness of the proposed design.

VI. SPEED-CONTROLLED MODE OF THE SENSORLESS DRIVE

In order to confirm the efficiency of the sensorless IM drive with the proposed estimators, simulation and experimental results are presented using different parameters and ratings. IFOC scheme of the sensorless IM drive is implemented in the simulation and experimental work. The proposed design in this paper is examined under two IM types: Type 1 IM and Type 2 IM. The parameters and ratings of the two IM types are given in Tables I and II in the Appendix. Type 1 IM is employed to examine the sensorless drive with the proposed AFO under the operation in the speed controlled mode, while
Type 2 IM is used under the operation in the torque control mode of the sensorless drive.

The simulation and experimental results of Figs. 9 and 10 are carried out using Type 1 IM operated under speed control mode, in which the estimated synchronous speed angle was used for dq-axis transformations under a standard IFOC structure. The estimated speed is also used as a feedback for the speed loop. A dc machine was used for mechanical loading of Type 1 IM. The speed control mode of a sensorless IM drive of Fig. 1 is built using three blocks. The first block (blue) is the speed control loop using a PI speed controller. The second block (red) takes the torque command from the output of the first block, and finally, a dc generator block (blue) is used for loading conditions. The gains of the speed controller are given in Table III in the Appendix.

A. Performance Under Slow-Speed Reversal Using Zero Feedback Gains

Simulation and experimental results illustrating the performance during slow-speed reversals are shown in Fig. 9(a) and (b) using $K = 0$, respectively. A step change of the load torque was applied at $t = 2$ s. The reference speed was slowly ramped from 10 rad/s ($t = 5$ s) to −10 rad/s ($t = 20$ s) and then back to 10 rad/s ($t = 35$ s). The sensorless drive operates first in the motoring mode, then in the regenerating
mode \((t \approx 17.5 \rightarrow 22.5 \text{ s})\), again in the plugging mode \((t \approx 22.5 \rightarrow 27.5 \text{ s})\), and finally again in the motoring mode, as shown in Fig. 9(a) and (b). It has been observed that the sensorless drive with \(K = 0\) and without the parameter estimators shows inaccurate estimation performance. Moreover, the sensorless drive was unstable with large estimation error. Slow-speed reversals require a very accurate stator resistance estimate, since the stator frequency remains in the vicinity of zero for a long time.

B. Performance Under Slow-Speed Reversal Using Proposed Feedback Gains

Corresponding simulation and experimental results using the proposed new feedback gains (26) and the parameters estimators are shown in Fig. 10(a) and (b), respectively. As shown, the sensorless drive with the proposed feedback gains and the parameters estimation shows a significant performance with excellent estimation accuracy. Fig. 10(a) and (b) is presented in comparison with [19]. It is found that the performance of the drive system with the proposed new feedback gains (26) is comparable with the previous works.

VII. Torque-Controlled Mode of the Sensorless Drive

In order to verify the effectiveness of the proposed AFO with stator and rotor resistance estimations, the torque control of a sensorless IM drive is built using the two red blocks, as shown in Fig. 1. The sensorless IM was operated in the torque-controlled mode and the load was running under speed-controlled operation. An experimental setup is also built, as shown in Fig. 11, using an IM interfaced with a digital signal processor (DSP) control board DSP-DS1103. The experimental tests are offered to validate the simulation
Fig. 15. (a) Simulation and (b) experimental results of the sensorless IM drive during sudden application of the rated load torque at a speed of 10 r/min (mechanical).

Fig. 16. (a) Simulation and (b) experimental results of the sensorless IM drive at zero stator frequency with a step speed change from 0 to −37.5 r/min mechanical at rated load condition.

results and the proposed theoretical analysis. The parameters of the IM are given in Table II in the Appendix. The adaptive PI gains of the three estimators are given in Table IV in the Appendix. The IM was operated under torque control with the estimated flux angle supplying the vector transformations to the rotor-oriented $dq$ frame. The machine was running under torque controlled with standard $idq$ current loops. It was connected to a speed-controlled load dynamometer whose speed is monitored.

