Development of 3D neutron noise simulator based on GFEM with unstructured tetrahedron elements

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A B S T R A C T

In the present study, the neutron noise, i.e. the stationary fluctuation of the neutron flux around its mean value is calculated based on the 2G, 3D neutron diffusion theory. To this end, the static neutron calculation is performed at the first stage. The spatial discretization of the neutron diffusion equation is performed based on linear approximation of Galerkin Finite Element Method (GFEM) using unstructured tetrahedron elements. Using power iteration method, neutron flux and corresponding eigen-value are obtained. The results are then benchmarked against the valid results for VVER-1000 (3D) benchmark problem. In the second stage, the neutron noise equation is solved using GFEM and Green's function method for the absorber of variable strength noise source. Two procedures are used to validate the performed neutron noise calculation. The calculated neutron noise distributions are displayed in the different axial layers in the reactor core and its variation in axial direction is investigated. The main novelty of the present paper is the solution of the neutron noise equations in the three dimensional geometries using unstructured tetrahedron elements.

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1. Introduction

The knowledge of the neutron noise, i.e. the difference between the time-dependent neutron flux and its steady state value with the assumptions that the processes are stationary and ergodic in time, is very useful for core monitoring, surveillance and diagnostics (Rácz and Pázsit, 1998; Hosseini and Vosoughi, 2014). Noise analysis method might be applied to determine dynamical parameters of the core, like the Moderator Temperature Coefficient (MTC) in a Pressurized Water Reactor (PWR) (Demazière and Pázsit, 2004) or the Decay Ratio in a Boiling Water Reactor (BWR). It can be applied to identify and localize the noise source such as unseated fuel assemblies, absorber of variable strength, vibrating absorber or vibrations of core internals (Hosseini and Vosoughi, 2013, 2014; Demazière and Andhull, 2005; Williams, 1974). One of the main advantages of the noise-based methods is that they can be used on-line without disturbing reactor operation (Sunder et al., 1991). Such monitoring techniques are of interest for the extensive program of power upgrades around the world. The other important advantage of noise analysis is that the calculations of neutron noise are performed in the frequency domain. This leads to reducing the dimension of the variable space of the noise equations. Namely, instead of solving the diffusion equations in both the time and space, after the Fourier transform one needs to solve them only in space whereas the frequency variable acts as a parameter in such a case (i.e. no any derivatives with respect to frequency). Since the system of noise equations is not an eigen-value problem, it is solved easier than the static ones. To calculate the neutron noise, firstly the neutron noise distribution induced by a point-like noise source has to be determined. Although many commercial codes for static calculations are available, there is no dedicated commercial code that is able to calculate the dynamical reactor transfer function. Recently, several researchers have tried to develop convenient methods for calculations of the dynamical reactor transfer function and noise analysis. Some selected researches are presented in the following:

1. Demazi'ere proposed a 2D, 2Group neutron noise simulator, which calculates the neutron noise and the corresponding adjoint function in the 2D heterogeneous systems. The simulator can only be applied to the western LWRs (PWR and...
2. Static calculation

In the absence of external neutron source, the multigroup neutron diffusion equation is as Eq. (1) (Duderstadt and Hamilton, 1976; Lamarsh, 1966):

$$-D_k \nabla^2 \phi_{eig}(\mathbf{r}) + \sum_{g=1}^{G} \Sigma_{eg} \phi_{eig}(\mathbf{r})$$

$$= \frac{\rho}{\mu \Sigma_f} \sum_{g=1}^{G} \phi_{eig}(\mathbf{r}) + \sum_{g=g'}^{G} \Sigma_{eg'} \phi_{eig}(\mathbf{r}), \quad g = 1, 2, \ldots, G.$$  (1)

where, all quantities are defined as usual. Removal cross section is expressed as $\Sigma_{eg} = \Sigma_{ag} + \sum_{g' \neq g} \Sigma_{eg'}. $

Eq. (1) is a linear partial differential equation which may be solved by different numerical methods. All of these methods transform the differential equation into a system of algebraic equations. Here, GFEM, a weighted residual method, is used to discretize the neutron diffusion equation (Hosseini and Vosoughi, 2013). To start the discretization, the whole solution volume is divided into the unstructured tetrahedron elements as shown in Fig. 1. These elements have been generated using Gambit mesh generator. In the linear approximation of shape function, the neutron flux in each element might be considered as Eq. (2) (Zhu et al., 2005):

$$\phi^{[e]}(x,y,z) = L_1(x,y,z)\phi_1 + L_2(x,y,z)\phi_2 + L_3(x,y,z)\phi_3 + L_4(x,y,z)\phi_4,$$  (2)

where $L_i, i = 1, 2, 3, 4$ are the components of the shape function in Eq. (3):

$$N^{[e]}(x,y,z) = [L_1(x,y,z) \quad L_2(x,y,z) \quad L_3(x,y,z) \quad L_4(x,y,z)].$$  (3)

The shape function components are defined as Eq. (4):

$$L_i(x,y,z) = \frac{a_i + b_i x + c_i y + d_i z}{6V}, \quad i = 1, 2, 3, 4.$$  (4)

with

$$6V = \det \begin{bmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{bmatrix}.$$  (5)

Fig. 1. The unstructured tetrahedron element.
in which, incidentally, the value $V$ represents the volume of the
tetrahedron. By expanding the other relevant determinants into
t heir cofactors we have:

$$
a_1 = \det \begin{bmatrix} x_2 & y_2 & z_2 \\
x_3 & y_3 & z_3 \\
x_4 & y_4 & z_4 \end{bmatrix} \quad b_1 = -\det \begin{bmatrix} 1 & y_2 & z_2 \\
x_3 & y_3 & z_3 \\
x_4 & y_4 & z_4 \end{bmatrix}$$

$$
c_1 = -\det \begin{bmatrix} x_2 & 1 & z_2 \\
x_3 & 1 & z_3 \\
x_4 & 1 & z_4 \end{bmatrix} \quad d_1 = -\det \begin{bmatrix} x_1 & y_2 & 1 \\
x_3 & y_3 & 1 \\
x_4 & y_4 & 1 \end{bmatrix}$$

(6)

with the other constants defined by cyclic interchange of the sub-
scripts in the order 1, 2, 3, 4.

Here, the linear approximation for shape function in each ele-
ment is considered (Zhu et al., 2005). The GFEM is a weighted
residual method in which the purpose is to minimize the residu-
al integral. The approximate solution of the neutron flux function,
$\phi(x,y,z)$, gives a residual as Eq. (7):

$$\nabla^2 \phi_g(r) + \Delta \phi_g(r) = \frac{X_e}{K_{eff}} \sum_{g=1}^{G} \Lambda_{e,g} \phi_g(r) - \sum_{g',g} \phi_{g',g} \phi_g(r)$$

$$R_g(r) = g = 1, 2, \ldots, G.$$  

(7)

In the weighted residual methods, the purpose is the minimiz-
ing the residual by multiplying with a weight $W(r)$ and integ-
rate over the domain as Eq. (8):

$$\int_V R(r)W(r)dv = 0$$  

(8)

The weighting function may be considered as $W(r) = W^I N(r)$. 
There are (at least) four sub-methods (Collocation, Sub-domain, 
LeastSquares, Galerkin) according to choices for $W^I$. Since
the value of $W^I$ is 1 (one) in the Galerkin method, the weighting
function is considered as Eq. (9):

$$W(r) = N(r),$$

(9)

where $N(r)$ is the global shape function.