A. Performance Under Very Low Speed

The performance of sensorless torque control of IM drives using the proposed three estimators at step speed change from zero to 10 r/min under rated load condition is shown in the simulation and experimental results of Fig. 12(a) and (b), respectively. The first subplot shows the reference speed (blue line), actual speed (black line), and estimated speed using AFO (red line), while the speed estimation error is depicted in the second subplot in r/min. It is observed that a considerable attenuation of the speed error to zero in steady state is achieved soon after nearly 0.3 s. This confirms the usefulness of the proposed design for the speed estimation at very low speeds. The third and fourth subplots of Fig. 12(a) and (b) show the load torque current ($iq$) and the stator currents in $\alpha\beta$ frames ($i_{ab}$), respectively. Finally, the fifth and sixth subplots in Fig. 12(a) and (b) show the rotor flux angles and the error between the reference and estimated flux angles to clarify the advantages of the proposed feedback gains design. The reference flux angle is obtained from the sensor-based current model (black line), and the estimated flux angle is obtained from the proposed AFO (red line). As shown, the estimated flux angle tracks the reference one smoothly with a good convergence.
TABLE II
RATED VALUES AND PARAMETERS OF TYPE 2 IM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated output power</td>
<td>5.5 kW</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>186 V</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>No. of pole pairs</td>
<td>2</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>0.294 Ω</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>0.14325 Ω</td>
</tr>
<tr>
<td>Stator self-inductance</td>
<td>57.3 mH</td>
</tr>
<tr>
<td>Rotor self-inductance</td>
<td>57.3 mH</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>56.43 mH</td>
</tr>
</tbody>
</table>

TABLE III
PI CONTROLLER GAINS OF THE SPEED CONTROLLER

<table>
<thead>
<tr>
<th>Controller</th>
<th>Proportional gain</th>
<th>Integral gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed Controller</td>
<td>8</td>
<td>15</td>
</tr>
</tbody>
</table>

TABLE IV
ADAPTIVE PI GAINS OF SPEED, STATOR RESISTANCE, AND ROTOR RESISTANCE ESTIMATORS

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Speed</th>
<th>Stator Resistance</th>
<th>Rotor Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed Estimator</td>
<td>K_{ps}</td>
<td>K_{im}</td>
<td>K_{rv}</td>
</tr>
<tr>
<td>K_{ps}</td>
<td>500</td>
<td>2500</td>
<td></td>
</tr>
<tr>
<td>K_{im}</td>
<td>200</td>
<td>1800</td>
<td></td>
</tr>
<tr>
<td>K_{rv}</td>
<td>250</td>
<td>2800</td>
<td></td>
</tr>
</tbody>
</table>

B. Performance During Very Low-Speed Reversal

The simulation and experimental tests illustrating the performance during very low-speed reversals are shown in Fig. 13(a) and (b). The IM drive system was operated at the speed reference of 10 r/min under rated load condition, and then, a sudden speed change to −10 r/min was applied. It is found that the sensorless torque control of the IM drive with the proposed feedback gain design of AFO and the parameters estimation shows a significant performance with excellent estimation accuracy during very low-speed reversals in the motoring and plugging modes of operations. In addition, the estimated rotor flux angle tracks the reference one smoothly with a good field orientation performance.

C. Performance Under Load Torque Disturbances at Very Low and Zero Speeds

Figs. 14(a) and (b) and 15(a) and (b) show the performance of the sensorless torque control under sudden application of the rated load torque at zero and very low-speed references. It is noted that the estimated speed follows the actual one smoothly. This proves that the proposed feedback gain design works appropriately, especially during the very low and zero speeds in motoring mode under sudden load disturbances. This also indicates that the torque control of sensorless IM drives works well. Moreover, the rotor flux angles show that the field orientation is maintained.

D. Performance Under Zero Stator Frequency

The zero stator frequency operating conditions under the rated load torque is a challenging for the state-of-the-art observers. The simulation and experimental results showing the performance of the sensorless torque control of IM drive during the zero stator frequency are given in Fig. 16(a) and (b).

This operating condition can be obtained by a speed reversal from zero speed to approximately −37.5 r/min under rated load condition. At this load condition, the speed −37.5 r/min equals the slip frequency, and the result is zero flux frequency with a constant rotor flux position, as shown in the fifth subplot of Fig. 16(a) and (b). As it is obvious, a stable AFO and a good speed estimation with high accuracy are achieved during the operation at zero stator frequency.

VIII. CONCLUSION

In this paper, an AFO has been proposed, which considerably improves the performance of the sensorless IM drives in the critical low and zero frequencies for all the four quadrants of operation. For the stability analysis and the design of adaptive PI gains of the three estimators, a cascade-decoupled MIMO structure has been simplified as an SISO structure of each estimator. The stability of the speed estimation has been analyzed using the Routh–Hurwitz criterion and root-locus plot. It has been found that the speed estimator is a nonminimum phase system due to the unstable zero with $K = 0$. Therefore, new feedback gains have been designed to stabilize the AFO in all the four quadrants of operation. The adaptive PI gains for the proposed estimators by using the root-locus method, to realize both fast dynamic response and good tracking performance, have been designed. The proposed method with a sensorless speed/torque control of the IM drive has been validated using simulations and experiments. The results show that the stability, robustness, and good estimation accuracy of the proposed AFO, especially at low and zero frequencies with less sensitivity to parameters variation, have been achieved. This confirms the validity of the proposed theoretical analysis.

APPENDIX

See Tables I–IV.

ACKNOWLEDGMENT

Authors would like to express their sincere gratitude to the reviewers for their constructive comments and valuable suggestions in order to improve this paper.

REFERENCES


Mohamed S. Zaky (M’16) was born in Menoufia, Egypt, in 1978. He received the B.Sc. (Hons.), M.Sc., and Ph.D. degrees from Menoufia University, Shebin El-Kom, Egypt, in 2001, 2005, and 2008, respectively, all in electrical engineering.

He became an Instructor with Menoufia University in 2002, and then an Assistant Lecturer in 2006. From 2008 to 2013, he was a Lecturer with the Department of Electrical Engineering, Faculty of Engineering, Menoufia University, where he has been an Assistant Professor since 2013. His current research interests include control of electrical machines, applications of power electronics, DSP-based real-time control, sensorless control, and renewable energy applications.

Mohamed K. Metwaly was born in Menoufia, Egypt, in 1974. He received the B.Sc. (Hons.) and M.Sc. degrees from Menoufia University, Shebin El-Kom, Egypt, in 1999 and 2003, respectively, and the Ph.D. degree (Hons.) from the Vienna University of Technology, Vienna, Austria, in 2009, all in electrical engineering.

He became an Instructor with Menoufia University in 2000, and then an Assistant Lecturer in 2003. From 2010 to 2015, he was a Lecturer with the Department of Electrical Engineering, Faculty of Engineering, Menoufia University, where he has been an Assistant Professor since 2015. His current research interests include ac machines control, power electronics, motor drives, the transient electrical behavior of ac machines, sensorless control techniques, and digital signals processing.
学霸图书馆
www.xuebalib.com

本文献由“学霸图书馆-文献云下载”收集自网络，仅供学习交流使用。

学霸图书馆（www.xuebalib.com）是一个“整合众多图书馆数据库资源，提供一站式文献检索和下载服务”的24小时在线不限IP图书馆。

图书馆致力于便利、促进学习与科研，提供最强文献下载服务。

图书馆导航：
图书馆首页 文献云下载 图书馆入口 外文数据库大全 疑难文献辅助工具