Multiplying Eq. (1) by the weighting function and integrating
the results over the solution space, Eq. (10) is obtained:

$$\int_V dV \nabla \phi_g(r) = \frac{X_e}{K_{eff}} \sum_{g=1}^{G} \Lambda_{e,g} \phi_g(r) - \sum_{g',g} \phi_{g',g} \phi_g(r)$$

$$- \sum_{g',g} \phi_{g',g} \phi_g(r) = 0$$

(10)

In the above equation, the differential part may be transformed
by applying the Divergence’s theorem:

$$\int_V dV \nabla W(r) (-D_k \nabla^2 \phi_g(r)) = \int_V dV \nabla W(r) \cdot \nabla \phi_g(r)$$

$$- \int_V dV \nabla \cdot (W(r) \nabla \phi_g(r))$$

$$= \int_V dV \nabla \nabla W(r) \cdot \nabla \phi_g(r) - \int_A \partial \phi_g(r) \frac{\partial dW}{\partial m}$$

(11)

where,

$$\frac{\partial \phi_g(r)}{\partial m} = \nabla \phi_g(r) \cdot \pi.$$

Here, $\pi$ is the normal unit vector on the volume $V$. Two types of boundary condi-
tions (B.C.) are considered. The first B.C. is no incoming neutrons at vacuum boundaries
(Marshak B.C.) which is expressed as Eq. (13):

$$\frac{\partial \phi_g(r)}{\partial m} = -\frac{\phi_g(r)}{2D_k}$$

(13)

The second B.C. is zero net current or perfect reflective bound-
ary condition which is described by Eq. (14):

$$\frac{\partial \phi_g(r)}{\partial m} = 0.$$  

(14)

Substituting Eqs. (9), (11), (13) and (14), and converting the
integration on the reactor domain to sum of the integration on
finite elements, the final form of Eq. (10) is obtained as Eq. (15):

$$\sum_{g=1}^{G} \int_V dV \nabla \phi_g(r) \nabla \phi_g(r)$$

$$+ \int_V dV \nabla \phi_g(r) \nabla \phi_g(r)$$

$$+ \int_V dV \nabla \phi_g(r) \nabla \phi_g(r)$$

(15)

When element matrices have to be evaluated it will follow that
we are faced with integration of quantities defined in terms of
volume coordinates over the tetrahedron region. It is useful to note
in this context the following exact integration expression:

$$\int_V L_1^2 L_2^2 L_3 dV = \frac{abc/3}{(a+b+c+d)^3} 6V$$

(16)

Evaluating the integrals of Eq. (15) for each element, using Eq.
(16), and assembling the local matrix into the global matrix, the
system of equations which is an eigenvalue problem is obtained.
Here, the problem is solved using the power iteration method
(Booth, 2006) and the neutron flux distribution in each energy
group and the multiplication factor are calculated.

3. Neutron noise calculation

3.1. Forward noise calculation

Here, the first-order forward noise approximation of two-group
forward diffusion equation is applied to noise calculation. The
global form of two-group noise equation which the neutron
noise sources is due to the variations of scattering, absorption and fission
macroscopic cross sections, is given in Eq. (17) (Demazière, 2004;
Hosseini and Vosoughi, 2012):

$$\nabla \bar{\phi}(\tau, \omega) + \bar{\phi}_{dyn}(\tau, \omega) \times \frac{\partial \phi_1(\tau, \omega)}{\partial \phi_2(\tau, \omega)}$$

$$= \bar{\phi}_{\omega-1-2}(\tau) \delta_{\omega-1-2}(\tau, \omega) + \bar{\phi}_0(\tau) \delta_{\omega-1-2}(\tau, \omega)$$

$$+ \bar{\phi}_0(\tau) \left[ \frac{\partial v_1 \delta_{\omega-1-2}(\tau, \omega)}{\partial \phi_2(\tau)} \right].$$

(17)

where $\partial \phi(\tau, \omega), \partial \delta_{\omega}(\tau, \omega), \partial \delta_{\omega}(\tau, \omega)$ and $\partial \delta_{\omega}(\tau, \omega)$ denote to
neutron noise and perturbations in scattering, absorption and fission
cross sections, respectively. Also, the above mentioned matrices and
vectors are expressed as Eqs. (18)–(23):

$$\bar{\phi} = \begin{bmatrix} D_1(\tau) & 0 \\ 0 & D_2(\tau) \end{bmatrix}$$

(18)

$$\bar{\phi}_{dyn}(\tau, \omega) = \begin{bmatrix} -\delta_{\omega}(\tau, \omega) \frac{\partial v_1 \delta_{\omega-1-2}(\tau, \omega)}{\partial \phi_2} (1 - \frac{\phi_1(\tau)}{\phi_2(\tau)}) \\ \delta_{\omega-1-2}(\tau) \delta_{\omega}(\tau, \omega) - (\delta_{\omega}(\tau, \omega) + \frac{\partial \phi_2(\tau)}{\partial \phi_2(\tau)}) \end{bmatrix}.$$  

(19)

$$\bar{\phi}_{\omega-1-2}(\tau) = \begin{bmatrix} \phi_1(\tau) \\ -\phi_2(\tau) \end{bmatrix}.$$  

(20)
\[ \frac{\partial \phi_i}{\partial t}(\tau, \omega) = \left[ \phi_i(\tau) \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right], \]

\[ \frac{\partial \phi_i}{\partial t}(\tau, \omega) = \left[ -\phi_i(\tau) \left(1 - \frac{i\omega \beta_{\text{eff}}}{\nu_1} \right) - \phi_{i+1}(\tau) \left(1 - \frac{i\omega \beta_{\text{eff}}}{\nu_1} \right) \right]. \] (22)

The coefficient \( \Sigma_1(\tau, \omega) \) applied in Eq. (19) is defined as Eq. (23):

\[ \Sigma_1(\tau, \omega) = \Sigma_{n1}(\tau) + \frac{i\omega}{\nu_1} \cdot \nu_1 \Sigma_{n1}(\tau) \left(1 - \frac{i\omega \beta_{\text{eff}}}{\nu_1 + \lambda} \right) \] (23)

In Eqs. (19)–(23), \( \nu_1 \) and \( \nu_2 \), \( \beta_{\text{eff}} \) and \( \lambda \) denote neutron velocity in energy group 1 and 2, frequency of occurrence of the noise source, effective delayed neutron fraction and decay constant, respectively. Here, it is assumed that the neutron noise source type is absorber of variable strength. To solve the forward noise equation \( \text{Eq. (17)} \), the Green’s function technique is applied. In the mentioned method, the Green’s function (neutron noise due to the point noise source with strength 1) is calculated from Eq. (24):

\[ \left[ \nabla \cdot \bar{D}(\tau) \nabla + \Sigma_{\text{eff}}(\tau, \omega) \right] \begin{bmatrix} G_{e-1}(\tau, \omega) \\ G_{e-2}(\tau, \omega) \end{bmatrix} = \begin{bmatrix} \delta(\tau - \tau_0) \\ 0 \end{bmatrix} \text{or} \begin{bmatrix} \delta(\tau - \tau_0) \\ \delta(\tau - \tau_0) \end{bmatrix} \] (24)

in which, \( G_{e-1}(\tau, \omega) \) and \( G_{e-2}(\tau, \omega) \) are the Green’s function components in groups 1 and 2 due to neutron noise source in group \( g \), respectively. The neutron noise source may be considered to be in fast or thermal energy group. The general form of neutron noise source is expressed as Eq. (25):

\[ S(\tau, \omega) = \begin{bmatrix} S_1(\tau, \omega) \\ S_2(\tau, \omega) \end{bmatrix} = \Sigma_{n1-2}(\tau) \delta(\tau - \tau_0) + \Sigma_{n1}(\tau) \begin{bmatrix} \delta(\tau - \tau_0) \\ 0 \end{bmatrix} \] (25)

The same applied discretisation method for the static calculation is used to the dynamical calculation. By considering the neutron noise source in group 2, the discrete form of Eq. (20) is obtained as Eqs. (26) and (27):

\[ \sum_{e=1}^{E} \int_{\Omega} \int_{(e)} d\Omega \nabla N^{(e)}(\tau) \nabla N^{(eGT)}(\tau) G_{e-1}^{(eGT)} = -\Sigma_{1}^{(e)} \int_{\Omega} \int_{(e)} d\Omega N^{(e)}(\tau) N^{(eGT)}(\tau) G_{e-1}^{(eGT)} \]

\[ + \frac{\nu_2 \Sigma_{1}^{(e)}(\tau)}{\beta_{\text{eff}}(\tau)} \int_{\Omega} \int_{(e)} \frac{d\Omega N^{(e)}(\tau) N^{(eGT)}(\tau) G_{e-1}^{(eGT)}}{2} = 0, \] (26)

\[ \sum_{e=1}^{E} \int_{\Omega} \int_{(e)} d\Omega \nabla N^{(e)}(\tau) \nabla N^{(eGT)}(\tau) G_{e-2}^{(eGT)} = -\Sigma_{1}^{(e)} \int_{\Omega} \int_{(e)} d\Omega N^{(e)}(\tau) N^{(eGT)}(\tau) G_{e-1}^{(eGT)} \]

\[ + \frac{\nu_2 \Sigma_{1}^{(e)}(\tau)}{\beta_{\text{eff}}(\tau)} \int_{\Omega} \int_{(e)} \frac{d\Omega N^{(e)}(\tau) N^{(eGT)}(\tau) G_{e-1}^{(eGT)}}{2} = 0. \] (27)

where \( d\Omega^{\text{out}} \) refers to boundary length with vacuum boundary condition in element \( e \). The integrals in Eqs. (26) and (27) may be solved using Eq. (16). Finally, the fast and thermal neutron noise distributions may be calculated through Eq. (28):

\[ \begin{bmatrix} \delta \phi_1(\tau, \omega) \\ \delta \phi_2(\tau, \omega) \end{bmatrix} = \begin{bmatrix} \int G_{e-1}(\tau, \omega) S_1(\tau, \omega) + \int G_{e-2}(\tau, \omega) S_2(\tau, \omega) \end{bmatrix} \text{d}\Omega \] (28)

In this study, it is assumed that the occurred perturbation is located in the thermal macroscopic absorption cross section. In such a case, there is only thermal neutron noise source \( \text{(Demazière, 2004; Hosseini and Vosoughi, 2012)} \) and Eq. (28) reduces to Eq. (29):

\[ \begin{bmatrix} \delta \phi_1(\tau, \omega) \\ \delta \phi_2(\tau, \omega) \end{bmatrix} = \begin{bmatrix} \int G_{e-2}(\tau, \omega) S_2(\tau, \omega) \end{bmatrix} \text{d}\Omega \] (29)

3.2. Adjoint noise calculation

The estimation of the neutron noise and the corresponding adjoint function has some practical applications. Hence, it is highly important to calculate the adjoint function of the neutron noise. Considering a point-like neutron source located at the position \( r_0 \), the adjoint noise equations may be derived by a similar process used to obtain the forward noise equations. The matrix form of adjoint noise equations might be expressed as Eq. (30) \( \text{(Hosseini and Vosoughi, 2012)} \):

\[ \nabla \cdot \bar{D}(\tau) \nabla + \Sigma_{\text{eff}}(\tau, \omega) \begin{bmatrix} \delta \phi_1(\tau, \omega) \\ \delta \phi_2(\tau, \omega) \end{bmatrix} = \begin{bmatrix} \delta(\tau - r_0) \\ \delta(\tau - r_0) \end{bmatrix} \] (30)

where \( \delta \phi_1(\tau, \omega) \) and \( \delta \phi_2(\tau, \omega) \) are the adjoint fast and thermal noise distribution, respectively. \( \Sigma_{\text{eff}}(\tau, \omega) \) is the transpose of the dynamical matrix \( \Sigma_{\text{eff}}(\tau, \omega) \). In addition, \( \delta(\tau - r_0) \) refers to Dirac delta function at the position \( r_0 \).

Like the forward noise calculations, Green’s function technique and GFEM are applied to adjoint noise calculations. The fast and thermal adjoint noise distributions are obtained from the adjoint noise calculation.

4. Main specification of the benchmark problem

The VVER-1000 is considered to validate the calculation against the available data for the mentioned benchmark problem \( \text{(Schulz, 1996)} \). The radial fuel assembly lattice pitch is 24.1 cm. This corresponds to the prototype VVER-1000 and is slightly different from the actual FA pitch of 23.6 cm in VVER-1000/V320, however it is acceptable for a mathematical benchmark. The core height is 355 cm, covered with axial and radial reflectors. The total height is 426 cm including 35.5 cm thick axial reflectors.

Fig. 2 shows the 1/12 of the benchmark core configuration. The boundary conditions of the reactor core comprise of the no incoming current boundaries. The material cross section of each assembly for the VVER-1000 is given in Table 1.

5. Numerical results and discussion

5.1. Results of static calculation

The developed program \( \text{(STA-GFEM (3D))} \) is used for static simulation of the VVER-1000 reactor core. Table 2 shows the calculated neutron multiplication factors with their Relative Percent
Error (RPE) (defined as Eq. (31)) vs. number of the unstructured tetrahedron elements.

\[
\text{RPE}(\%) = \frac{\text{calculated value} - \text{reference value}}{\text{reference value}} \times 100.
\]  

Fig. 3 shows the arrangement of unstructured tetrahedron elements in the VVER-1000 reactor core. The power distribution (corresponding to 171964 elements in core modeling) in 1/12th of the reactor core is compared to the reference data (Schulz, 1996) in Table 3. The axial distribution of relative power in the different layers of the reactor core is also given in the mentioned table. As shown, the results of STA-GFEM (3D) and reference values are in a good agreement.

As shown in Table 3, the calculations are repeated for three different numbers of the elements in order to analyze the sensitivity of the calculations to number the elements. As expected, differences between the calculated neutron multiplication factor and reference value decreases as the number of elements is increased. The calculated RPEs for neutron multiplication factor and power

<table>
<thead>
<tr>
<th>Number of elements</th>
<th>Number of unknowns</th>
<th>(k_{\text{eff}})</th>
<th>RPE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>44,006</td>
<td>7979</td>
<td>1.04825</td>
<td>-0.1220</td>
</tr>
<tr>
<td>75,040</td>
<td>13,340</td>
<td>1.04893</td>
<td>-0.0572</td>
</tr>
<tr>
<td>171,964</td>
<td>30,133</td>
<td>1.04906</td>
<td>-0.0448</td>
</tr>
</tbody>
</table>

The reference effective multiplication factor is \(k_{\text{eff}} = 1.04953\) (Schulz, 1996).
Table 3
The distribution of the relative power in the 1/12th of the VVER-1000 reactor core.

| F.A. | Layer | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | Qmean | Qmean (Ref.) | RPE (%) |
|------|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---------|-----------|--------|
| 1    |       | 0.891 | 1.914 | 2.369 | 1.838 | 1.465 | 1.052 | 0.703 | 0.469 | 0.361 | 0.158 | 1.1219 | 1.1199 | 0.1786 |
| 2    |       | 0.054 | 1.411 | 1.798 | 1.712 | 1.394 | 0.985 | 0.652 | 0.426 | 0.268 | 0.113 | 0.9413 | 0.9475 | -0.6544 |
| 3    |       | 0.668 | 1.451 | 1.900 | 1.925 | 1.569 | 1.040 | 0.675 | 0.437 | 0.258 | 0.104 | 1.0026 | 1.0138 | -1.1048 |
| 4    |       | 0.649 | 1.409 | 1.835 | 1.843 | 1.509 | 1.030 | 0.670 | 0.433 | 0.258 | 0.105 | 0.9741 | 0.9821 | -0.8146 |
| 5    |       | 0.880 | 1.918 | 2.542 | 2.617 | 2.100 | 1.007 | 0.781 | 0.506 | 0.293 | 0.117 | 1.2761 | 1.3112 | -2.6769 |
| 6    |       | 0.701 | 1.528 | 2.019 | 2.070 | 1.677 | 1.049 | 0.663 | 0.426 | 0.248 | 0.099 | 1.0479 | 1.0571 | -0.8703 |
| 7    |       | 0.691 | 1.512 | 2.015 | 2.097 | 1.724 | 1.074 | 0.684 | 0.438 | 0.252 | 0.100 | 1.0587 | 1.0603 | -0.1509 |
| 8    |       | 0.703 | 1.537 | 2.045 | 2.119 | 1.707 | 0.859 | 0.529 | 0.338 | 0.195 | 0.078 | 1.0109 | 1.0091 | -0.1784 |
| 9    |       | 0.873 | 1.904 | 2.531 | 2.620 | 2.139 | 1.318 | 0.819 | 0.519 | 0.299 | 0.119 | 1.3142 | 1.3135 | 0.0533 |
| 10   |       | 0.762 | 1.668 | 2.232 | 2.344 | 2.014 | 1.447 | 0.964 | 0.616 | 0.354 | 0.140 | 1.2542 | 1.2495 | 0.3762 |
| 11   |       | 0.665 | 1.437 | 1.921 | 2.011 | 1.701 | 1.160 | 0.758 | 0.484 | 0.278 | 0.110 | 1.0516 | 1.0481 | 0.3339 |
| 12   |       | 0.659 | 1.442 | 1.926 | 2.013 | 1.693 | 1.140 | 0.736 | 0.468 | 0.269 | 0.107 | 1.0453 | 1.0409 | 0.4227 |
| 13   |       | 0.416 | 0.911 | 1.221 | 1.288 | 1.125 | 0.839 | 0.570 | 0.365 | 0.209 | 0.083 | 0.7027 | 0.6981 | 0.6589 |
| 14   |       | 0.627 | 1.370 | 1.836 | 1.935 | 1.684 | 1.245 | 0.842 | 0.539 | 0.309 | 0.123 | 1.0509 | 1.0452 | 0.5454 |
| 15   |       | 0.692 | 1.514 | 2.029 | 2.137 | 1.856 | 1.365 | 0.920 | 0.589 | 0.338 | 0.134 | 1.1572 | 1.1508 | 0.5361 |
| 16   |       | 0.567 | 1.243 | 1.664 | 1.753 | 1.521 | 1.118 | 0.752 | 0.481 | 0.276 | 0.109 | 0.9484 | 0.9434 | 0.5300 |
| 17   |       | 0.364 | 0.796 | 1.068 | 1.129 | 0.993 | 0.749 | 0.513 | 0.330 | 0.189 | 0.075 | 0.6207 | 0.6161 | 0.0075 |

Fig. 4. The fast neutron flux distribution in the layer with \( Z = 106.5 \) cm of the VVER-1000 reactor core.

Fig. 5. The thermal neutron flux distribution in the layer with \( Z = 106.5 \) cm of the VVER-1000 reactor core.
distribution in the present study are in the range of reported results in the similar work (Schulz, 1996).

5.2. Validation of the noise calculation

The comparison between static calculations and noise calculations in zero frequency is the first benchmarking process of noise calculations. As expected, at zero frequency and without the neutron noise source the fast and thermal forward noises approach to static corresponding fluxes (Hosseini and Vosoughi, 2012). Figs. 4 and 5 show the obtained fast and thermal forward noises in the middle layer of the VVER-1000 reactor core at zero frequency without the neutron noise source. The static fast and thermal neutron fluxes have the same distribution displayed in Figs. 4 and 5.

Considering the features of the inner product of two vectors, the calculated adjoint noise can be benchmarked against the formerly calculated forward noise. As mentioned in the other similar works (Demazière and Pázsit, 2004; Hosseini and Vosoughi, 2012), if the neutron noise source is assumed to be a point source located in the position $r_s$ and adjoint noise source as a Dirac function located in the position $r_0$, Eq. (31) could be applied to benchmark the forward noise calculation:

$$\gamma^2 \delta \phi_2 \left( r_s, r_0, \omega \right) = \delta \phi_1 \left( r_0, r_s, \omega \right) + \delta \phi_2 \left( r_0, r_s, \omega \right)$$

in which, $\gamma$ stand for $\phi_{20} (r_s)$, respectively.

Therefore, the calculated adjoint noise can be benchmarked against the forward noise by comparing the calculated amplitude and phase of these two approaches in all the elements inside the reactor core. Figs. 6 and 7 show the magnitude (absolute value) and phase of the right hand side of Eq. (31) in the layer with $Z = 106.5$ cm, respectively. Also, the absolute and phase of the left hand side of Eq. (31) are displayed in the Figs. 8 and 9, respectively.

As shown in the Figs. 6–9, the magnitude and phase of the left and right hand sides have a good agreement with each other. The neutron noise and corresponding adjoint distributions have the same agreement in the other layers. To avoid the duplication, the results obtained from Eq. (31) for the other layers have not been presented.

5.3. Results of noise calculation

To investigate the variation of calculated neutron noise by DYN-GFEM (3D) in axial direction, the noise calculation is performed for source type of absorber of variable strength located...
Fig. 8. The magnitude of the left hand side of Eq. (31) in the layer with $Z = 106.5$ cm of the VVER-1000 reactor core.

Fig. 9. The phase of the left hand side of Eq. (31) in the layer with $Z = 106.5$ cm of the VVER-1000 reactor core.

Fig. 10. Magnitude of neutron noise in layer with $Z = 35.5$ cm.

Fig. 11. Magnitude of thermal neutron noise in layer with $Z = 71$ cm.
at $X = 7.5312$ cm, $Y = 2.6089$ cm and $Z = 9.516$ cm. The variation in the macroscopic cross section should be such that it still is assumed as perturbation. In the first approximation, the $\delta \sum$ is considered as perturbation when $\frac{\delta \sum}{\sum}$ be negligible (in the present

![Fig. 12. Magnitude of thermal neutron noise in the layer with $Z = 106.5$ cm.](image1.png)

![Fig. 13. Magnitude of thermal neutron noise in the layer with $Z = 142$ cm.](image2.png)

![Fig. 14. Magnitude of thermal neutron noise in the layer with $Z = 177.5$ cm.](image3.png)

![Fig. 15. Magnitude of thermal neutron noise in the layer with $Z = 213$ cm.](image4.png)

![Fig. 16. Magnitude of thermal neutron noise in the layer with $Z = 248.5$ cm.](image5.png)

![Fig. 17. Magnitude of thermal neutron noise in the layer with $Z = 284$ cm.](image6.png)
study, $\sum_{\lambda} \approx 10^{-4}$). The small variation in the neutron flux due to perturbation is the neutron noise.

Figs. 10–18 display the magnitude of neutron noise distribution in different layers of the VVER-1000 reactor core. As shown in these figures, the magnitudes of neutron noise in the layers $Z = 35.5$ cm and $Z = 71$ cm have a similar behavior of neutron noise in 2D geometry (Demazière and Andhill, 2005; Hosseini and Vosoughi, 2012). By increasing the distance between the selected layer and noise source, the calculated neutron noise takes the different distribution and its values decreases remarkably.

6. Conclusion

In the present study, 3D, 2G static and dynamic simulators based on GFEM was developed using tetrahedron unstructured elements. The advantage of unstructured tetrahedron to structured ones is the possibility of implementing smaller elements in the boundary layers, regions with high flux gradient or fuel assembly containing neutron noise source. This leads to obtaining the desired accuracy with low cost of calculations. Due to finite element method capability, the STA-GFEM (3D) and DYN-GFEM (3D) is applicable for both rectangular and hexagonal reactor cores. In the present study, the results of calculation for the VVER-1000 reactor core were only reported. To solve the neutron noise equation, the Green’s function technique was applied. The neutron noise induced by the absorber of variable strength noise source was calculated using the developed simulator. The results of static simulator were benchmarked against the valid reported data for the VVER-1000 reactor core. The variation of neutron noise in axial direction was investigated. It was concluded that the induced neutron noise have a different distribution for the layers far from the noise source.

Overall, a reader can conclude that the developed computer code is a more reliable tool for static and dynamical calculations in 3D geometry. STA-GFEM (3D) is applicable to neutronic calculation of the reactor core in the design studies of nuclear reactors. DYN-GFEM (3D) may be used as a reliable tool for neutron noise analysis in 3D geometries.

References


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